MODELING THE EFFECT OF THE INCLINATION ANGLE ON NATURAL CONVECTION FROM A FLAT PLATE
The Case of a Photovoltaic Module

by

Bojan D. PEROVIĆ [a], ***Jelena Lj. KLIMENTA*** [b], ***Dragan S. TASIĆ*** [c], ***Joan L. G. PEUTEMAN*** [d], ***Dardan O. KLIMENTA*** [a], **and *Ljiljana N. ANDJELKOVIĆ*** [a]

[a] Faculty of Technical Sciences, University of Pristina in Kosovska Mitrovica, Kosovska Mitrovica, Serbia
[b] Independent Consultant in the Field of Urban and Spatial Planning, Nis, Serbia
[c] Faculty of Electronic Engineering, University of Nis, Nis, Serbia
[d] Kulab, KU Leuven, Ostend, Belgium

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The main purpose of this paper is to show how the inclination angle affects natural convection from a flat-plate photovoltaic module which is mounted on the ground surface. In order to model this effect, novel correlations for natural convection from isothermal flat plates are developed by using the fundamental dimensionless number. On the basis of the available experimental and numerical results, it is shown that the natural convection correlations correspond well with the existing empirical correlations for vertical, inclined, and horizontal plates. Five additional correlations for the critical Grashof number are derived from the available data, three indicating the onset of transitional flow regime and two indicating the onset of flow separation. The proposed correlations cover the entire range of inclination angles and the entire range of Prandtl numbers. This paper also provides two worked examples, one for natural convection combined with radiation and one for natural convection combined with forced convection and radiation.

Key words: *empirical correlation, flat plate, fundamental dimensionless number, inclination angle, natural convection, photovoltaic module*

Introduction

In order to produce the maximum amount of electricity, a photovoltaic (PV) module must receive the highest possible amount of sunlight. When sunlight is at its maximum, temperature reaches the highest level of influence on the efficiency of a PV module [1-3]. This occurs when PV modules are perpendicular to the incoming sunrays. The amount of sunlight can be also controlled with the inclination of PV modules, which is usually set at a nearly optimal angle [4]. Hence, the inclination angle, ψ, affects the efficiency. It is also known that the efficiency of a PV module depends on heat losses due to natural convection, forced convection, and radiation [5]. Moreover, natural convection is significantly affected by the inclination angle, ψ. Therefore, modeling the effect of the inclination angle on natural convection from a PV module is of considerable interest to the researchers [5, 6].

*Corresponding author, e-mail: dardan.klimenta@pr.ac.rs

It is common practice for natural convection from PV modules to be analyzed by using the empirical correlations for vertical, inclined, and horizontal isothermal flat plates. Many studies have been devoted to natural convection from isothermal plates and surfaces at inclination angles between the vertical ($\psi = 0°$) and horizontal ($\psi = 90°$) limits [7-13]. Some of these studies are experimental [7-10], while others are numerical [11-13]. Most of the correlations between the average Nusselt number and the Rayleigh number are in the form $\mathrm{Nu} = C\mathrm{Ra}^n$, where C and n are the dimensionless parameter and exponent which depend on the flow conditions [14, 15]. There are, however, some correlations which are in the form Nu $C\Pi_N^n$, where $_N$ is the fundamental dimensionless number for natural convection [15-17]. To the best knowledge of the authors, natural convection correlations for inclined and horizontal isothermal plates based on the fundamental dimensionless number were not described elsewhere in the literature. Development of such correlations and their further application to the flat-plate PV modules are the objectives of this paper.

This paper introduces a set of new correlations for simultaneous natural convection from both surfaces of an isothermal plate with arbitrary inclination between 0 and 90 . This paper also proposes two correlations that apply only to laminar natural convection from upward-facing (UF) surface of a heated plate when its downward-facing (DF) surface is thermally insulated. All the correlations are in the form $\mathrm{Nu}(\psi)=C(\psi)\ \Pi_N^n$, where C depends on the inclination angle ψ, $n = 1/4$ for the laminar flow and $n = 1/3$ for the turbulent flow and the flow separation. For any positive angle of inclination, the dimension representing the height of a plate in the direction of gravity is adopted as the characteristic length, L. Excluding the limits on inclination, there are no other limitations with regard to the correlations. Moreover, based on [9, 10, 13, 14, 18, 19], five equations are constructed for the calculation of the critical Grashof numbers as functions of the inclination angle, ψ, and the Prandtl number. While three of these five equations indicate the onset of transition from laminar to turbulent flow regime, the fourth and fifth equations indicate the onset of flow separation (also boundary-layer separation, stall).

The natural convection correlations cover the entire ranges of inclination angles and Prandtl numbers and they are applied to estimate the associated heat transfer coefficients for two different PV modules, both mounted on the ground surface and tested under different ambient conditions. The transitional flow regime is modeled by the correlations which are the same as those used for the turbulent flow regime. The effects of the solar radiation and heat losses due to forced convection and radiation from the outer surfaces of the PV modules are taken into consideration. Calculation results of the two worked examples and the corresponding experimental data [4, 20, 21] have served to validate the proposed modeling concept.

Experimental and numerical results used in the present study

To derive the correlations for natural convection, the authors use the experi- mental results of Hassan and Mohamed [8], Lim *et al*. [10], and Heo and Chung [22], the numerical results of Corcione *et al*. [13] and one worked example [14]. Heo and Chung [22] reported on the vertical isothermal cylinders, while the others reported on the vertical, inclined, and horizontal isothermal flat plates.

Table 1 provides the data of different rectangular flat plates and different circular cylinders which were experimentally, numerically, and theoretically analyzed. The inclinations of the plates change from the vertical to the horizontal orientation, while the cylinders only have a vertical position. The parameters indicated in tab. 1 have the following meanings: L is the plate height, W – the plate width, D – the cylinder diameter, and Gr – the Grashof number.

