TIME-SPACE DEPENDENT FRACTIONAL VISCOELASTIC MHD FLUID FLOW AND HEAT TRANSFER OVER ACCELERATING PLATE WITH SLIP BOUNDARY

by

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The MHD flow and heat transfer of viscoelastic fluid over an accelerating plate with slip boundary are investigated. Different from most classical works, a modified time-space dependent fractional Maxwell fluid model is proposed in depicting the constitutive relationship of the fluid. Numerical solutions are obtained by explicit finite difference approximation and exact solutions are also presented for the limiting cases in integral and series forms. Furthermore, the effects of parameters on the flow and heat transfer behavior are analyzed and discussed in detail.

Key words: *MHD flow, Maxwell fluid, time-space fractional derivatives, slip boundary*

Introduction

Non-Newtonian fluids do not satisfy the linear relationship between stress tensor and the rate of deformation tensor, it has received much attention due to the various applications in engineering and industry, including food stuffs, molten plastics, pulps, petroleum drilling, and other similar activities. As an important class of non-Newtonian fluids, viscoelastic fluids show the properties of both elasticity and viscosity. Plenty of models have been proposed to describe the response characteristics of these fluids, among which the Maxwell model has been studied most widely [1-5].

Fractional calculus has been applied successfully in describing the complex viscoelastic fluids [6]. Generally, these governing equations are derived from classical equations which are modified by replacing the time ordinary derivatives of stress with the fractional calculus operators. This kind of generalization allows us to define non-integer order integrals or derivatives precisely. With the development of research, the interest in viscoelastic fluids has considerably increased. Nazar *et al.* [7] studied the velocity field and the shear stresses of generalized Maxwell fluid on oscillating rectangular duct. Jamil *et al.* [8] and Fetecau *et al.* [9] discussed the unsteady flow of Maxwell fluid with fractional derivative. Yang and Zhu [10] studied the flow of a viscoelastic fluid in a pipe. Cao *et al.* [11] derived in time domain the fundamental solution and relevant properties of the fractional order weighted distributed parameter Maxwell model. Hayat *et al.* [12] studied the heat and mass transfer effects in 3-D flow of Maxwell fluid

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over a stretching surface with convective boundary conditions. Vieru *et al.* [13] investigated the time-fractional free convection flow of an incompressible viscous fluid near a vertical plate with Newtonian heating and mass diffusion. Mustafa *et al.* [14] addressed the flow of Maxwell fluid due to constantly moving flat radiative surface with convective boundary condition. Some attempts concerning this field we refer to the recent papers [15-18].

However, most of the analytical solutions of the fractional fluid model containing complex series or special functions, which is not conducive to approximate calculation. The finite difference method, because of its flexibility, continues to be an efficient and reliable method. The finite different method is now found large applications in solving fractional differential equations [19-21].

Motivated from the afore mentioned studies, the aim of this paper is to extend the results of Zheng *et al.* [22] to consider the MHD flow and heat transfer of viscoelastic fluid over an accelerating plate with slip boundary. A modified time-space dependent fractional Maxwell fluid model is proposed in depicting the constitutive relationship of the fluid. Moreover, for the limiting cases $\beta \to 1$, the similar solutions are obtained by means of Laplace transform, which are presented in terms of series. Finally, the effects of different parameters on velocity and temperature fields are investigated and analyzed.

The basic equations

Consider the flow and heat transfer of a modified Maxwell fluid, which depicts by the time-space dependent fractional derivative (in the constitutive relationship), ignoring the pressure gradient, the governing equation can be written:

$$\left(1+\frac{\lambda}{\sigma_1^{1-\alpha}}\mathrm{D}_t^{\alpha}\right)\frac{\partial u(y,t)}{\partial t}=\sigma_2 \nu^{\beta}\left[\mathrm{D}_y^{\beta+1}u(y,t)\right]-M\left(1+\frac{\lambda}{\sigma_1^{1-\alpha}}\mathrm{D}_t^{\alpha}\right)u(y,t) \tag{1}$$

$$\left(1+\frac{\lambda_T}{\sigma_1^{1-\gamma}}\mathrm{D}_t^{\gamma}\right)\frac{\partial T(y,t)}{\partial t}=\sigma_2\frac{k_T^{\beta}}{\rho^{\beta}c_p^{\beta}}\mathrm{D}_y^{\beta+1}T(y,t) \tag{2}$$

