# INFLUENCE OF NON-LINEAR CONVECTION AND THERMOPHORESIS ON HEAT AND MASS TRANSFER FROM A ROTATING CONE TO FLUID FLOW IN POROUS MEDIUM

by

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In this paper, we study the effects of thermophoresis and non-linear convection on mixed convective flow of viscous incompressible rotating fluid due to rapidly rotating cone in a porous medium, whose surface temperature and concentration are higher than the temperature and concentration of its surrounding fluid. The governing equations for the conservation of mass, momentum, energy, and concentration are transformed, using similarity transformations and the solutions are obtained by employing shooting method that uses Runge-Kutta method and Newton-Raphson method. A comparison of the present results with previously published work for special cases shows a good agreement. The effects of temperature and concentration, ratio of angular velocities, relative temperature difference parameter, thermophoretic coefficients on velocity, temperature, and concentration profiles as well as tangential and circumferential skin friction coefficients, Nusselt number, and Sherwood number results are discussed in detail. The results indicate that the temperature is more influential compared to concentration. Also, the wall thermophoretic deposition velocity changes according to different values of pertinent parameter. Applications of the study arise in aerosol technology, space technology, astrophysics, and geophysics, which related to temperature-concentration-dependent density.

Keywords: convection, thermophoresis, rotating flow, rotating cone

# Introduction

Transport process of the mixed convective heat and mass transfer through fluid saturated porous medium play an important role in enormous practical applications in modern industry and engineering fields, such as geothermal energy technology, design of building component, solar power collectors, food industries, oil recovery modeling, thermal insulating systems, nuclear reactors, electronic equipment, and compact heat exchangers. These applications can be found to date in the recent books Nield and Bejan *et al.* [1] and Ingham and Pop [2]. Recently, Rashidi *et al.* [3-5] studied the combined free and forced convection flow past a different geometries embedded in a porous medium.

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Mixed convective heat and mass transfer over a rotating vertical cone embedded in a fluid saturated porous medium is an important phenomenon for spin stabilized missiles, geothermal reservoirs, rotating heat exchanger, and containers of nuclear waste disposal. Earlier, Hering and Grosh [6] investigated combined convection about a rotating vertical cone. They reported the results of buoyancy parameter which gives rise to three convective regions, as purely forced, purely free and combined convection flows. Himasekhar et al. [7] studied analytically heat transfer and buoyancy induced flow around a rotating vertical cone. Saravanan [8] theoretically investigated the effect of magnetic field on the onset of convection in a fluid saturated anisotropic rotating porous medium. Mohiddin et al. [9] studied numerically the unsteady free convective heat and mass transfer in a Walters-B viscoelastic flow over a vertical cone. Rashidi et al. [10] and Rashidi and Erfani [11] studied analytically the convective heat transfer of magnetized micropolar fluid and viscous dissipative fluid of slip flow induced by rotating disc. Chamkha and Ahmed [12] investigated unsteady MHD double diffusive convective flow in a rotating sphere at different wall conditions. Chamkha and Rashad [13] studied chemical reaction and cross-diffusion effects on heat and mass transfer flow over a rotating vertical cone. Hariprasad Raju et al. [14] studied chemical reaction and thermophoretic effect on convective flow of a viscous fluid over a rotating cone.

Combined heat and mass transfer flow of rotating flow in a porous media has been growing interest the last several decades due to its practical applications in engineering and industrial applications. Salah *et al.* [15] reported the exact solutions of unsteady MHD convective rotating flow of second grade fluid in a porous medium. Bhadauria *et al.* [16] investigated non-linear thermal instability in a rotating flow. They studied the stability of the system and presented stream lines for different slow times as a function of modulation amplitude. Rashad [17] and Bhuvanavijaya and Mallikarjuna [18] studied the effects of variable properties on unsteady and steady mixed convective flow of a rotating flow on a stretching sheet and vertical plate in a porous medium.

Boussinesq approximation is applicable in cases with a moderate influence of temperature and concentration gradients on the fluid density. Therefore, the density is considered as constant everywhere except in the buoyancy force term. Since the temperature and concentration difference between ambient fluid and cone surface was appreciably large, the mathematical model developed by using a linear density temperature and concentration relation becomes more inaccurate. The heat produced by the viscous dissipation and thermal stratifica-

tion are also another reasons for the density temperature concentration relationship to become non-linear. The applications related to temperature-concentration-dependent density relation is of immense important in industrial and geothermal engineering, for instance, design of thermal system, cooling transpiration, cooling of electric components, turbine blades, drying of the surfaces, areas of reactor safety, solar collectors, combustion, metallic foams and sponges. In view of the previous applications, the authors envisage to investigate the effect of non-linear convection and thermophoretic on heat and mass transfer flow of rotating fluid over a rotating vertical cone in a fluid saturated Darcy porous medium [19, 20].



