TRANSVERSE VIBRATION OF AN AXIALLY MOVING SLENDER FIBER OF VISCOELASTIC FLUID IN BUBBFIL SPINNING AND STUFFER BOX CRIMPING

by

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Transverse vibration of an axially moving slender fiber of viscoelastic fluid is studied. The governing equations are derived under the assumptions of one-dimensional steady and incompressible flow and linear Euler-Bernoulli bar. Effect of the moving velocity of the liquid fiber on natural frequencies is discussed, and the critical velocities of moving fibers are derived, below which transverse vibration is exponentially damped.

Key words: *axially moving fiber, natural frequency, mode shape, variational principle, crimped fiber, bubbfil spinning, bubble electrospinning*

Introduction

There is a large body of literature on the flow of slender viscous jets in electrospinning and bubble electrospinning and other fiber spinning [1-3]. Most of the previous work was focused on fabrication of various functional fibers. Figure 1 shows a viscous charged jet in electrospinning, a liquid fiber is extruded from a nozzle and stretched under the tension applied on the surface by the electrostatic force. The bubbfil spinning [1-3], see fig. 2, which is developed from the bubble electrospinning, is to stretch a thin polymer membrane from a broken bubble to produce nanofibers and transverse vibration of the moving jet can produce crimped nanofibers [4], its mechanism is similar to that of stuffer box crimping [5, 6], see fig. 3.

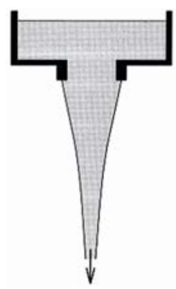

Figure 1. Viscoelastic jet in electrospinning

The transverse vibration of liquid fiber has not been studied previously, and its effect on properties of its products has been omitted. For example, the stuffer box texturing [7] benefits greatly from the transverse vibration of the liquid fiber, and its crimp characteristic corresponds to its wave number. This paper is to elucidate its mechanism for fabricating crimped fibers by transverse vibration.

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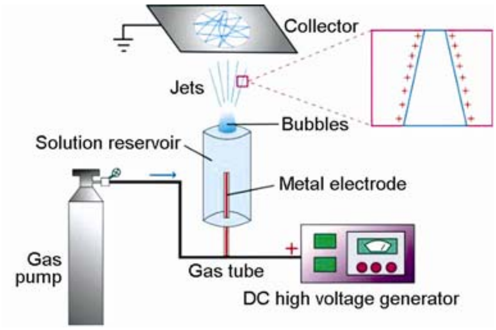

Figure 2. Viscoelastic jet in bubbfil spinning

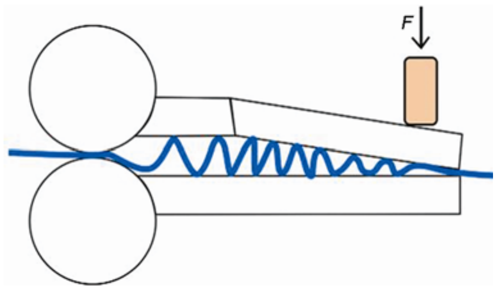

Figure 3. Crimping mechanism in the stuffer box

The stuffer box crimping is a widely used method for fabrication of crimped fibers [7]. Figure 3 reveals the process of crimp formation. Heated and softened viscoelastic fibers are fed into a chamber until they meet the wall of the upper doctor blade. A moving slender fiber of viscoelastic fluid in the stuffer box vibrates transversally, its wave-like configuration corresponds to its wave number when it is solidified.

Governing equations

The flow of slender viscoelastic jets is complex [5, 6], and some assumptions have to be made to simplify the governing equation. In this paper the following assumptions are adopted:

(1) The slender fiber of viscoelastic fluid moves axially at a constant speed (u),
(2) The flow is steady and incompressible and can be modeled by 1-D model of fluid mechanics,
(3) The fiber is subject to a constant compressive force (P),
(4) The initial fiber tension is T, and
(5) The fiber can be considered as a linear Euler-Bernoulli bar.

Under such assumptions, the force balance of liquid fiber gives [3]:

$$u\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\partial \tau}{\partial z} \tag{1}$$

where u is the velocity, ρ – the liquid density, and τ – the viscous force.

$$\tau = \mu\frac{\partial u}{\partial z} + \alpha\left(\frac{\partial u}{\partial z}\right)^n \tag{2}$$

where μ is the viscosity coefficient, and α and n are constants.

Conservation of mass gives [3]:

$$\pi r^2 \rho u = Q \tag{3}$$

where Q is the flow rate, which keeps unchanged during the spinning process.