In the relationship, $u(y,t)$ is the velocity, $T(y,t)$ is the temperature, D_t^{α} and D_y^{β} are fractional calculus operators based on Caputo definition and Riemann-Liouville definition, respectively, in the form:

$$\mathrm{D}_t^{\alpha}f(t)=\frac{1}{\Gamma(\alpha-1)}\int_0^t\frac{f'(\tau)}{(t-\tau)^{\alpha}}\mathrm{d}\tau \quad 0\le\alpha<1 \tag{3}$$

$$\mathrm{D}_y^{\beta}f(y)=\frac{1}{\Gamma(1-\beta)}\frac{\mathrm{d}}{\mathrm{d}y}\int_0^y\frac{f(\xi)}{(y-\xi)^{\beta}}\mathrm{d}\xi \quad 0\le\beta<1 \tag{4}$$

where $\Gamma(\cdot)$ is the gamma function, D_t^{γ} – the fractional calculus operators based on Caputo definition as eq. (3), $\nu=\mu/\rho$ – the kinematic viscosity, ρ – the constant density of the fluid, and λ, λ_T – the relaxation times. Parameter σ_1 and σ_2 characterize the fractional structures [23] and they are introduced for dimensional balance, $M=\sigma_0 B_0^2/\rho$ where σ_0 is the electric conductivity, B_0 – the magnetic intensity, k_T – the thermal conductivity, and c_p – the specific heat capacity of fluid.

This model is reduced to the generalized Maxwell model [22] when $\beta=1$ and to the ordinary Maxwell model when $\alpha=1$, $\beta=1$.

Statement of the problem and solutions

It is assumed that the fluids are static on the plate at first, and at the time $t = 0+$, the plate achieves an accelerated velocity in the x-direction with slip boundary. The shear stress results in the motion of the fluid. The governing equation is given by eq. (1). Accordingly, the initial and boundary conditions are:

$$\left(1+\frac{\lambda}{\sigma_1^{1-\alpha}}\mathrm{D}_t^\alpha\right)u(0,t)=A\left(1+\frac{\lambda}{\sigma_1^{1-\alpha}}\mathrm{D}_t^\alpha\right)t+\theta\left.\frac{\partial u}{\partial y}\right|_{y=0}\quad t>0 \tag{5}$$

$$u(y,0)=\frac{\partial u(y,0)}{\partial t}=0\quad y>0 \tag{6}$$

$$u(y,t),\ \partial_y u(y,t)\to 0\quad \text{as}\quad y\to\infty\quad t>0 \tag{7}$$

where A is the constant acceleration and θ is the slip coefficient. If $\theta = 0$ then the no-slip boundary condition is obtained. If θ is finite, fluid slip occurs at the wall but its effect depends on the length scale of the flow.

We assume T_∞ is the temperature of the fluid at the moment $t = 0$, $T_\infty + (T_w - T_\infty) f(t)$ denotes the temperature of the plate for $t \ge 0$ (with $f(t)$ be a known function). The corresponding initial and boundary conditions for energy equation are:

$$T(0,t)=T_\infty+(T_w-T_\infty)f(t)\quad t\ge 0 \tag{8}$$

$$T(y,0)=T_\infty,\quad \frac{\partial T(y,0)}{\partial t}=0\quad y>0 \tag{9}$$

$$T(y,t)\to T_\infty,\quad \partial_y T(y,t)\to 0\quad \text{as}\quad y\to\infty \tag{10}$$