Figure 1. Physical configuration and co-ordinate system

#### **Problem formulation**

We consider steady, 2-D, incompressible rotating Newtonian fluid over a vertical rotating cone in a fluid saturated porous medium. The physical configuration and co-ordination of the system is given in fig. 1. We consider the rectangular curvilinear co-ordinate system. We assume the velocity components u, v, and w along tangential (x-axis), azimuthal or circumferential (y-axis), and normal (z-axis) directions, respectively. The cone surface is maintained with variable temperature and concentration, which are greater than free stream fluid temperature and concentration. All the fluid and porous medium properties are assumed to be constant. We assume that the fluid and the porous medium are to be locally thermodynamic equilibrium with solid matrix. Using previous assumptions, Boussinesq, and boundary-layer approximations the governing equations for conservation of mass, momentum, energy, and species are:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} + \frac{u}{x} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} - \frac{v^2}{x} = -\frac{v_e^2}{x} + v\frac{\partial^2 u}{\partial z^2} - \frac{v}{K}u + g\cos(\alpha) \left[\beta_0 \left(T - T_\infty\right) + \beta_1 \left(T - T_\infty\right)^2 + \beta_2 \left(C - C_\infty\right) + \beta_3 \left(C - C_\infty\right)^2\right]$$
(2)

$$u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} + \frac{uv}{x} = v\frac{\partial^2 v}{\partial z^2} - \frac{v}{K}v$$
(3)

$$\left(u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}\right) = \frac{k_e}{\rho c_p} \frac{\partial^2 T}{\partial z^2}$$
(4)

$$u\frac{\partial C}{\partial x} + w\frac{\partial C}{\partial z} + \frac{\partial}{\partial z}\left(Cv_t\right) = D\frac{\partial^2 C}{\partial z^2}$$
(5)

The corresponding boundary conditions are:

$$u = 0, \quad v = r \ \Omega_1, \quad w = 0, \quad T = T_w(x), \quad C = C_w(x) \quad \text{at} \quad z = 0$$
  
$$u = 0, \quad v = 0, \quad T = T_{\infty}, \quad C = C_{\infty} \qquad \text{as} \quad z \to \infty$$
(6)

where  $r = x \sin \alpha$  is the radius of the cone,  $\Omega_1$ ,  $\Omega_2$  are the angular velocities of the cone and free stream, respectively,  $\beta_o$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$  are thermal and solutal diffusivities,  $\rho$  – the fluid density,  $\mu$  – the dynamic viscosity,  $c_p$  – the specific heat at constant pressure, g – the acceleration due to gravity. Also  $\alpha$  represents the cone apex half angle, K – the permeability of the porous medium,  $k_e$  – the effective thermal conductivity, and D – the molecular diffusivity.

The thermophoretic velocity  $v_t$  which appear in eq. (5) recommended by Talbot *et al.* [21]:

$$v_t = -k\nu \frac{\partial T}{\partial z} \tag{7}$$

where k is the thermophoretic coefficient, whose values range from 0.2 and 1.2 and kv is the thermophoretic diffusivity.

# Non-dimentionalisation

Now we introduce the following non-dimensional variables to get the non-dimensional governing equations:

$$\eta = \left[\frac{\Omega \sin(\alpha)}{\nu}\right]^{1/2} z, \ u = x\Omega \sin(\alpha)F(\eta), \ v = x\Omega \sin(\alpha)G(\eta), \ w = \left[\nu\Omega \sin(\alpha)\right]^{1/2}H(\eta)$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \ \phi(\eta) = \frac{C - C_{\infty}}{C_{w} - C_{\infty}}, \ T_{w}(x) - T_{\infty} = \frac{\left(T_{L} - T_{\infty}\right)x}{L}, \ C_{w}(x) - C_{\infty} = \frac{\left(C_{L} - C_{\infty}\right)x}{L}$$
(8)