Table 1. Summary of data relevant to plates and cylinders used in the present study

ψ [ ]	Object	L [m]	W or D [m]	Pr [–]	Gr [–]	Analysis and reference
0	Plate	4	10	0.7	$3.744\ 10^{11}$	Theoretical, [14]
0, 15, 30, 45, 60, 75	Plate	0.504	0.2	0.71	$2.394\ 10^{6}$	Experimental, [8]
15, 30, 45, 60	Plate	0.504	0.2	0.71	$2.394\ 10^{6}$	Numerical, [13]
0, 5, 10, … , 75	Plate	Unknown	Unknown	0.71	14084.51	Numerical, [13]
0, 5, 10, … , 75	Plate	Unknown	Unknown	7	1428.571	Numerical, [13]
0, 5, 10, … , 75	Plate	Unknown	Unknown	70	142.857	Numerical, [13]
0, 10, 20, … , 90	Plate	0.1	0.03	2094	$8.06\ 10^{7}$	Experimental, [10]
0, 10, 20, … , 90	Plate	0.35	0.03	2094	$3.45\ 10^{9}$	Experimental, [10]
0	Cylinder	0.1	0.067	2094	$8.06\ 10^{7}$	Experimental, [22]
0	Cylinder	0.25	0.067	2094	$1.26\ 10^{9}$	Experimental, [22]
0	Cylinder	0.45	0.067	2094	$7.34\ 10^{9}$	Experimental, [22]

Theoretical background

Natural convection from inclined PV modules

Figure 1 shows cross-sectional views of a PV module at inclinations of $= 0°$, $0° < \psi < 90°$, and $\psi = 90°$ in combination with the corresponding air-flows along its surfaces. As the PV module rotates from the vertical to the horizontal position, its one surface faces upwards while another one faces downwards. In general, a PV module is not placed vertically or horizontally. The module is inclined at an angle ψ to optimize the output power [5]. The average Nusselt number for the DF surface of a PV module inclined at an angle up to 75° can be estimated by replacing the acceleration of gravity, g, with gcos ψ in the correlation for natural convection from a vertical plate [5, 13]. Since the buoyancy force is mainly into the DF surface of a PV module, a laminar flow prevails up to very high Rayleigh numbers [23]. According to [9, 10, 14, 19], laminar natural convection from the UF surface of a PV module can also be affected by the flow separation.

The significance of the natural convection heat transfer from a PV module can be determined by the ratio of Gr/Re^2, where Re is the Reynolds number. If $Gr/Re^2 \gg 1$or $Gr/Re^2 \ll 1$, the convective heat transfer is dominated by natural or forced convection, respectively [5]. A

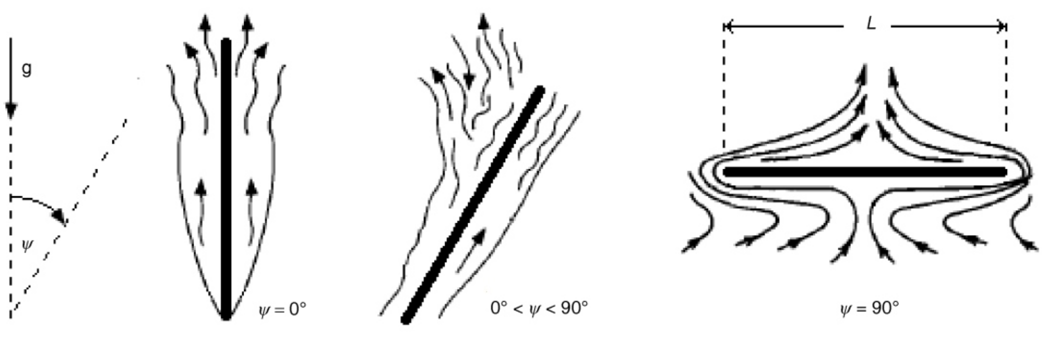

Figure 1. Air-flows along a PV module for different inclination angles

value of $\mathrm{Gr/Re}^2 \sim 1$ means both natural and forced convection heat transfers have significant effects [5].

Correlations for natural convection from isothermal plates

In order to calculate the natural convection heat transfer from vertical, inclined, and horizontal isothermal flat plates, the authors derived correlations of the following form:

$$\mathrm{Nu}(\psi) \quad C(\psi)(\ _N)^n \tag{1}$$

where $_N$ is defined as [15]

$$_N \quad \frac{\mathrm{Ra}}{1 \quad \frac{0.492}{\mathrm{Pr}}} \tag{2}$$

These correlations are similar to the ones for vertical plates and cylinders from [15]. In accordance with [15-17], eq. (1) can be applied to any characteristic length, angle of inclination between 0 and 90° from the vertical, and Prandtl number between 0.001 and positive infinity.

The correlations (1) are derived based on the existing empirical correlations [14, 15], experimental data [10, 22], and numerical results [12]. All the new correlations, together with the limitations, flow regimes, references, and corresponding indications, are presented in tab. 2. It can be shown that eqs. (6)-(12) can be reduced exactly to correlations for the vertical position or to correlations for the horizontal position. It can also be seen that eqs. (11) and (18) are very similar to eqs. (10) and (17), respectively. The correlations (1) agree well with the existing ones. The parameters $\mathrm{Gr_{cr1}}$, $\mathrm{Gr_{cr2}}$, and $\mathrm{Gr_{cr3}}$, which are given in tab. 2, represent the critical Grash of numbers indicating the onset of transition from laminar to turbulent flow regime, *i. e.* the first appearance of instability of laminar flow over an inclined flat plate. In tab. 2 are also shown $\mathrm{Gr_{cr4}}$ and $\mathrm{Gr_{cr5}}$ indicating the onset of flow separation. The $\mathrm{Gr_{cr1}}$, $\mathrm{Gr_{cr2}}$, and $\mathrm{Gr_{cr4}}$ correspond to the UF surface, while $\mathrm{Gr_{cr3}}$ and $\mathrm{Gr_{cr5}}$ correspond to the DF surface.