Employing the non-dimensional quantities:

$$u^*=\frac{1}{\sigma_2(A\nu)^{\frac{\beta}{3}}}u,\quad y^*=\frac{\sigma_2(A\nu)^{\frac{\beta}{3}}}{\nu}y,\quad t^*=\frac{\sigma_2^{\beta+2}(A\nu)^{\frac{\beta(\beta+1)}{3}}}{\nu}t,$$

$$\lambda^*=\left[\frac{\sigma_2^{\beta+2}(A\nu)^{\frac{\beta(\beta+1)}{3}}}{\nu}\right]^\alpha\sigma_1^{\alpha-1}\lambda,\quad M^*=\frac{\nu}{\sigma_2^{\beta+2}(A\nu)^{\frac{\beta(\beta+1)}{3}}}M,\quad T^*=\frac{T-T_\infty}{T_w-T_\infty},$$

$$\lambda_T^*=\left[\frac{\sigma_2^{\beta+2}(A\nu)^{\frac{\beta(\beta+1)}{3}}}{\nu}\right]^\gamma\sigma_1^{\gamma-1}\lambda_T,\quad a^*=\frac{k_T}{\mu c_p} \tag{11}$$

Dimensionless motion equations can be derived (for brevity the dimensionless mark "*" is omitted here):

$$\left(1+\lambda\mathrm{D}_t^\alpha\right)\frac{\partial u(y,t)}{\partial t}=\mathrm{D}_y^{\beta+1}u(y,t)-M\left(1+\lambda\mathrm{D}_t^\alpha\right)u(y,t) \tag{12}$$

$$\left(1+\lambda_T\mathrm{D}_t^\gamma\right)\frac{\partial T(y,t)}{\partial t}=a^\beta\mathrm{D}_y^{\beta+1}T(y,t) \tag{13}$$

The corresponding initial and boundary conditions become:

$$\left(1+\lambda \mathrm{D}_t^{\alpha}\right)u(0,t) = A\left(1+\lambda \mathrm{D}_t^{\alpha}\right)t + \theta \left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{14}$$

$$u(y,0) = \frac{\partial u(y,0)}{\partial t} = 0\,,\;\; u(y,t)\,,\;\; \partial_y u(y,t) \to 0 \;\; \text{as} \;\; y \to \infty \tag{15}$$

$$T(0,t) = f(t) \tag{16}$$

$$T(y,0) = \frac{\partial T(y,0)}{\partial t} = 0\,,\;\; T(y,t)\,,\;\; \partial_y T(y,t) \to 0 \;\; \text{as} \;\; y \to \infty \tag{17}$$

Disperse space and time at grid points and time instants, define:

$$y_j = 0 + jh\,,\;\; h > 0 \quad j = 0, 1, 2, \ldots \tag{18}$$

$$t_n = n\tau \quad \tau > 0 \quad n = 0, 1, 2, \ldots \tag{19}$$

where h and τ are the length of space and time steps, respectively. Define u_i^j and T_i^j as the numerical approximation to $u(y_i, t_n)$ and $T(y_i, t_n)$, we obtain:

$$\mathrm{D}_t^{\alpha} u(y,t)\Big|_{y_i}^{t_n} = \frac{1}{\tau\Gamma(1-\alpha)}\left[b_0 u_i^j - \sum_{k=1}^{j-1}(b_{j-k-1} - b_{j-k})u_i^k - b_{j-1}u_i^0\right] + O(\tau^{2-\alpha}) \quad 0<\alpha<1 \tag{20}$$

$$\mathrm{D}_t^{\alpha} u(y,t)\Big|_{y_i}^{t_n} = \frac{1}{\tau\Gamma(2-\alpha)}\left[c_0 \frac{u_i^j - u_i^{j-1}}{\tau} - \sum_{k=1}^{j-1}(c_{j-k-1} - c_{j-k})\frac{u_i^k - u_i^{k-1}}{\tau}\right] + O(\tau^{3-\alpha}) \quad 1<\alpha<2 \tag{21}$$

$$\mathrm{D}_y^{\beta} u(y,t)\Big|_{y_i}^{t_n} = \frac{1}{h^{\beta}}\sum_{k=0}^{i+1} d_k u_{i-k+1}^j + O(h) \quad 0<\beta<1 \tag{22}$$

Introduce the coefficients:

$$b_k = \frac{\tau^{1-\alpha}}{1-\alpha}\left[(k+1)^{1-\alpha} - k^{1-\alpha}\right] \tag{23}$$

$$c_k = \frac{\tau^{2-\alpha}}{2-\alpha}\left[(k+1)^{2-\alpha} - k^{2-\alpha}\right] \tag{24}$$

$$d_0 = 1, \quad d_k = \left(1 - \frac{\beta+1}{k}\right)d_{k-1} \quad k = 1, 2, \ldots \tag{25}$$