where *L* being the cone slant height,  $T_L$  being the cone surface temperature, and  $C_L$  being the cone surface concentration at the base (x = L). Using eqs. (7) and (8), the eqs. (1)-(6) reduces to:

$$F = -\frac{1}{2}H' \tag{9}$$

$$H''' - HH'' - \mathrm{Da}^{-1}H + \frac{1}{2}H'^{2} - 2\left[G^{2} - (1 - \lambda)^{2}\right] - 2g_{s}\left[\left(\theta + \alpha_{1}\theta^{2}\right) + N\left(\phi + \alpha_{2}\phi^{2}\right)\right] = 0$$
(10)

$$G'' - HG' + H'G - \mathrm{Da}^{-1}G = 0$$
(11)

$$\theta'' - \Pr\left(H\theta' - \frac{1}{2}H'\theta\right) = 0 \tag{12}$$

$$\phi'' + \operatorname{Sc}\left(\frac{1}{2}H'\phi - H\phi'\right) + \frac{N_t \operatorname{Sc}K}{\theta N_t + 1} \left(\phi'\theta' + \phi\theta'' - \frac{N_t \phi\theta'^2}{\theta N_t + 1}\right) = 0$$
(13)

Boundary conditions

$$H = 0, \quad H' = 0, \quad G = \lambda, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0$$
  
$$H' = 0, \quad G = 1 - \lambda, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \to \infty$$
 (14)

where

$$\mathrm{Da}^{-1} = \frac{\nu}{K\Omega\sin\alpha}$$

is the inverse of Darcy number,

$$Gr = \frac{g\beta_T (T_\omega - T_\infty)L^3 \cos\alpha}{v^2}$$

is the Grashof number,

$$N = \frac{\beta_{c} \left( C_{w} - C_{\infty} \right)}{\beta_{t} \left( T_{w} - T_{\infty} \right)}$$

is the buoyancy ratio,

$$\alpha_1 = \frac{\beta_1}{\beta_0} (T_w - T_\infty), \quad \alpha_2 = \frac{\beta_3}{\beta_2} (C_w - C_\infty)$$
$$\operatorname{Re} = \frac{\Omega L^2 \sin \alpha}{\nu}$$

is the local Reynolds number,

$$g_s = \frac{\mathrm{Gr}}{\mathrm{Re}^2}$$

is the mixed convection parameter,

$$\Pr = \frac{\mu c_p}{k_e}$$

is the Prandtl number,

$$Sc = \frac{v}{D}$$

is the Schmidt number,

$$N_t = \frac{T_w - T_\infty}{T_\infty}$$

is the temperature difference parameter,  $\Omega_1$  and  $\Omega_2$  are the angular velocities of the cone and free stream fluid, respectively,  $\Omega_1 + \Omega_2 = \Omega$  and  $\lambda = \Omega_1/\Omega$ .

# Skin-friction, Nusselt number, and Sherwood number

The physical parameters of interest, local skin friction coefficients in x- and y-directions, local Nusselt number, local Sherwood number, and wall thermophoretic deposition velocity in non-dimensional form are given by:

$$C_{fx} \operatorname{Re}^{1/2} = -H''(0), \quad 2^{-1} \operatorname{Re}^{1/2} C_{fy} = -G'(0), \quad \operatorname{Re}^{-1/2} \operatorname{Nu}_{x} = -\theta'(0)$$

$$\operatorname{Re}^{-1/2} \operatorname{Sh}_{x} = -\phi'(0), \quad V_{t} = \left(\frac{-k N_{t}}{N_{t}+1}\right)\theta'(0)$$
(15)

#### Numerical method of solution

The set of eqs. (9)-(13) with boundary conditions (14) are solved by using shooting method that uses Runge-Kutta method and Newton-Raphson method, Srinivasacharya *et al.* [22]. At first eqs. (9)-(13) are converted into a system of differential equations of first order, by assuming:

$$H = X_1, \ G = X_4, \ \theta = X_6, \ \phi = X_8, \ \text{we get}$$
  
$$H' = X_2, \ H'' = X_3, \ G' = X_5, \ \theta' = X_7, \ \phi' = X_9$$
(16)

$$H''' = X_1 X_3 - \frac{1}{2} X_2^2 + \text{Da}^{-1} X_2 + 2 \left[ X_4^2 - (1 - \lambda)^2 \right] + 2g_s \left[ X_6 + \alpha_1 X_6^2 + N \left( X_8 + \alpha_2 X_8^2 \right) \right]$$
(17)