The constant speed of the liquid fiber replies $\tau = 0$, as a result the following Bernoulli equation is obtained:

$$\frac{1}{2}u^2 + \frac{P}{\rho} = \tilde{B} \tag{4}$$

or

$$P = B - \frac{1}{2}\rho u^2 \tag{5}$$

where $\tilde{B}$ is the Bernoulli constant, and $B = \rho\tilde{B}$.

According to eq. (3), the cross-section area of the liquid fiber keeps unchanged during the spinning process. The axial compressive loading of the liquid fiber can be approximated by the fluid pressure. Figure 4 shows the transverse vibration of the moving fiber, and figs. 5 and 6 are photos of the produced crimped fibers.

The kinetic energy of the moving fiber is [5, 6]:

$$E = \frac{1}{2}\rho A \int_0^L \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} \right)^2 \mathrm{d}x \tag{6}$$

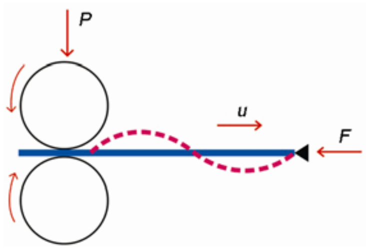

Figure 4. Axially moving liquid fiber

Figure 5. Crimped fibers by stuffer box crimping

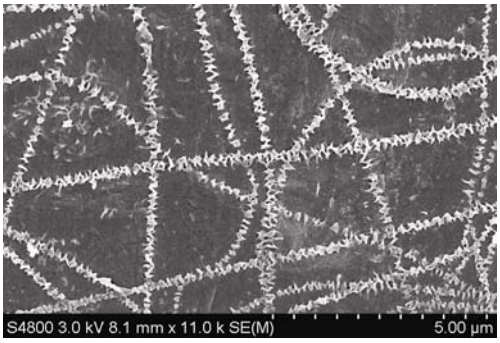

Figure 6. Crimped nanofibers by bubbfil spinning

The potential energy is:

$$U = \int_0^L A(T-P)\varepsilon_L + \frac{1}{2} EI \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \mathrm{d}x \tag{7}$$

where w is the transverse displacement, E – the modulus of elasticity, I – the moment of inertia, and A – the section area. ε_L strain is:

$$\varepsilon_L = \frac{\Delta S - \Delta x}{\Delta x} \tag{8}$$

where ΔS is the length of the curve:

$$\Delta S = \sqrt{1 + \left(\frac{\mathrm{d}w}{\mathrm{d}x} \right)^2} \Delta x \approx \left[1 + \frac{1}{2} \left(\frac{\mathrm{d}w}{\mathrm{d}x} \right)^2 \right] \Delta x \tag{9}$$

The strain is obtained as:

$$\varepsilon_L = \frac{\Delta S - \Delta x}{\Delta x} = \frac{1}{2} \left(\frac{\mathrm{d}w}{\mathrm{d}x} \right)^2 \tag{10}$$

The variational principle [8-11] for the moving fiber is:

$$J(w) = \int_0^L \left\{ \frac{1}{2}\rho A \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} \right)^2 - \left[\frac{1}{2} A(T - B + \frac{1}{2}\rho u^2) \left(\frac{\partial w}{\partial x} \right)^2 + \frac{1}{2} EI \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \right] \right\} \mathrm{d}x \tag{11}$$

Its Euler-Lagrange equation can be obtained, which reads:

$$\rho A\left(\frac{\partial^2 w}{\partial t^2}+2u\frac{\partial^2 w}{\partial t\partial x}+u^2\frac{\partial^2 w}{\partial x^2}\right)+A\left(-T+B-\frac{1}{2}\rho u^2\right)\frac{\partial^2 w}{\partial x^2}+EI\frac{\partial^4 w}{\partial x^4}=0 \tag{12}$$

or

$$\rho A\left(\frac{\partial^2 w}{\partial t^2}+2u\frac{\partial^2 w}{\partial t\partial x}+\frac{1}{2}u^2\frac{\partial^2 w}{\partial x^2}\right)+A(B-T)\frac{\partial^2 w}{\partial x^2}+EI\frac{\partial^4 w}{\partial x^4}=0 \tag{13}$$

Natural frequency and critical velocities

The solution of eq. (13) can be presented in the form:

$$w(x,t)=W(x)\cos\omega t \tag{14}$$

where W is the normal function, and ω – the natural frequency.