If the Grashof number is larger than the appropriate critical Grashof number $\mathrm{Gr_{cr1}}$, $\mathrm{Gr_{cr2}}$ or $\mathrm{Gr_{cr3}}$, the Nusselt number starts deviating from laminar behavior. Moreover, if Grashof number is larger than $\mathrm{Gr_{cr4}}$ or $\mathrm{Gr_{cr5}}$, Nusselt number is also affected by the flow separation [9, 10, 14]. An increase in the inclination angle, ψ, decreases $\mathrm{Gr_{cr1}}$, $\mathrm{Gr_{cr2}}$, and $\mathrm{Gr_{cr4}}$ and increases $\mathrm{Gr_{cr3}}$ and $\mathrm{Gr_{cr5}}$. Also, an increase in the Prandtl number has a destabilizing effect on the flow and decreases the critical Grashof numbers [11]. A set of experimental data concerning the critical Grashof numbers $\mathrm{Gr_{cr1}}$, $\mathrm{Gr_{cr2}}$, $\mathrm{Gr_{cr3}}$, $\mathrm{Gr_{cr4}}$, and $\mathrm{Gr_{cr5}}$, which mark the first appearance of instability or the onset of flow separation at different inclinations and Prandtl number, was reported in [9, 10, 14, 18, 24-27].

According to [9], after the flow separation takes place the Nusselt number for the UF surface of a horizontal plate may be considered to be in agreement with Churchill and Chu correlation for transitional and turbulent flows along the surfaces of a vertical plate. This is also in agreement with Black and Norris [24] correlation for a turbulent flow along the UF surface of a horizontal plate. Since the flow separation from a DF surface is not expected [8], it is assumed that the corresponding Nusselt number correlations agree with those for laminar natural convection from the DF surfaces of horizontal and inclined plates. This assumption is well verified in experiments of Lim *et al*. [10]. Moreover, the experiments of Lim *et al*. [10], as well as of Heo and Chung [22], have shown that the flow separation from the surfaces of a vertical plate can be modeled by using the Churchill and Chu correlation for the laminar flow along the surfaces of a vertical plate. While eqs. (11) and (18) are derived based on the experimental results for the flat

Table 2. Correlations for natural convection from isothermal plates and surfaces

Nu(ψ C ψ n			Plate temperature is constant			Reference	Eqs.
$_N$ = Ra/(1+0.492/Pr)			0.001 Pr +∞				
Geometry		Range of Gr	Flow regime	Correlation			
Plate	Sur.			$C(\psi)$	n		
Vertical $\psi = 0$	SF [a]	Gr $\mathrm{Gr_{cr4}}$	Laminar	0.67	1/4	[15]	(3)
		$\mathrm{Gr_{cr1}}$ Gr > $\mathrm{Gr_{cr4}}$	Separation	0.057	1/3	[10, 14, 24] [c]	(4)
		Gr > $\mathrm{Gr_{cr1}}$	Turbulent	0.1335	1/3	[14] [d]	(5)
Inclined, 0 < ψ < 90°	UF	Gr $\mathrm{Gr_{cr4}}$ for ψ < 21.42°[b], Gr $\mathrm{Gr_{cr1}}$ for 21.42° ψ < 30° or Gr $\mathrm{Gr_{cr2}}$ for ψ 30°	Laminar	0.376 + 0.294 (cos ψ)$^{1/4}$	1/4	[12, 14, 15] [e]	(6)
				0.616 + 0.054 (cos)$^{1/4}$	1/4	[14, 15] [f]	(7)
		$\mathrm{Gr_{cr1}}$ Gr > $\mathrm{Gr_{cr4}}$ for ψ < 21.42° Gr > $\mathrm{Gr_{cr4}}$ for ψ 21.42°	Separation	0.057 + 0.098 (sin ψ)$^{1/3}$	1/3	[10, 14, 24] [c]	(8)
		Gr > $\mathrm{Gr_{cr1}}$ for ψ < 21.42°,$\mathrm{Gr_{cr1}}$ < Gr $\mathrm{Gr_{cr4}}$ for 21.42° ψ < 30° or $\mathrm{Gr_{cr2}}$ < Gr $\mathrm{Gr_{cr4}}$ for ψ 30°	Turbulent	0.1335 + 0.0456 (sin ψ	1/3	[14] [d]	(9)
	DF	Gr $\mathrm{Gr_{cr5}}$	Laminar	0.308 + 0.362 (cos ψ)$^{1/4}$	1/4	[12, 14, 15] [e]	(10)
		$\mathrm{Gr_{cr3}}$ Gr > $\mathrm{Gr_{cr5}}$	Separation	0.046 + 0.011 (cos ψ)$^{1/3}$	1/3	[10, 14, 24] [c]	(11)
		Gr > $\mathrm{Gr_{cr3}}$	Turbulent	0.036 + 0.0975 (cos ψ)$^{1/3}$	1/3	[14] [d]	(12)
Horizontal ψ = 90°	UF	Gr $\mathrm{Gr_{cr2}}$	Laminar	0.376	1/4	[12, 14, 15] [e]	(13)
				0.616	1/4	[14, 15] [f]	(14)
		Gr > $\mathrm{Gr_{cr4}}$	Separation	0.155	1/3	[10, 14, 24] [c]	(15)
		$\mathrm{Gr_{cr2}}$ < Gr $\mathrm{Gr_{cr4}}$	Turbulent	0.1791	1/3	[14] [d]	(16)
	DF	Gr $\mathrm{Gr_{cr5}}$	Laminar	0.308	1/4	[12, 14, 15] [e]	(17)
		$\mathrm{Gr_{cr3}}$ Gr > $\mathrm{Gr_{cr5}}$	Separation	0.046	1/3	[10, 14, 24] [c]	(18)
		Gr > $\mathrm{Gr_{cr3}}$	Turbulent	0.036	1/3	[14] [d]	(19)