$$G'' = X_1 X_5 - X_2 X_4 + \mathrm{Da}^{-1} X_4 \tag{18}$$

$$\theta'' = \Pr\left(X_1 X_7 - \frac{1}{2} X_2 X_6\right)$$
(19)

$$\phi'' = \operatorname{Sc}\left(X_1 X_9 - \frac{1}{2} X_2 X_8\right) - \frac{N_t \operatorname{Sc}k}{X_6 N_t + 1} \left[X_7 X_9 + X_8 \operatorname{Pr}\left(X_1 X_7 - \frac{1}{2} X_2 X_6\right) - \frac{N_t X_8 X_7^2}{X_6 N_t + 1}\right]$$
(20)

with Boundary conditions:

$$X_{1}(0) = 0, \ X_{2}(0) = 0, \ X_{4}(0) = \lambda, \ X_{6}(0) = 1, \ X_{8}(0) = 1$$
  
$$X_{2}(\infty) = 0, \ X_{4}(\infty) = 1 - \lambda, \ X_{6}(\infty) = 0, \ X_{8}(\infty) = 0$$
(21)

We assume the initial conditions for  $X_3(0)$ ,  $X_5(0)$ ,  $X_7(0)$  which are not given at z = 0(initial conditions) and then the eqs. (16)-(21) are integrated using fourth order Runge-Kutta method from  $\eta = 0$  to  $\eta_{\text{max}}$  over successive step lengths  $\Delta \eta$ , where  $\eta_{\text{max}}$  is  $\eta$  at  $\infty$  and chosen large enough so that the solution shows little further change for  $\eta$  larger than  $\eta_{\text{max}}$ . We employed ODE45 solver in MATLAB® to solve these nine first ordered coupled non-linear ODE. The accuracy of the assumed values for  $X_3(0)$ ,  $X_5(0)$ ,  $X_7(0)$  is then checked by comparing the calculated values of  $X_2$ ,  $X_4$ ,  $X_6$ ,  $X_8$  at  $\eta = \eta_{\text{max}}$  with their given value at  $\eta = \eta_{\text{max}}$ . If a difference exists, another set of initial values for  $X_3(0)$ ,  $X_5(0)$ ,  $X_7(0)$  are assumed and the process is repeated. In principle, a trial and error-method can be used to determine these initial values, but it is tedious.

Alternatively, we used Newton-Raphson method to accurately find the initial values of  $X_3(0)$ ,  $X_5(0)$ ,  $X_7(0)$  and then integrate eqs. (16)-(20) by using fourth order Runge-Kutta method. This process is continued until the agreement between the calculated and the given condition at  $\eta = \eta_{\text{max}}$  is within the specified degree of accuracy  $10^{-5}$ .

# **Code validation**

In the absence of mass transfer with thermophoresis, and inverse Darcy parameter and without rotating fluid for linear convection, the non-linear ODE (16)-(20) with corresponding boundary conditions (21) exactly coincides with those of Hering and Grosh [6] and Himasekhar *et al.* [7]. The comparison results found very good agreement with earlier existing results as shown in tabs. 1 and 2.

Table 1. The values of -H''(0), -G'(0), and  $-\theta'(0)$  for different values of  $g_s = \text{Gr/Re}^2$ , for Pr = 0.7,  $\alpha_1 = 0$ ,  $\alpha_2 = 0$ ,  $\text{Da}^{-1} = 0$ ,  $\lambda = 1$ , and N = 0 (in the absence of concentration equation without rotating flow)

Gr/Re <sup>2</sup>	<i>–H</i> "(0)		-G'(0)		$-\theta'(0)$	
	Hering and Grosh [6]	Present work	Hering and Grosh [6]	Present work	Hering and Grosh [6]	Present work
0	1.0205	1.0203	0.61592	0.61583	0.42852	0.42842
1.0	2.2078	2.2075	0.85076	0.85080	0.61202	0.61213
100	46.052	46.0523	2.4738	2.47382	1.7946	1.79459

Pr	<i>–H</i> ″(0)		-G'(0)		- heta'(0)	
	Himasekhar et.al [7]	Present work	Himasekhar et.al [7]	Present work	Himasekhar et.al [7]	Present work
1.0	1.1282	1.12824	0.6437	0.64374	0.5457	0.54573
2.0	1.1120	1.11203	0.6335	0.63347	0.7450	0.74502
10	1.0702	1.07018	0.6202	0.62021	1.4106	1.41066

