

BIOCONVECTION HEAT TRANSFER OF A NANOFLUID OVER A STRETCHING SHEET WITH VELOCITY SLIP AND TEMPERATURE JUMP

by

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Original scientific paper

<https://doi.org/10.2298/TSCI150424128S>

This paper presents an investigation for bioconvection heat transfer of a nanofluid containing gyrotactic microorganisms over a stretching sheet, in which the effects of radiation, velocity slip, and temperature jump are taken into account. The non-linear governing equations are reduced into four ordinary differential equations by similarity transformations and solved by homotopy analysis method, which is verified with numerical results in good agree. Results indicate that the density of motile microorganisms and gyrotactic microorganisms increase with bioconvection Rayleigh number, while decrease with increasing in bioconvection Peclet number and bioconvection Lewis number. It is also found that the Nusselt number, Sherwood number, and gyrotactic microorganisms density depend strongly on the buoyancy, nanofluids, and bioconvection parameters.

Key words: *nanofluid, gyrotactic microorganisms, bioconvection, velocity slip, temperature jump*

Introduction

As a new generation of highly efficient heat transfer fluid, nanofluids have been extensively investigated by many researchers. The term *nanofluids* was coined by Choi [1] at the ASME Winter Annual Meeting, which refers to a liquid containing a dispersion of submicronic solid particles (nanoparticles) with typical length on the order of 1-50 nm. Nanofluids have the higher heat conductivity efficiency than pure fluid due to a volume fraction (usually < 5%) of metal nanoparticles. Heat and mass transfer of nanofluids have been widely investigated.

Nazar *et al.* [2] studied the unsteady boundary-layer flow of a nanofluid over a stretching sheet caused by an impulsive motion or a suddenly stretched surface using the Keller-box method. Buongiorno [3] developed a new correlation explanation for convective heat transport of nanofluids considering the Brownian diffusion. Chamkha and Ismael [4] considered the steady-conjugate natural convection heat transfer of three types of nanofluids in a square porous cavity which was heated by a triangular solid wall under a wide range of considered parameter. Xu *et al.* [5] studied the mixed convection flow of nanofluids caused by both the external pressure and the buoyancy force in a vertical channel, the effects of the Prandtl number and other parameters were analyzed.

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In [6] the effect of a convective surface on the heat transfer characteristics of nanofluids over a static or moving wedge in the presence of thermal radiation was investigated with three types of nanoparticles considered. Alam and Hossain [7] considered the effects of viscous dissipation and Joule heating on the heat and mass transfer of a 2-D steady MHD forced convection flow of nanofluids over a non-linear stretching sheet and studied numerically.

Recently, Kuznetsov and Nield [8] analytically studied the natural convective boundary-layer flow of nanofluids past a vertical plate. Unlike the commonly employed constant conditions, a convective heating boundary condition was used in study Makinde and Aziz [9]. Dulal [10] numerically studied the flow and heat transfer of an incompressible viscous fluid past an unsteady stretching permeable sheet. Anbuechzian *et al.* [11] studied the flow of nanofluids caused by buoyancy along a vertical plate in a porous medium. Mushtaq *et al.* [12] investigated radiation effects to 2-D stagnation-point flow of viscous nanofluid due to solar energy. Malvandi *et al.* [13] numerically simulated unsteady stagnation point flow of nanofluids with a slip boundary condition, the results show dual solution would exist when the unsteadiness parameter was negative. The parameters of thermophoresis, Brownian motion and the velocity slip played a vital role in the transport process of various nanofluids. Behseresht *et al.* [14] displayed that the heat transfer associated with nanoparticles migration was negligible compared with heat conduction and convection on the natural convection heat transfer of nanofluids in a saturated porous medium. Noghrehabadi *et al.* [15] pointed that the Reynolds number for the temperature profile could be significantly affected by Prandtl number. Rahman *et al.* [16] numerically investigated the steady boundary-layer flow and heat transfer of nanofluids past a permeable exponentially shrinking/stretching surface with second order slip velocity.