[a] An abbreviation of sideward-facing.
[b] The inclination angle of 21.42° comes from $\mathrm{Gr_{cr1}} = \mathrm{Gr_{cr4}}$.
[c] Correlations derived based on experiments with inclined plates conducted by Lim *et al*. [10], experiments with inclined cylinders conducted by Heo and Chung [22], and correlations provided by Churchill and Chu (for a laminar flow along the surfaces of a vertical plate) [14], Black and Norris (for a turbulent flow along the UF surface of a horizontal plate) [24] and McAdams (for a laminar flow along the DF surface of a horizontal plate) [14].
[d] Correlations derived based on correlations provided by Churchill and Chu (for transitional and turbulent flows along the surfaces of a vertical plate) [14], Fujii and Imura (for a turbulent flow along the UF surface of a horizontal plate) [9, 14] and McAdams (for a laminar flow along the DF surface of a horizontal plate) [14].
[e] Correlations derived based on correlations provided by Arpaci *et al*. (for a laminar flow along the surfaces of a vertical plate) [15] and McAdams (for a laminar flow along the surfaces of a horizontal plate) [14], and numerical study conducted by Wei *et al*. (for a laminar flow along the surfaces of a horizontal plate) [12].
[f] Correlations derived based on correlations provided by Arpaci *et al*. (for a laminar flow along the surfaces of a vertical plate) [15] and McAdams (for a laminar flow along the surfaces of a horizontal plate) [14]. These two correlations apply only to laminar natural convection from the UF surface of a heated plate when its DF surface is thermally insulated.

plate with $H/W = 11.67$, all other correlations are derived from the available data on flat plates with H/W 3.33. Therefore, the ratio H/W 3.33 can also be introduced as a limiting criterion for eqs. (3)-(19) and the five critical Grashof numbers.

In accordance with [9, 10, 13, 14, 18, 19], the following equations are derived to calculate the critical Grashof numbers Gr_{cr1}, Gr_{cr2}, Gr_{cr3}, Gr_{cr4}, and Gr_{cr5} as functions of the inclination angle ψ and the Prandtl number:

$$Gr_{cr1} = \frac{1}{Pr} 10^{(24.258\cos\psi - 13.028)} \text{ for UF surface and } 0° < \psi < 30° \tag{20}$$

$$Gr_{cr2} = \frac{1}{Pr} 10^{(5\cos\psi + 3.65)} \text{ for UF surface and } 30° \le \psi \le 90° \tag{21}$$

$$Gr_{cr3} = \frac{1.7 \times 10^{11}}{Pr} e^{(\psi \ln 2)/90} \text{ for UF surface and } 0° \le \psi \le 90° \tag{22}$$

$$Gr_{cr4} = \frac{1}{Pr} 10^{(5\cos\psi + 4.9)} \text{ for UF surface and } 0° < \psi \le 90° \tag{23}$$

$$Gr_{cr5} = \frac{10^{9.9}}{Pr} e^{(\psi \ln 2)/90} \text{ for UF surface and } 0° \le \psi < 90° \tag{24}$$

as well as for $H/W \ge 3.33$ and $0.001 \le Pr \le +\infty$, as shown in fig. 2.

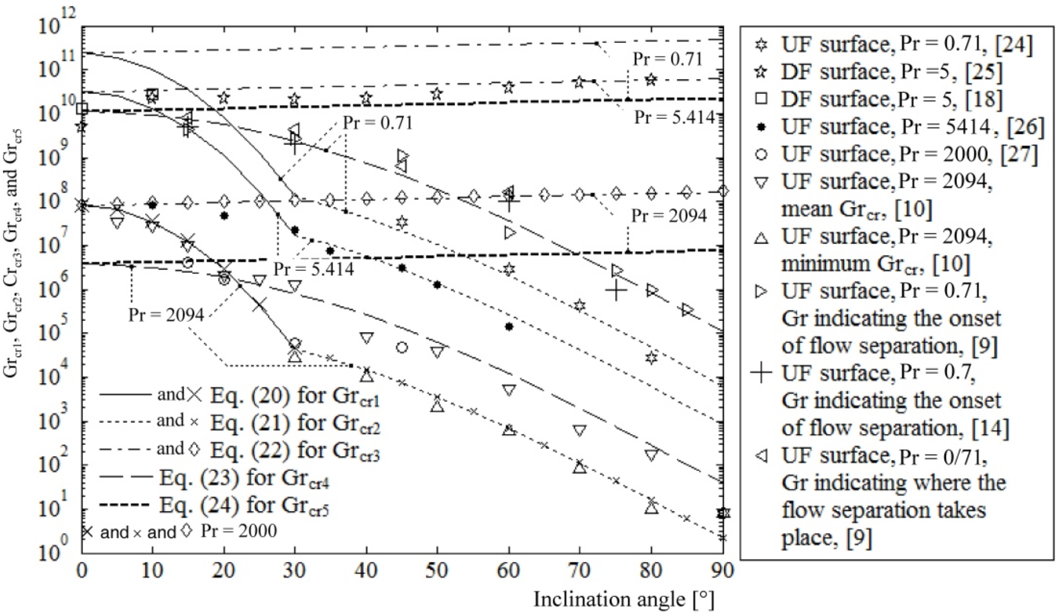

Figure 2. A comparison between the critical Grashof numbers Gr_{cr1}, Gr_{cr2}, Gr_{cr3}, Gr_{cr4}, and Gr_{cr5} and the associated experimental data for different angles of inclination and Prandtl numbers

Furthermore, the Grashof number for a vertical cylinder is defined using its height, L, as a characteristic length. If the thermal boundary layer thickness is not large compared to the cylinder diameter, D, the natural convection heat transfer may be calculated with the same correlations used for vertical flat plates. According to [22], a vertical isothermal cylinder may be treated as a vertical isothermal flat plate when the following criterion for transversal curvature effect is valid in the range of Prandtl number from 0.01 to 4173:

$$\sqrt[4]{Gr_L}\frac{D}{L} \ge 11.474 + \frac{48.92}{Pr^{1/2}} - \frac{0.006085}{Pr^2} \tag{25}$$

This implies that the experimental data on vertical cylinders, which are displayed in tab. 2, can be used in the present study.

Heat transfer model

This section provides guidance for the modeling of the heat transfer along the surfaces of the PV module, as shown in fig. 3. The average values of the heat transfer coefficients are calculated using the law of conservation of energy, correlations for natural convection given in tab. 2 and a number of the existing correlations for forced convection and radiation. All the parameters displayed in this section and their meanings are included in the Nomenclature. Figure 3 shows the sky and the ground are modeled as two unbounded parallel flat plates. The four sides of the PV module are assumed to be adiabatic and the two surfaces of the PV module are assumed to be at a constant temperature $T_{PV} = (T_{UF} + T_{DF})/2$. The ground temperature T_g is assumed to equal the sky temperature T_{sky}, which is modeled by the correlation of Swinbank [28].

