

REACTION-DIFFUSION EQUATION FOR CONTAMINANT ION FLOW USING POINT-SOURCE APPROXIMATION

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Short paper
DOI: 10.2298/TSCI1504452S

We developed a new model for fluid movement around the wellbore by introducing a Dirac function, which simplifies a water-injection well as a point source. The effect of porosity on the reaction and diffusion of contaminant ions is also taken into account in the model, making it more reliable in theoretical prediction.

Key words: contaminant ions, Dirac function, point-source, flow simulation

Introduction

In the process of water-injection operation, the generated precipitates will change the original pore structure, deteriorating the permeability property [1, 2]. Bedrikovetsky *et al.* [3] established a chemical kinetic model to predict the distribution and transportation of contaminants. This paper will introduce a point source to simplifying the model.

Establishment of percolation model by using point-source approximation

As shown in fig. 1, an injection well can be simplified as a point source. Then we introduce the Dirac delta function (impulse function) to express the point-source well:

$$\delta(x - x_1) = \begin{cases} 0, & x \neq x_1 \\ \infty, & x = x_1 \end{cases}, \quad \int_{-\infty}^{+\infty} \delta(x - x_1) dx = \int_{x_1^-}^{x_1^+} \delta(x - x_1) dx = 1 \quad (1)$$

Figure 2 shows the cylindrical co-ordinate system that was established with the point source as its origin. In this co-ordinate system, the circumferential velocity is equal to zero.

The radial velocity, V_r , can be expressed as:

$$V_r = \frac{Q}{2\pi rh} = -\frac{K(r, t)}{\mu} \frac{dp}{dr} \quad (2)$$

where Q [m^3t^{-1}] (a constant) is the flow rate of the injected fluids, $Q = R\tau$, τ – the volume of porous media, and h – the formation thickness. R [$\text{m}^3\text{t}^{-1}\text{m}^{-3}$] is the injection intensity of the volume of injected fluids per unit time and per unit volume of porous media.

The transportation continuity of the contaminant ions can be expressed as:

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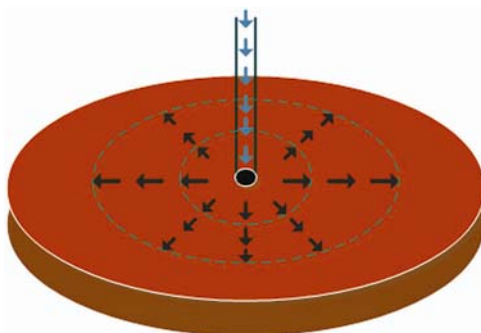


Figure 1. Diagrammatic sketch of point source well

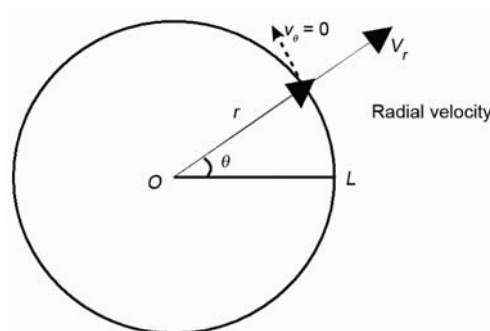


Figure 2. The cylindrical co-ordinate system

$$\phi \frac{\partial c}{\partial t} + V_r \frac{\partial c}{\partial r} - \phi D \left(\frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} \right) - W \delta(r) = 0 \quad (3)$$

where W is a term associated with the point source, $W = Rc_R$, in which c_R [molm⁻³] is the ion concentration (B^{2-}) in the injected fluids, and ϕ – the porosity.

According to eq. (3), the ion concentrations distributed in the porous media can be expressed in the equations:

$$\begin{cases} \phi \frac{\partial c_A}{\partial t} = \phi D \frac{\partial^2 c_A}{\partial r^2} + \frac{\phi D}{r} \frac{\partial c_A}{\partial r} - V_r \frac{\partial c_A}{\partial r} - K_a c_A c_B \\ \phi \frac{\partial c_B}{\partial t} = \phi D \frac{\partial^2 c_B}{\partial r^2} + \frac{\phi D}{r} \frac{\partial c_B}{\partial r} - V_r \frac{\partial c_B}{\partial r} - K_a c_A c_B + W \delta(r) \\ \phi \frac{\partial c_{AB}}{\partial t} = K_a c_A c_B, \quad V_r = \frac{Q}{2\pi r h} \end{cases} \quad (4)$$

where c_A , c_B , and c_{AB} [molm⁻³] are the ions concentration of A^{2+} , B^{2-} , and the precipitate AB , respectively.

Conclusions

We established a new model for fluid seepage by modifying the traditional model to introduce a term to accommodate the reactions between ions and to consider the influence of the porosity of the porous media. In addition, we substituted a Dirac function for the point-source well. The model can describe the distribution and transportation of contaminants, while illustrating the dynamic changes in the porosity, permeability, and pressure of a porous medium.

Acknowledgment

The work is supported by the Open Fund (Number: PLN1421) of the State Key Laboratory of Oil and Gas Reservoir Geology, Exploitation (Southwest Petroleum University), SWPU Science & Technology Fund (No. 2013XJZ029).

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