The time first-order derivative can be approximated by the Euler backward difference scheme:

$$\frac{\partial u(y_i, t_j)}{\partial t} = \frac{u(y_i, t_j) - u(y_i, t_{j-1})}{\tau} + O(\tau) \tag{26}$$

The explicit finite difference approximation for eqs. (12) and (13) are:

$$\frac{u_i^j - u_i^{j-1}}{\tau} + \frac{\lambda}{\tau^2\Gamma(2-\alpha)}\left[c_0(u_i^j - u_i^{j-1}) - \sum_{k=1}^{j-1}(c_{j-k-1} - c_{j-k})(u_i^k - u_i^{k-1})\right] =$$

$$=\frac{1}{h^{\beta+1}}\sum_{k=0}^{i+1}d_k u_{i-k+1}^j - Mu_i^j - \frac{M\lambda}{\tau\Gamma(1-\alpha)}\left[b_0 u_i^j - \sum_{k=1}^{j-1}(b_{j-k-1}-b_{j-k})u_i^k - b_{j-1}u_i^0\right] \tag{27}$$

$$\frac{T_i^j - T_i^{j-1}}{\tau} + \frac{\lambda_T}{\tau^2\Gamma(2-\gamma)}\left[c_0(T_i^j - T_i^{j-1}) - \sum_{k=1}^{j-1}(c_{j-k-1}-c_{j-k})(T_i^k - T_i^{k-1})\right] = \frac{a^\beta}{h^{\beta+1}}\sum_{k=0}^{i+1}d_k T_{i-k+1}^j \tag{28}$$

Special cases

Letting $\beta \to 1$ in eq. (12), we attain the velocity field equation:

$$\left(1+\lambda D_t^\alpha\right)\frac{\partial u(y,t)}{\partial t} = \frac{\partial^2 u(y,t)}{\partial y^2} - M\left(1+\lambda D_t^\alpha\right)u(y,t) \tag{29}$$

Denoting by:

$$\overline{u}(y,s) = L\{u(y,t)\} = \int_0^\infty e^{-st}u(y,t)\mathrm{d}t \tag{30}$$

The image function of $u(y,t)$ and applying the Laplace transform to eq. (29):

$$\frac{\partial^2 \overline{u}}{\partial y^2} = (s+M)\left(1+\lambda s^\alpha\right)\overline{u} \tag{31}$$

According to the boundary conditions:

$$\overline{u}(0,s) = \frac{A}{s^2} + \frac{\theta}{\left(1+\lambda s^\alpha\right)}\left.\frac{\partial \overline{u}}{\partial y}\right|_{y=0} \tag{32}$$

$$\overline{u}(y,s)\,,\quad \frac{\partial \overline{u}(y,s)}{\partial y} \to 0 \quad \text{as} \quad y \to \infty \tag{33}$$

we obtain:

$$\overline{u} = \frac{A}{s^2\left[1+\theta\sqrt{\dfrac{s+M}{1+\lambda s^\alpha}}\right]}\exp\left\{-\sqrt{(s+M)(1+\lambda s^\alpha)}\,y\right\} \tag{34}$$

In order to avoid the complexity of calculating the residues and contour integrals, we apply discrete inverse Laplace transform to get the velocity and express eq. (34) as series form:

$$\overline{u} = A\sum_{l=0}^{\infty}\frac{(-\theta)^l}{l!}\sum_{k=0}^{\infty}\frac{(-y)^k}{k!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{\lambda^{\frac{k-l}{2}-n}\Gamma\left(m-\dfrac{l+k}{2}\right)\Gamma\left(n-\dfrac{k-l}{2}\right)}{\Gamma\left(-\dfrac{l+k}{2}\right)\Gamma\left(\dfrac{l-k}{2}\right)s^{\left(m-\frac{l+k}{2}\right)+\alpha\left(n-\frac{k-l}{2}\right)+2}} \tag{35}$$