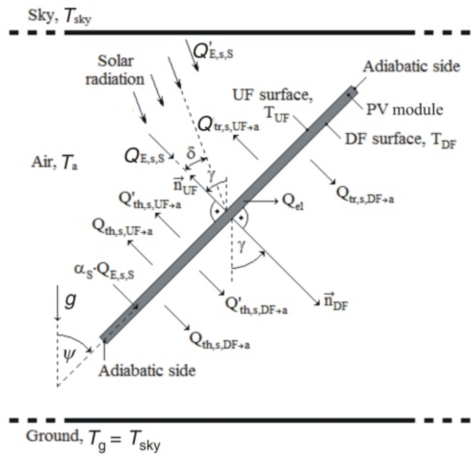

Figure 3. Heat transfer along the surfaces of the PV module

The heat transfer coefficients corresponding to the processes $Q_{th,s,UF \quad a} \; \psi$, $Q'_{th,s,UF \quad a}$, $Q_{tr,s,UF \quad a}$, $Q_{th,s,DF \quad a}$, $Q'_{th,s,DF \quad a}$, and $Q_{tr,s,DF \quad a}$ are $h_{UF} = h_{UF}$ (ψ , h'_{UF}, $h_{r,UF}$, $h_{DF} = h_{DF}$ (ψ , h'_{DF}, and $h_{r,DF}$, respectively.

Assuming a number of the aforementioned parameters are known, heat losses due to natural convection need to be estimated for different inclinations ψ.

The iterative procedure for computing h_{UF} and h_{DF} requires knowledge of T_{PV}, h'_{UF}, $h_{r,UF}$, h'_{DF}, and $h_{r,DF}$, which are initially unknown. To obtain an initial estimate of T_{PV}, the same numerical value should be taken for all unknown coefficients (for example 12 $Wm^{-2}K^{-1}$). Therefore, an initial estimate of T_{PV} can be obtained from [29]:

$$\alpha_S Q_{E,s,S} \quad Q_{th,s,UF \quad a} \quad Q'_{th,s,UF \quad a} \quad Q_{tr,s,UF \quad a} \quad Q_{th,s,DF \quad a} \quad Q'_{th,s,DF \quad a} \quad Q_{tr,s,DF \quad a} \quad Q_{el}/S \tag{26}$$

More precisely:

$$T_{PV} \quad \frac{\alpha_S \; Q_{E,s,S} \quad Q_{el} \; / \; S \quad (h_{UF} \quad h'_{UF} \; h_{DF} \quad h'_{DF}) \; T_a \quad h_{r,UF} \; T_{sky} \quad h_{r,DF} \; T_g}{h_{UF} \quad h'_{UF} \; h_{r,UF} \quad h_{DF} \quad h'_{DF} \; h_{r,DF}} \tag{27}$$

where

$$_S Q_{th,s,UF \quad a} \quad h_{UF}(T_{PV} \quad T_a), \quad Q'_{th,s,UF \quad a} \quad h'_{UF}(T_{PV} \quad T_a), \quad Q_{tr,s,UF \quad a} \quad h_{r,UF}(T_{PV} \quad T_{sky}),$$
$$h_{r,UF} \quad \varepsilon_{UF}\sigma_{SB}(T_{PV}^2 \quad T_{sky}^2)(T_{PV} \quad T_{sky}), Q_{th,s,DF \quad a} \quad h_{DF}(T_{PV} \quad T_a), Q'_{th,s,DF \quad a} \quad h'_{DF}(T_{PV} \quad T_a),$$
$$Q_{tr,s,DF \quad a} \quad h_{r,DF}(T_{PV} \quad T_g), h_{r,DF} \quad \varepsilon_{DF}\sigma_{SB}(T_{PV}^2 \quad T_g^2)(T_{PV} \quad T_g), Q_{el} \quad \eta_{el}\alpha_S Q_{E,s,S} S,$$
$$Q_{E,s,S} \quad Q'_{E,s,S} \cos\delta, \quad \text{and} \quad T_{sky} \quad T_g \quad 0.0552 T_a^{1.5}.$$

For forced convection from the UF and DF surfaces of the PV module and any direction of the wind, the following correlations can be used [30]:

$$\mathrm{Nu} \quad 0.664\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3} \quad \text{for} \quad \mathrm{Re} \quad v_w L/\nu \quad 5 \;\; 10^5 \tag{28}$$

$$\mathrm{Nu} \quad (0.037\,\mathrm{Re}^{4/5} \quad 871)\,\mathrm{Pr}^{1/3} \quad \text{for} \quad \mathrm{Re} \quad v_w L/\nu \quad 5 \;\; 10^5 \tag{29}$$

where Nu is the corresponding Nusselt number, Re – the Reynolds number, v_w – the wind velocity, and ν – the kinematic viscosity of the air. For the case of radiation, the corresponding heat transfer coefficients are non-linear and calculated assuming the optically thin limit. More details on this heat transfer model can be found in [29], where a similar model was presented.

Results and discussion

Validation of the proposed correlations

Figure 4 shows the results obtained by fitting eqs. (3), (4), (6), (8), (10), and (11) to the numerical and experimental results in [8, 10, 13, 22]. According to this figure, in almost all cases, the Nusselt number reaches its maximum for the inclination of $\psi = 0°$. The Nusselt number gradually decreases as the inclination angle ψ increases and reaches its minimum for $\psi = 90°$. An exception occurs in cases of the laminar and turbulent flows with the flow separation from the UF surfaces when the Nusselt number has its minimum for $\psi = 0°$ and its maximum for $\psi = 90°$, as shown in figs. 4(b) and 4(c). The correlations for laminar natural convection without the flow separation from the UF and DF surfaces reveal similar trends with very small differences. The relative errors between the numerical or experimental data and the data obtained by the proposed correlations for inclined plates are lower than 19.31% for a laminar re-