Table 2. The values of -H''(0), -G'(0), and  $-\theta'(0)$  for different values of Prandtl number for  $g_s = \text{Gr/Re}^2 = 0.1$ ,  $\text{Da}^{-1} = 0$ ,  $\alpha_1 = 0$ ,  $\alpha_2 = 0$ ,  $\lambda = 1$ , and N = 0(in the absence of concentration equation without rotating flow)

# **Results and discussion**

In order to discuss the results, the computational calculations are presented in the form of non-dimensional velocity, temperature and concentration profiles, as well as skinfriction, local rate of heat and mass transfer and wall thermophoretic velocity. In this problem we considered the fluid as air (Pr = 0.72) and the species is hydrogen (Sc = 0.22) and mixed convective parameter  $g_s = 1$  represents the equal influence of free and forced convection, is responsible for the flow. The effects of the angular velocity ratio, non-linear temperature, concentration and thermophoretic coefficients are shown in figs. 2-20 for various values of fluid and physical properties.

Figures 2-5 show the effects of the non-linear temperature,  $\alpha_1$ , and concentration,  $\alpha_2$ , on tangential and circumferential velocity, temperature and concentration profiles, respectively. Increasing non-linear temperature parameter  $\alpha_1$  ( $\alpha_2$  is fixed) serves to enhance the tangential flow velocity, fig. 2, *i. e.* accelerates the flow. This effect is accentuated close to the cone where a peak in velocity arises.



The influence of non-linear concentration parameter  $\alpha_2$  ( $\alpha_1$  is fixed) on tangential velocity reported similar behavior to that of  $\alpha_1$ . Thus, hydrodynamic boundary-layer thickness is increased near to the surface for different values of  $\alpha_1$  and  $\alpha_2$ . From the fig. 3, we notice that



increasing  $\alpha_1$  leads to depreciate the circumferential velocity profile near to the cone surface until getting to certain point and then enhanced when  $\alpha_2$  is fixed. The similar results are reported for different values of  $\alpha_2$  when  $\alpha_1$  is fixed. The effects of non-linear temperature and concentration parameter on the boundary-layer flow are represented in figs. 4 and 5. It is noticed from these figures that temperature and concentration profiles are decreased with increasing values of  $\alpha_1$  and  $\alpha_2$ . It is important to note that the influence of non-linear concentration parameter  $\alpha_2$  on velocity, temperature and concentration profiles seems to be more prominent compared with that of non-linear temperature parameter  $\alpha_1$ .

Figures 6-9 are plotted to see the variation of the ratio of angular velocities,  $\lambda$ . The value  $\lambda = 0$  represents the fluid is rotating and cone is at rest while the fluid and cone are rotating with same angular velocities in the same direction for  $\lambda = 0.5$ . From fig. 6 we observed that for  $\lambda > 0.5$ , the tangential velocity decreases and the opposite behavior exist for  $\lambda < 0.5$ . It should be noted that for certain values of physical parameters, these results are opposite to the finding by Nadeem and Saleem [23] for unsteady flow of non-Newtonian nanofluid. We observed from fig. 7 that, circumferential velocity increases for larger values of  $\lambda$  for both cases of  $\lambda < 0.5$  and  $\lambda > 0.5$  near to the cone surface, ( $0 < \eta < 1.2$ ) when  $\eta$  range is  $1.2 < \eta < 5$ , the



**Figure 6. Tangential velocity profile** (*F*) **for different values of**  $\lambda$  (for color image see journal web site)



Figure 7. Circumferential velocity profile (*G*) graph for different values of  $\lambda$  (for color image see journal web site)

circumferential velocity profile reported opposite results. It is noticed from figs. 8 and 9 that, temperature and concentration profiles are decreased as  $\lambda$  increases for the case of  $\lambda < 0.5$ . however, these results are reported opposite for the case  $\lambda > 0.5$ .



(for color image see journal web site)

different values of  $\lambda$ (for color image see journal web site)



Figures 10 and 11 show the effect of non-linear convection parameters  $\alpha_1$  and  $\alpha_2$ on tangential and circumferential skin friction coefficients. From these figures we notice that

tangential and circumferential skin friction coefficients are increased for larger values of  $\alpha_1$ and  $\alpha_2$ . The influence of non-linear parameters  $\alpha_1$  and  $\alpha_2$  on local rate of heat and mass transfer are given in figs. 12 and 13, respectively. It is seen from these figures that, increasing  $\alpha_1$ and  $\alpha_2$ , local rate of heat transfer (Nusselt number) and mass transfer (Sherwood number) results are increased which similar to that of skin friction coefficients as shown in figs. 10 and 11. It is to be noted that influence of  $\alpha_1$  produces more dominant results compared to the results of  $\alpha_2$ .