Applying the discrete inverse Laplace transform, we have:

$$u = A\sum_{l=0}^{\infty}\frac{(-\theta)^l}{l!}\sum_{k=0}^{\infty}\frac{(-y)^k}{k!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{\frac{k-l}{2}-n}t^{\left(m-\frac{l+k}{2}\right)+\alpha\left(n-\frac{k-l}{2}\right)+1}}{n!}\cdot$$

$$\cdot\frac{\Gamma\left(m-\dfrac{l+k}{2}\right)\Gamma\left(n-\dfrac{k-l}{2}\right)}{\Gamma\left(-\dfrac{l+k}{2}\right)\Gamma\left(\dfrac{l-k}{2}\right)\Gamma\left[\left(m-\dfrac{l+k}{2}\right)+\alpha\left(n-\dfrac{k-l}{2}\right)+2\right]} \tag{36}$$

which is similar to the solution, eq. (25) of [24] when $\theta = M = 0$ and $\alpha = 1$.

In the same way, we obtain the solution of eq. (13) (let $\lambda_T = 0$, see eq. (16) in [25]).

$$T=\sum_{k=0}^{\infty}\frac{(-y)^k}{a^{\frac{k}{2}}k!}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{\lambda_T^{\frac{k}{2}-n}\Gamma\left(n-\dfrac{k}{2}\right)}{\Gamma\left(-\dfrac{k}{2}\right)\Gamma\left[-\dfrac{k}{2}(\gamma+1)+\gamma n\right]}\int_0^t f(t-s)s^{\frac{k}{2}(-\gamma-1)+\gamma n-1}\mathrm{d}s \tag{37}$$

Results and discussion

In this paper, the flow and heat transfer for modified Maxwell fluid with time-space fractional derivatives are studied, where the flow is due to an infinite constantly accelerating plate with slip boundary. The generalization here is a type of new fractional operators and defined in the Caputo and Riemann-Liouville sense. For values of the parameters of the fluid, the velocity field and the temperature field distributions are shown as in figs. 1-10.

For the sake of the simplicity, we take $A = 2$ in all figures.

Figures 1 and 2 show the velocity field distribution with the fractional parameters. It is seen that the smaller the α, the more slowly the velocity decays. However, one can see that an increase in material parameter β has quite the opposite effect to that of α. Meanwhile, the results also indicate the influence of the magnetic parameter, which decreases the velocity.

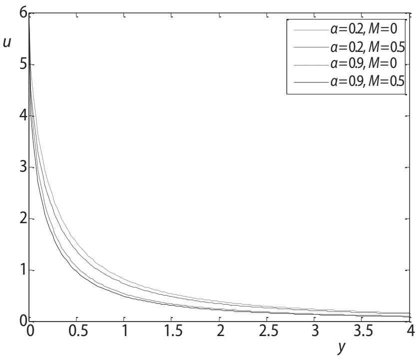

Figure 1. Velocity profiles for different values of α and M
$\lambda = 0.1, \beta = 0.5, t = 3, \theta = 0$

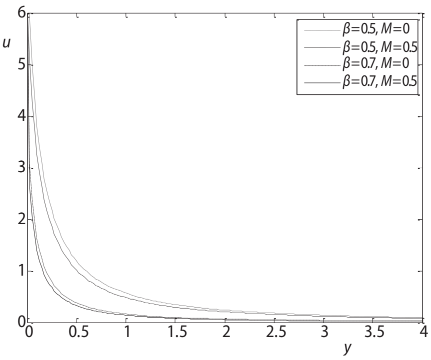

Figure 2. Velocity profiles for different values of β and M
$\lambda = 0.1, \alpha = 0.6, t = 3, \theta = 0$

Figure 3 shows the velocity field distribution with the change of relaxation parameter, the results indicate that the greater the value of λ, the more rapidly the velocity declines. Figure 4 shows the effects of slip parameter on velocity field distribution, we can see that the changes of velocity with different values of slip coefficient. The result indicates that the increasing in the slip parameter at the wall result in the decreases in velocity profiles.

Figure 5 is the velocity profile $u(y,t)$ *vs.* the time. Results indicate that with the increasing the value of t, the velocity rapidly speeds up. As seen from figs.7-9, the bigger the value of β is, the more slowly the velocity decays. However, one can see that an increase in material parameter γ (or λ_T) has quite the opposite effect to that of β .