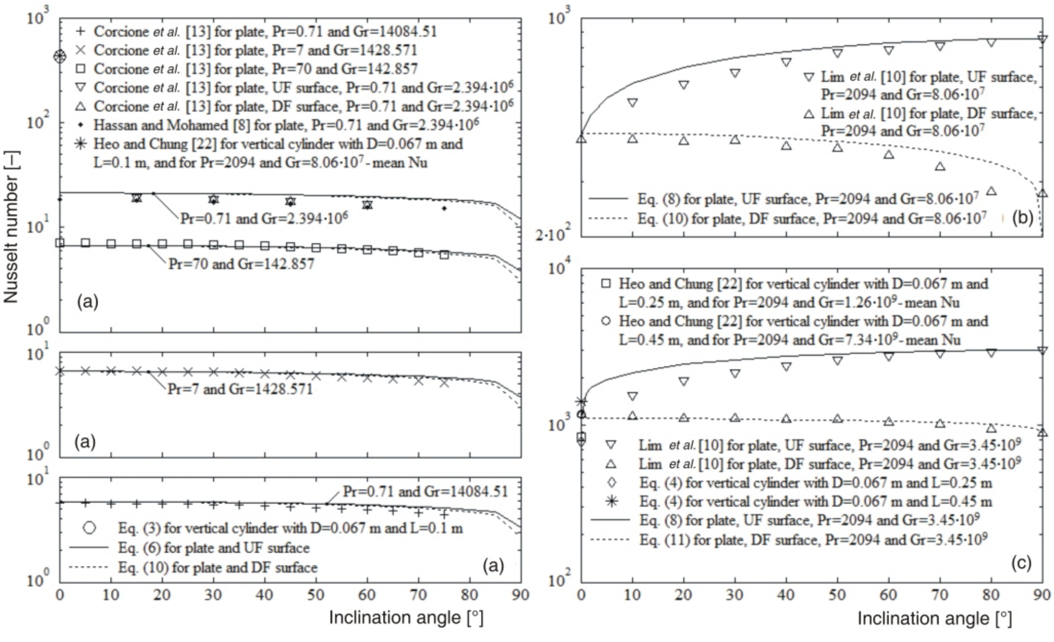

Figure 4. A comparison between the Nusselt number calculated using the equations from tab. 2 and relevant data from the literature: (a) in case when the flows are laminar; (b) in case when the flows are laminar and affected by the flow separation; and (c) in case when the flows are turbulent and affected by the flow separation

gime without and with the flow separation, figs. 4(a) and 4(b), and lower than 28.24% for a turbulent regime with the flow separation, fig. 4(c). Maximum relative errors are obtained for the experimental results of Lim *et al*. [10], *i. e*. for the plates with dimensions of 0.1 0.03 m^2 and 0.35 0.03 m^2. For other plates, the errors are definitely smaller, *i. e*. from 0 to about 15%.

The influence of the inclination angle on the laminar natural convection from an isothermal plate for Pr = 0.71 and Gr = 2.394 10^6 is shown in fig. 5(a). Figure 5(a) illustrates the comparisons between eqs. (6), (7), and (10) on one side and the correlations of Churchill and Chu in [14], Raithby and Hollands [23], Al-Araby and Sakr [31], and Corcione *et al*. [13] on the other.

There are a large number of correlations for laminar natural convection available in the literature since the inclined plate surrounded by air is frequently encountered in practice. In order to provide a comparison, the present paper selects some of the most commonly used empirical correlations for the average Nusselt number. Equations (6), (7), and (10) follow the trends of selected correlations, although giving different relative errors. The smallest errors are between these three equations and the correlation of Raithby and Hollands [23]. For instance, eqs. (6), (7), and (10) for $\psi = 75°$ give errors with respect to the correlation of Raithby and Hollands [23] which equal 29.91, 45.36, and 25.7%, respectively. These errors originate from the fact that the correlation of Raithby and Hollands [23] represents the arithmetic mean of the average Nusselt number for the UF surface of the plate and the average Nusselt number for the DF surface of the plate, of course, comparing to the eqs. (6), (7), and (10). Moreover, these equations give the expected trends but overestimate the Nusselt numbers calculated using the correlation of Raithby and Hollands [23].

The effect of the inclination angle on the turbulent natural convection from an isothermal plate of dimensions 4 m by 10 m for Pr = 0.7 and Gr = 3.744 10^{11} is shown in fig. 5(b). Figure 5(b) compares the eqs. (9) and (12) with the correlations of Fujii and Imura [9, 14], Churchill and Chu in [14], McAdams in [14], and Black and Norris [24]. In some of these correlations, the Rayleigh number is modified with sin ψ or cos ψ.

In accordance with fig. 5(b), eq. (9) follows the trend of the correlation of Fujii and Imura [9, 14] between the inclinations of 20-90°, as well as the trend of the correlation of Black and Norris [24] between the inclinations of 45-90°. A maximum relative error of 5.59% is obtained for the UF surface. Figure 5(b) also shows that eq. (12) follows the trend of a modified

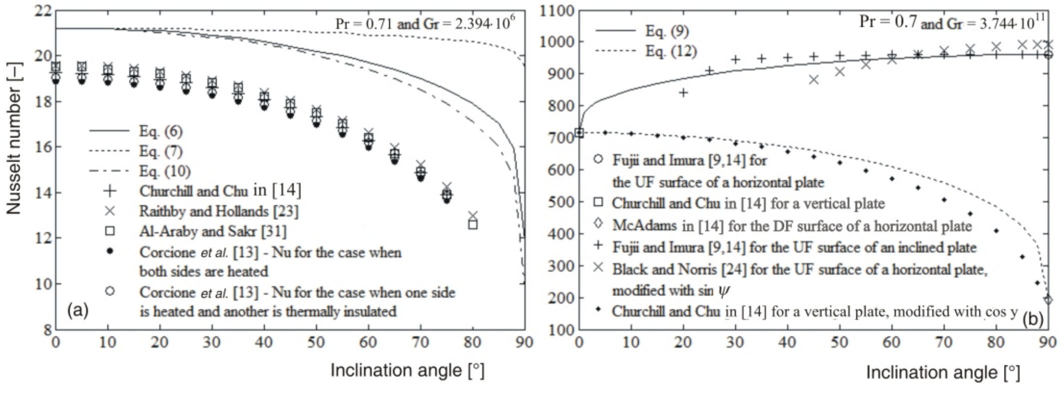

Figure 5. A comparison between the equations from tab. 2 and the existing correlations: (a) for laminar natural convection and (b) turbulent natural convection

correlation of Churchill and Chu in [14]. The standard correlation of Churchill and Chu [14] for transitional and turbulent flows along the surfaces of a vertical plate is modified with cos ψ. The relative errors between the eq. (12) and the modified correlation of Churchill and Chu in [14] are lower than 18.8% for 0° ψ 80° (*i. e.* between 0-18.8%).