Bioconvection is induced by swimming of motile microorganisms, leading the increase of the density of the base fluid. Few studies exist on nanofluids containing gyrotactic microorganisms over a convectively heated stretching sheet. Kuznetsov [17] studied both non-oscillatory and oscillatory nanofluid biothermal convection in a horizontal layer of finite depth and analyzed the dependence of the thermal Rayleigh number on the nanoparticle Rayleigh number and the bioconvection Rayleigh number. Khan and Makinde [18] investigated MHD flow of nanofluids with heat and mass transfer along a vertical stretching sheet in the presence of motile gyrotactic microorganisms. Xu and Pop [19] obtained a more physically realistic result using a passively controlled nanofluid model by an analysis of bioconvection flow of nanofluids in a horizontal channel. In a recent paper, Khan *et al.* [20] investigated the effects of both Navier slip and magnetic field on boundary-layer flow of nanofluids containing gyrotactic microorganisms over a vertical plate. Their results show that the bioconvection parameters tend to reduce the local concentration of motile microorganisms. Xu and Pop [21] presented an analysis on the mixed convection flow of a nanofluid over a stretching surface with uniform free stream containing nanoparticles and gyrotactic microorganisms.

In the present paper, we investigate the bioconvection and radiation heat transfer of a nanofluid containing gyrotactic microorganisms over a stretching sheet, in which the effects of Brownian motion and thermophoresis are considered according to Rosseland's approximation [22] as well as velocity slip and temperature jump. We obtain the analytical solutions by using the homotopy analysis method (HAM). The HAM as introduced by Liao [23-27], has been adopted to solve the highly non-linear and coupled differential equations. Many studies have confirmed the effectiveness of this method by contrast.

Mathematical formulation

We consider in this paper the steady incompressible viscous fluid containing gyrotactic microorganisms near a stagnation-point. In the co-ordinate system, the origin is a fluid stagnation

point and the x-axis is the flat direction, that is $y = 0$. The flow region is confined to $y > 0$ and the stretching surface temperature, nanoparticle volume fraction, and microorganisms fraction are defined to have constant values T_w , C_w , and n_w , respectively, while at a large value of y , temperature, nanoparticle volume fraction, and microorganisms fraction have constant values T_∞ , C_∞ , and n_∞ , respectively. Flow induced by bioconvection only take place in a dilute suspension of nanoparticles so that the nanoparticle volume fraction, C , is lower than 0.01. The boundary-layer governing equations considering thermal radiation and bioconvection are given:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} - \mu \frac{u}{k} + \frac{(1 - C_\infty) \rho_{f\infty} \beta (T - T_\infty) \mathbf{g}}{\rho_f} - \frac{[(\rho_p - \rho_{f\infty})(C - C_\infty) + (n - n_\infty) \gamma (\rho_{m\infty} - \rho_f)] \mathbf{g}}{\rho_f} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] - \frac{\partial q_r}{\partial y} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} = D_n \frac{\partial^2 n}{\partial y^2} - \frac{b W_c}{C_w - C_\infty} \frac{\partial \left(\frac{n \partial C}{\partial y} \right)}{\partial y} \tag{5}$$

The slip boundary conditions for the governing equations are:

$$u = ax + \frac{2 - \sigma_v}{\sigma_v} \lambda_0 \frac{\partial u}{\partial y} \Big|_{y=0}, \quad v = 0, \tag{6}$$

$$T = T_w + \frac{2 - \sigma_T}{\sigma_T} \frac{2r}{r + 1} \frac{\lambda_0}{\text{Pr}} \frac{\partial T}{\partial y} \Big|_{y=0}, \quad C = C_w, \quad n = n_w, \quad y = 0,$$

$$u = 0, \quad v = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad n \rightarrow n_\infty \quad \text{when} \quad y \rightarrow \infty$$