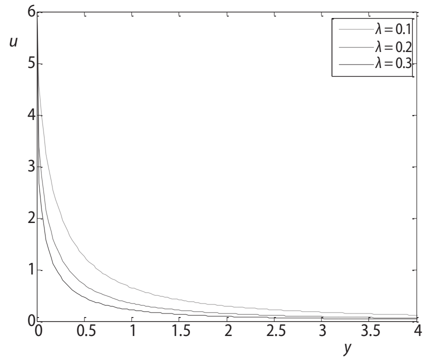

Figure 3. Velocity profiles for different values of λ
$\alpha = 0.6, \beta = 0.5, t = 3, \theta = 0, M = 0$

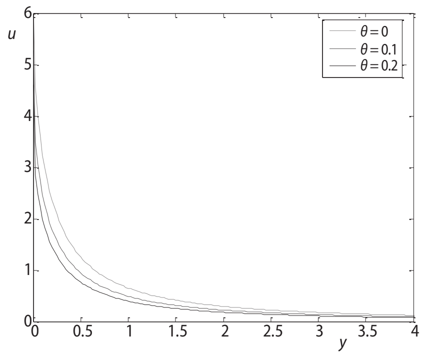

Figure 4. Velocity profiles for different values of θ
$\alpha = 0.6, \beta = 0.5, t = 3, \lambda = 0.1, M = 0$

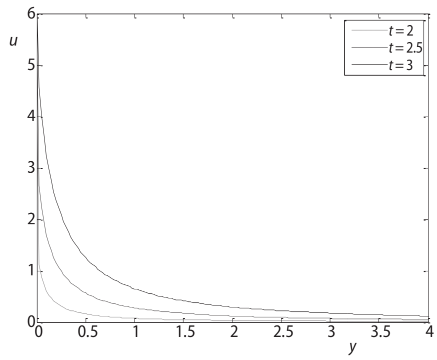

Figure 5. Velocity profiles for different values of t
$\alpha = 0.6, \beta = 0.55, \lambda = 3, \theta = 0, M = 0$)

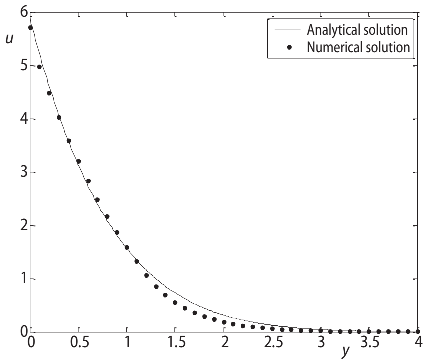

Figure 6. Velocity profiles for generalized Maxwell fluids with
$\alpha = 0.2, \beta = 1, \lambda = 0.1, M = 0.1, \theta = 0.001$

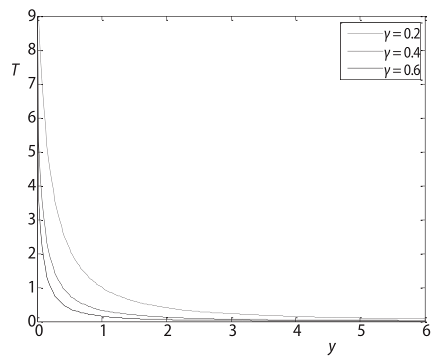

Figure 7. Temperature profiles for different values of γ
$\beta = 0.6, \lambda_T = 3, t = 3, a = 0.5, f(t) = t^2$

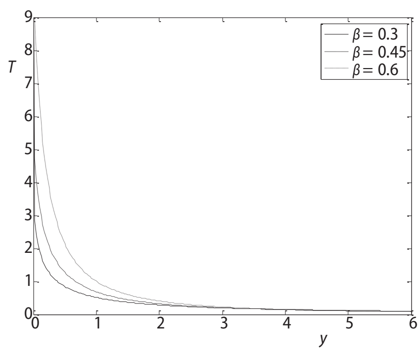

Figure 8. Temperature profiles for different values of β
$\gamma = 0.2, \lambda_T = 3, t = 3, a = 0.5, f(t) = t^2$

Figures 6 and 10 present the comparisons of numerical and exact analytical solutions for both velocity and temperature fields. The reliability and efficiency of the numerical solutions are verified by analytical results with good agreement.