Worked examples

In this paper, the proposed correlations are applied to study two different PV modules. Heat transfer processes associated with these two PV modules are presented in fig. 3. Each PV module has a width, $W = W_{PV}$, and a height, $L = L_{PV}$ – characteristic length. The PV modules are mounted on the ground surface at different inclinations from the vertical, ψ. Each PV module has a solar-to-electric power conversion efficiency, η_{el}, a solar absorption coefficient of the UF surface, α_S, a thermal emission coefficient of the UF surface, ε_{UF}, and a thermal emission coefficient of the DF surface, ε_{DF}. The PV modules are situated in environments having different temperatures, T_a, and a pressure of 1 atmosphere. The PV modules are exposed to the effects of direct solar irradiance, $Q'_{E,s,S}$, and wind velocity, v_w. Heat exchange between the first PV module and its environment occurs through natural convection and radiation. In the case of the second PV module, heat exchange occurs through mixed convection and radiation. The PV modules are also assumed to be solid, isotropic, and homogeneous materials. The heat transfer coefficients due to convection and radiation (h_{UF}, h'_{UF}, $h_{r,UF}$, h_{DF}, h'_{DF}, and $h_{r,DF}$) need to be calculated for various inclination angles.

Example 1. $W = W_{PV} = 0.5442$ m, $L = L_{PV} = 0.43$ m, $\eta_{el} = 0.14$, $\alpha_S = 0.97$, $\varepsilon_{UF} = 0.91$, $\varepsilon_{DF} = 0.85$, $T_a = 22$ °C, $\psi = 53°$, $Q'_{E,s,S} = 920$ Wm^{-2}, and $v_w = 0$ ms^{-1} [20].

Example 2. $W = W_{PV} = 0.468$ m, $L = L_{PV} = 0.624$ m, $\eta_{el} = 0.113$, $\alpha_S = 0.97$, $\varepsilon_{UF} = 0.91$, $\varepsilon_{DF} = 0.85$, $T_a = 36$ °C, $\psi = 42°$, $Q'_{E,s,S} = 997\cos(48°)$ Wm^{-2}, and $v_w = 2.3$ ms^{-1} [4, 21].

Tables 3 and 4 summarize the analytical results obtained for these two examples. Table 3 corresponds to the first PV module, while tab. 4 corresponds to the second one. For each inclination angle ψ a set of 12 to 14 iterations was needed. The heat transfer coefficients are calculated using a MATLAB program.

Table 3. Heat transfer coefficients for the first PV module at different inclination angles

ψ [°]	δ [°]	T_{PV} [C]	h_{UF} and h'_{UF} [Wm^{-2}K^{-1}]	$h_{r,UF}$ [Wm^{-2}K^{-1}]	h_{DF} and h'_{DF} [Wm^{-2}K^{-1}]	$h_{r,DF}$ [Wm^{-2}K^{-1}]
0	–53	38.907	3.787 and 0	5.367	3.787 and 0	5.014
10	–43	43.424	4.003 and 0	5.496	4.001 and 0	5.133
20	–33	47.057	4.134 and 0	5.601	4.128 and 0	5.232
30	–23	48.878	4.991 and 0	5.654	4.152 and 0	5.282
40	–13	50.490	5.174 and 0	5.702	4.142 and 0	5.326
50	–3	51.247	5.287 and 0	5.724	4.074 and 0	5.347
53	0	52.312	4.512 and 0	5.756	4.074 and 0	5.376
60	7	52.101	4.578 and 0	5.750	3.974 and 0	5.371
70	17	51.084	4.609 and 0	5.719	3.765 and 0	5.342
80	27	49.261	4.564 and 0	5.665	3.434 and 0	5.292
90	37	47.830	4.504 and 0	5.623	1.928 and 0	5.253

Table 4. Heat transfer coefficients for the second PV module at different inclination angles

ψ [°]	δ [°]	T_{PV} [C]	h_{UF} and h'_{UF} [Wm^{-2}K^{-1}]	$h_{r,UF}$ [Wm^{-2}K^{-1}]	h_{DF} and h'_{DF} [Wm^{-2}K^{-1}]	$h_{r,DF}$ [Wm^{-2}K^{-1}]
0	–42	45.711	2.976 and 7.519	6.122	2.976 and 7.519	5.719
10	–32	47.404	3.090 and 7.519	6.174	3.089 and 7.519	5.767
20	–22	48.676	3.155 and 7.518	6.212	3.150 and 7.518	5.803
30	–12	49.242	3.883 and 7.518	6.230	3.150 and 7.518	5.819
40	–2	49.560	3.985 and 7.518	6.240	3.117 and 7.518	5.828
42	0	49.569	3.998 and 7.518	6.240	3.105 and 7.518	5.828
50	8	49.666	3.390 and 7.518	6.243	3.053 and 7.518	5.831
60	18	49.055	3.426 and 7.518	6.224	2.925 and 7.518	5.814
70	28	48.004	3.392 and 7.519	6.192	2.736 and 7.519	5.784
80	38	46.534	3.285 and 7.519	6.147	2.455 and 7.519	5.742
90	48	44.848	3.115 and 7.518	6.096	1.338 and 7.518	5.694

It can be noticed from tabs. 3 and 4 that the heat losses due to natural convection from the surfaces of the PV modules change between: (1) 13.863-20.175% for the UF surface of the first PV module; (2) 8.126-16.092% for the DF surface of the first PV module; (3) 6.775-9.451% for the UF surface of the second PV module; and (4) 3.082-7.504% for the DF surface of the second PV module. The heat losses due to natural convection are expressed as a percentage of the total heat loss. Trends of the heat transfer coefficients h_{UF} and h_{DF} are similar to those obtained by Lim *et al.* [10] for inclined flat plates. Moreover, the results obtained for $\psi = 53°$ and $\psi = 42°$ correspond well with the experiments performed by Date *et al.* [20] and Ali *et al.* [21], respectively.