The differential equation for the mode shape of vibration, after using the Galerkin technology, is obtained as:

$$\rho A\left(-\omega^2+\frac{1}{2}u^2W''\right)+A(B-T)W''+EIW^{(4)}=0 \tag{15}$$

The expression of the normal function:

$$W=w_{\max}\sin\left(\frac{\pi}{L}x\right) \tag{16}$$

leads to the following fundamental frequency of vibration:

$$\omega_1=\sqrt{\left(\frac{1}{2}u^2+\tilde{B}-\frac{T}{\rho}\right)\frac{\pi^2}{L^2}+\frac{EI}{\rho A}\frac{\pi^4}{L^4}} \tag{17}$$

Condition of stability is:

$$\left(\frac{1}{2}u^2+\tilde{B}-\frac{T}{\rho}\right)\frac{\pi^2}{L^2}+\frac{EI}{\rho A}\frac{\pi^4}{L^4}=0 \tag{18}$$

The first critical velocity is:

$$u_{1cr}=\sqrt{2\left(\frac{T}{\rho}-\tilde{B}-\frac{EI}{\rho A}\frac{\pi^2}{L^2}\right)} \tag{19}$$

above which the fundamental transverse vibration occurs.

The mode shape can also be expressed in the form:

$$W=w_{\max}\sin\left(\frac{2\pi}{L}x\right) \tag{20}$$

which corresponds to the second frequency of vibration, and leads to the following expressions for second natural frequency and second critical velocity:

$$\omega_2=\sqrt{\left(\frac{1}{2}u^2+\tilde{B}-\frac{T}{\rho}\right)\frac{4\pi^2}{L^2}+\frac{EI}{\rho A}\frac{16\pi^4}{L^4}} \tag{21}$$

$$u_{1cr} = \sqrt{2\left(\frac{T}{\rho} - \tilde{B} - \frac{EI}{\rho A}\frac{4\pi^2}{L^2}\right)} \quad (22)$$

Conclusion

The velocity of the moving liquid fiber affects greatly the natural frequencies of transverse vibration. If transverse vibration occurs just before fiber's solidification, a crimped fiber can be obtained, which can remarkably improve surface-to-volume ratio. If a smooth fiber is to be fabricated, transverse vibration should be completely avoided, this requires that the velocity of the moving liquid fiber is less than the critical velocity. In transportation of fibers/yarn/fabrics/woven, transverse vibration should also be shunned. It should also be emphasized that the crimped fibers are discontinuous.

Acknowledgments

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References

[1] Chen, R. X., *et al.,* Mini-Review on Bubbfil Spinning Process for Mass-Production of Nanofibers, *Materia, 19* (2014), 4, pp. 325-343
[2] Chen, R. X., *et al.,* Bubbfil Spinning for Mass-Production of Nanofibers, *Thermal Science, 19* (2014), 5, pp. 1718-1719
[3] He, J.-H., *et al.,* Review on Fiber Morphology Obtained by Bubble Electrospinning and Blown Bubble Spinning, *Thermal Science, 16* (2012) 5, pp. 1263-1279
[4] Chen, R. X., *et al.,* Mechanism of Nanofiber Crimp, *Thermal Science, 17* (2013), 5, pp. 1473-1477
[5] Huang, J. X., *et al.,* Effect of Temperature on Nonlinear Dynamical Property of Stuffer Box Crimping and Bubble Electrospinning, *Thermal Science, 18* (2014), 3 pp. 1049-1053
[6] Huang, H., *et al.,* A Mathematical Model for an Axially Moving Slender Fiber of Viscoelastic Fluid: Part 1 Fabrication Of Crimped Fiber, *SYLWAN, 158* (2014), 5, pp. 285-293
[7] Singh, R. K., Vohra, J. N., Study of Process Mechanics and Yarn Characteristics Using a Fabricated Stuffer-Box Crimper, *Text. Res. J., 46* (1976), Mar., pp. 164-170
[8] Li, Z. B., Liu, J., Variational Formulations for Soliton Equations Arising in Water Transport in Porous Soils, *Thermal Science, 17* (2013), 5, pp. 1483-1485
[9] Fei, D. D., *et al*., A Short Remark on He-Lee Variational Principle for Heat Conduction, *Thermal Science, 17* (2013), 5, pp. 1561-1563
[10] Li, X. W., *et al*., On the Semi-Inverse Method and Variational Principle, *Thermal Science, 17* (2013), 5, pp. 1565-1568
[11] Washizu, K., *Variational Methods in Elasticity and Plasticity,* Pergamon Press, Oxford, UK, 1982

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