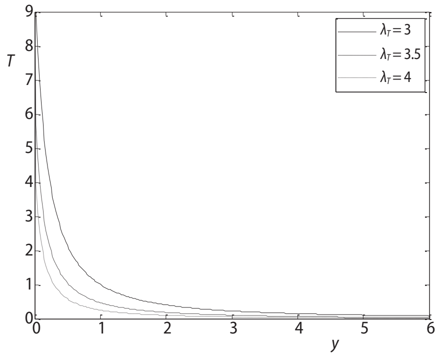

Figure 9. Temperature profiles for different values of λ_T $\beta = 0.6, \gamma = 0.2, t = 3, a = 0.5, f(t) = t^2$

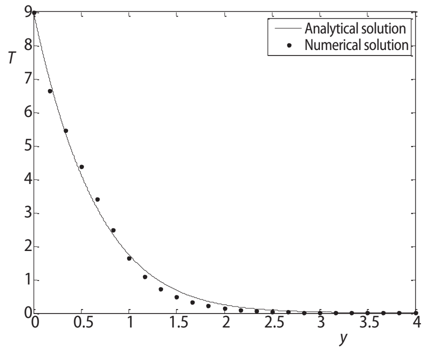

Figure 10. Temperature profiles for generalized Maxwell fluids with $\gamma = 0.5, \beta = 1, a = 10, f(t) = t^2$

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Nomenclature

- B_0 – magnetic induction, [T]
- c_p – specific heat of fluid at a constant pressure, [Jkg^{-1}K^{-1}]
- h – space step, [m]
- k_T – thermal conductivity, [Wm^{-1}K^{-1}]
- M – magnetic field intensity, [Am^{-1}]
- T_w – temperature at the wall, [K]
- T_∞ – temperature at infinity, [K]
- t – time, [s]
- y – space, [m]

Greek symbols

- α, β, γ – order of fractional derivative, [–]
- θ – slip parameter,[–]
- λ, λ_T – relaxation time, [s]
- μ – dynamic viscosity, [Nsm^{-2}]
- ν – kinematic viscosity, [m^{2}s^{-1}]
- ρ – constant density of the fluid, [kgm^{-3}]
- σ_1 – parameter for dimensional balance, [s]
- σ_2 – parameter for dimensional balance, [s$^{\beta-1}$m$^{1-\beta}$]
- σ_0 – electric conductivity, [Sm^{-1}]
- τ – time step, [s]

References

[1] Renardy, M., Stability of Creeping Flows of Maxwell Fluids, *Archive for Rational Mechenics and Analysis, 198* (2010), 2, pp. 723-733
[2] Savelev, E., Renardy, M., Control of Homogeneous Shear Flow of Multimode Maxwell Fluids, *Journal of Non-Newtonian Fluid Mechanics, 165* (2010), 3-4, pp. 136-142
[3] Karra, S., *et al.*, On Maxwell Fluids with Relaxation Time and Viscosity Depending on the Pressure, *International Journal of Non-Linear Mechanics, 46* (2011), 6, pp. 819-827
[4] Hayat, T., *et al.*, Radiation Effects on MHD Flow of Maxwell Fluid in a Channel with Porous Medium, *International Journal of Heat and Mass Transfer, 54* (2011), 4, pp. 854-862
[5] Salah, F., *et al.*, New Exact Solution for Rayleigh-Stokes Problem of Maxwell Fluid in a Porous Medium and Rotating Frame, *Results in Physics, 1* (2011), 1, pp. 9-12
[6] Podlubny, I., *Fractional Differential Equations,* Academic Press, New York, USA, 1999
[7] Nazar, M., *et al.*, Flow through an Oscillating Rectangular Duct for Generalized Maxwell Fluid with Fractional Derivatives, *Commun Nonlinear Sci Numer Simulat., 17* (2012), 8, pp. 3219-3234