In order to discuss the influence of thermophoresis on particle deposition onto a rotating vertical cone surface, the tangential and circumferential skin friction coefficients and



Figure 12. Nusselt number  $-\theta''(0)$  for different values of  $\alpha_1$  and  $\alpha_2$ 

Figure 13. Sherwood number  $-\phi''(0)$  for different values of  $a_1$  and  $a_2$ 

Nusselt number and Sherwood number are shown in figs. 14-17, respectively, for representative relative temperature difference parameter,  $N_t$ , and thermophoretic coefficient, k. It is observed from these figures that tangential and skin friction coefficients and Nusselt number results are increased for increasing values of  $N_t$  and k for both cases of  $\lambda > 0.5$  and  $\lambda < 0.5$ , but the opposite results are reported on Sherwood number, *i. e.*, the Sherwood number results are decreased with increasing values of  $N_t$  and k as shown in fig. 17. It is worth to mention that these results are more pronounced for  $\lambda > 0.5$  compared to  $\lambda < 0.5$ .



Figure 14. Tangential skin friction coefficients  $(C_{fx})$  for different values of  $N_t$ 

Figure 15. Circumfertial skin friction coefficient  $(C_{fy})$  for different values of  $N_t$ 

Figures 18 and 19 display non-dimensional wall thermophoretic velocity,  $V_{tw}$ , for different values of non-linear parameters  $\alpha_1$ ,  $\alpha_2$  and ratio of angular velocities,  $\lambda$ . From these figures we say that the wall thermophoretic velocity decreases for increasing values of  $\alpha_1$  and  $\alpha_2$ . It is also observed from these figures that increasing  $\lambda$  leads to decrease the wall thermophoretic velocity deposition. Variation of wall thermophoretic velocity,  $V_{tw}$ , for various values of  $N_t$  and k is shown in fig. 20. This figure shows the usual decreasing effect of  $N_t$  and thermophoretic coefficient on wall thermophoretic velocity,  $V_{tw}$ .



Figure 16. Nusselt number  $-\theta''(0)$  for different values of  $N_t$ 



Figure 18. Wall thermophoretic velocity  $(V_{tw})$  for different values of  $\alpha_1$ 

# Conclusion

Effects of non-linear convection and thermophoretic on heat and mass transfer flow of a Newtonian incompressible rotating fluid over a rotating vertical cone have been analyzed. In order to determine velocity, temperature, and concentration profiles as well as skin friction coefficients, Nusselt number and Sherwood number, we used similarity transformations and numerical method (shooting method). The effects of non-linear convection parameter, ratio of angular velocities, relative temperature difference parameter, and thermophoretic coefficient are examined. These



Figure 17. Sherwood number  $-\phi''(0)$  for different values of  $N_t$ 



Figure 19. Wall thermophoretic velocity  $(V_{tw})$  for different values of  $\alpha_2$ 



Figure 20. Wall thermophoretic velocity  $(V_{tw})$  for different values of  $N_t$ 

results are reported in the form of graphs. The major conclusions of the present investigation are given as follows: Increasing non-linear temperature and concentration parameters leads to enhance the tangential velocity profiles but circumferential velocity, temperature, and concentration profiles are decreased. The tangential and circumferential skin friction coefficients, Nusselt number, and Sherwood numbers results are increased for larger values of non-linear temperature and concentration parameter. Tangential velocity, temperature, and concentration profiles are decreased with increasing values of ratio of angular velocity  $\lambda$  for  $\lambda < 0.5$ . For  $\lambda > 0.5$  the opposite results are reported. But circumferential velocity profiles produces opposite to that of tangential velocity profile for larger values of  $\lambda$ . The tangential and circumferential skin friction coefficients, Nusselt number and results are decreased for larger values of relative temperature difference parameter and thermophoretic coefficient. But opposite results are produced on Sherwood number. The wall thermophoretic velocity decreases with increasing of non-linear temperature difference parameter, thermophoretic coefficient, and ratio of angular velocity.

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