Conclusions

The main conclusions arising from the present paper are as follows.

A set of new correlations for natural convection without and with the flow separation, based on the fundamental dimensionless number, is derived and successfully validated. The correlations apply to vertical, inclined, and horizontal flat plates as well as surfaces.

In order to calculate the critical Grashof numbers, the following five correlations are used: (1) a new correlation indicating the onset of transitional/turbulent flow along the UF surface of a flat plate for $0° < \psi < 30°$, (2) a known correlation of Corcione *et al.* [13] indicating the onset of transitional/turbulent flow along the UF surface of a flat plate for $30°$ ψ $90°$, (3) a modified correlation of Warneford [18] indicating the onset of transitional/turbulent flow along the DF surface of a flat plate for $0°$ ψ $90°$, (4) a new correlation indicating the onset of flow separation along the UF surface of a flat plate for $0° < \psi$ $90°$, and (5) a new correlation indicating the onset of flow separation along the DF surface of a flat plate for $0°$ ψ $90°$.

All the proposed correlations for natural convection and the critical Grashof numbers are limited by the ratio of H/W 3.33 and generalized to the entire range of Prandtl numbers (0.001 Pr $+\infty$) with a satisfactory accuracy.

Introducing the proposed correlations into the heat transfer model, a simple algorithm to estimate the average heat transfer coefficients due to natural convection, forced convection, and radiation is developed. The heat transfer coefficients due to natural convection are available for different inclination angles.

The proposed heat transfer model that is based on the Swinbank correlation [28] gives better results than the models developed using the radiation shape factors.

The heat transfer coefficients due to natural convection from the inclined PV modules have trends which are in the line with the expectations.

The heat losses due to natural convection from the UF and DF surfaces of the PV modules are between 6.775-20.175% and 3.082-16.092% of the total heat loss, respectively. Lower values of the heat losses due to natural convection correspond to the case when the transfer of heat is affected by the wind.

The presented analytical results correspond well with existing experimental data.

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Nomenclature

C – parameter in eq. (1), [–]
D – cylinder diameter, [–]
g – acceleration of gravity, [ms^{-2}]
Gr – Grashof number, [–]
Gr_{cr} – critical Grashof number, [–]
h_{DF}, h_{UF} – coefficients due to natural convection from the DF and UF surfaces, respectively, [Wm^{-2}K^{-1}]
h'_{DF}, h'_{UF} – coefficients due to forced convection from the DF and UF surfaces, respectively, [Wm^{-2}K^{-1}]
$h_{r,DF}$, $h_{r,UF}$ – coefficients due to radiation from the DF and UF surfaces, respectively, [Wm^{-2}K^{-1}]
L – plate height, cylinder height or characteristic length, [m]
L_{PV} – height of a PV module, [m]
n – exponent in eq. (1), [–]
$\vec{n}_{DF}$ – a normal to the DF surface, [–]
$\vec{n}_{UF}$ – a normal to the UF surface, [–]
Nu – Nusselt number, [–]
Pr – Prandtl number, [–]
$Q_{E,s,S}$ – solar irradiance component which is normal to the UF surface, [Wm^{-2}]
$Q'_{E,s,S}$ – direct solar irradiance, [Wm^{-2}]
Q_{el} – total electric power generated by a PV module, [W]
$Q_{th,s,DF\ a}$ – natural convection from the DF surface to the air, [Wm^{-2}]
$Q'_{th,s,DF\ a}$ – forced convection from the DF surface to the air, [Wm^{-2}]
$Q_{th,s,UF\ a}$ – natural convection from the UF surface to the air, [Wm^{-2}]
$Q'_{th,s,UF\ a}$ – forced convection from the UF surface to the air, [Wm^{-2}]
$Q_{tr,s,DF\ a}$ – net radiation transfer from the DF surface to the ambient, [Wm^{-2}]
$Q_{tr,s,UF\ a}$ – net radiation transfer from the UF surface to the ambient, [Wm^{-2}]
Ra – Rayleigh number, [–]
Re – Reynolds number, [–]
S – active area of a PV module, [m^2]
T_a – air temperature, [K] or [C]
T_{DF} – temperature of the DF surface, [K]
T_g – ground temperature, [K]
T_{PV} – average temperature of a PV module, [K] or [C]
T_{sky} – sky temperature, [K]
T_{UF} – temperature of the UF surface, [K]
v_w – wind velocity, [ms^{-1}]
W – plate width, [m]
W_{PV} – width of a PV module, [m]

Greek symbols

α_S – solar absorption coefficient of the UF surface of a PV module, [–]
δ – angle between the sunrays and the normal n_{UF}, [ ]
ε_{DF} – thermal emission coefficient of the DF surface of a PV module, [–]
ε_{UF} – thermal emission coefficient of the UF surface of a PV module, [–]
γ – angle between the vertical and the normal n_{UF} or n_{DF}, [ ]
h_{el} – solar-to-electric power conversion efficiency, [–]

ν – kinematic viscosity of the air, [m^2s^{-1}]
$_N$ – fundamental dimensionless number for natural convection, [–]
σ_{SB} – Stefan-Boltzmann constant, [$Wm^{-2}K^{-4}$]
ψ – angle of inclination from the vertical, [ ]

Subscripts

a – air or ambient
cr – critical
DF a – from the downward-facing surface towards the air or the ambient
E – irradiation
el – electrical
g – ground
L – cylinder height as a characteristic length
N – natural convection
r – radiation
S – solar
s – per square meter
sky – sky
th – convection or natural convection
tr – thermal radiation
UF a – from the upward-facing surface towards the air or the ambient
w – wind

Superscript

′ – forced convection

Acronyms

DF – downward-facing
PV – photovoltaic
SB – Stefan-Boltzmann
SF – sideward-facing
UF – upward-facing

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