[8] Jamil, M., *et al.*, New Exact Analytical Solutions for Stokes' First Problem of Maxwell Fluid with Fractional Derivative Approach, *Computers and Mathematics with Applications, 62* (2011), 3, pp. 1013-1023
[9] Fetecau, C., *et al.*, Flow of Fractional Maxwell Fluid between Coaxial Cylinders, *Archive of Applied Mechanics, 81* (2011), 8, pp. 1153-1163
[10] Yang, D., Zhu, K. Q., Start-up Flow of a Viscoelastic Fluid in a Pipe with a Fractional Maxwell's Model, *Computers and Mathematics with Applications, 60* (2010), 8, pp. 2231-2238
[11] Cao, L. L., *et al.*, Time Domain Analysis of the Fractional Order Weighted Distributed Parameter Maxwell Model, *Computers and Mathematics with Applications, 66* (2013), 5, pp. 813-823
[12] Hayat, T., *et al.*, Soret and Dufour Effects in Three-dimensional Flow of Maxwell Fluid with Chemical Reaction and Convective Condition, *International Journal of Numerical Method for Heat & Fluid Flow, 25* (2015), 1, pp. 98-120
[13] Vieru, D., *et al.*, Time-Fractional Free Convection Flow near a Vertical Plate with Newtonian Heating and Mass Diffusion, *Thermal Science, 19* (2015), Suppl. 1, pp. S85-S98
[14] Mustafa, M., *et al.*, Sakiadis Flow of Maxwell Fluid Considering Magnetic Field and Convective Boundary Conditions, *AIP Advances, 5* (2015), 2, pp. 027106
[15] Fetecau, C., *et al.*, Unsteady Flow of a Generalized Maxwell Fluid with Fractional Derivative due to a Constantly Accelerating Plate, *Computers and Mathematics with Applications, 57* (2009), 4, pp. 596-603
[16] Jamil, M., Fetecau, C., Helical Flows of Maxwell Fluid between Coaxial Cylinders with Given Shear Stresses on the Boundary, *Nonlinear Analysis: Real World Applications, 11* (2010), 5, pp. 4302-4311
[17] Laadj, T., Renardy, M., Initial Value Problems for Creeping Flow of Maxwell Fluids, *Nonlinear Analysis, 74* (2011), 11, pp. 3614-3632
[18] Liu, Q. S., *et al.*, Time Periodic Electroosmotic Flow of the Generalized Maxwell Fluids between Two Micro-Parallel Plates, *Journal of Non-Newtonian Fluid Mechanics, 166* (2011), 9-10, pp. 478-486
[19] Yuste, S. B., Weighted Average Finite Difference Methods for Fractional Diffusion Equations, *Journal of Computational Physics, 216* (2006), 1, pp. 264-274
[20] Jia, J. H., Wang, H., Fast Finite Difference Methods for Space-Fractional Diffusion Equations with Fractional Derivative Boundary, *Journal of Computational Physics, 293* (2015), July, pp. 359-369
[21] Xu, Y. F., *et al.*, Numerical and Analytical Solutions of New Generalized Fractional Diffusion Equation, *Computers and Mathematics with Applications, 66* (2013), 10, pp. 2019-2029
[22] Zheng, L. C., *et al.*, Exact Solutions for Generalized Maxwell Fluid Flow due to Oscillatory and Constantly Accelerating Plate, *Nonlinear Analysis: Real World Applications, 11* (2010), 5, pp. 3744-3751
[23] Gomez-Aguilar, J. F., *et al.*, A Physical Interpretation of Fractional Calculus in Observables Terms: Analysis of the Fractional Time Constant and the Transitory Response, *Revista Mexicana de Física, 60* (2014), 1, pp. 32-38
[24] Fetecau, C., *et al.*, A Note on the Flow Induced by a Constantly Accelerating Plate in an Oldroyd-B Fluid, *Applied Mathematical Modelling, 31* (2007), 4, pp. 647-654
[25] Fetecau, C., *et al.*, General Solutions for Magnetohydrodynamic Natural Convection Flow with Radiative Heat Transfer and Slip Condition over a Moving Plate, *Zeitschrift fuer Naturforschung A, 68a* (2013), 10-11, pp. 659-667

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