where a is a positive constant, σ_v and σ_T are the tangential momentum accommodation coefficient and the thermal accommodation coefficient, respectively, [22], λ_0 is the molecular mean free path, and r – the specific heat ratio [28]. The T is the temperature, C and n are the densities of nanoparticle and motile microorganisms, respectively, $\tau = (\rho c_p)_p / (\rho c_p)_f$ is the ratio of effective heat capacity of the nanoparticle material to the heat capacity of the fluid, ρ_p – the density of nanoparticles, $\rho_{m\infty}$ – the microorganism density, ρ_f – the base fluid density, γ – the average volume of a microorganism, D_B – the Brownian diffusion coefficient, D_T – the thermophoretic diffusion coefficient, D_n – the diffusivity of microorganisms, b – the chemotax constant, and W_c – the maximum cell swim-ming speed. The $\partial q_r / \partial y = (-16 \sigma_q T_\infty^3 / 3k) \partial^2 T / \partial y^2$ is obtained according to Rosseland's approximation and the Taylor expansion, σ_q and k are Stefan-Boltzmann constant and absorption coefficient, respectively. Now eq. (3) reduces to:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma_q T_\infty^3}{3\rho c_p k} \right) \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (7)$$

The physical flow model and co-ordinate system is shown in fig. 1.

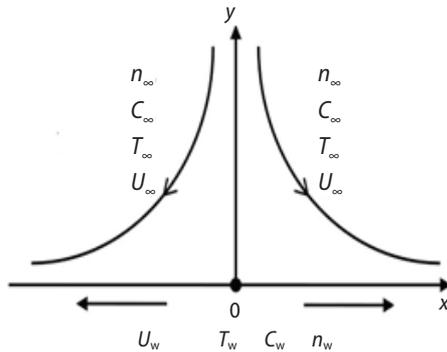


Figure 1. Flow configuration and co-ordinate system

Introducing the stream function and similarity transformation:

$$\eta = \frac{y \text{Ra}_x^{1/4}}{x}, \quad \psi = \bar{\alpha} \text{Ra}_x^{1/4} f(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$

$$\chi(\eta) = \frac{n - n_\infty}{n_w - n_\infty}, \quad \text{Ra}_x = \frac{(1 - C_\infty)\beta g \Delta T_w x^3}{\rho_f \bar{\alpha} \mu}$$

where $\bar{\alpha} = \alpha + 16\sigma_q T_\infty^3 / 3\rho c_p k$, ψ – the stream function satisfying $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, η – the

similarity variable, f – the dimensionless stream function, θ – the dimensionless temperature function, and ϕ and χ are dimensionless nanoparticle fraction and microorganisms fraction functions, respectively. Equations (1)-(5) and (7) can be reduced into the following similarity equations:

$$f''' + \frac{3}{4} \left(\frac{1}{\text{Pr}} + N \right) f f'' + \theta - Nr\phi - Rb\chi - Af' = 0 \quad (8)$$

$$\theta'' + \frac{3}{4} f \theta' + Nb\theta'\phi' + Nt(\theta')^2 = 0 \quad (9)$$

$$\phi'' + \frac{3}{4} \text{Le} f \phi' + Nt/Nb\theta'' = 0 \quad (10)$$

$$\chi'' + \frac{3}{4} Lb f \chi' - \text{Pe}(\phi'\chi' + \phi''\chi + \sigma\phi'') = 0 \quad (11)$$

and the reduced boundary conditions are:

$$f'(0) = \lambda_1 + \lambda_2 f''(0), \quad f(0) = 0, \quad \theta(0) = 1 + \delta\theta'(0), \quad \phi(0) = \chi(0) = 1 \quad (12)$$

$$f' = 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0, \quad \chi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (13)$$

where $\lambda_1 = ax^2 \text{Ra}_x^{-1/2} / (\alpha + N)$ the stretching velocity parameter, $\lambda_2 = \lambda_0(2 - \sigma_v)\text{Ra}_x^{1/4} / \sigma_v x$ – the velocity slip parameter, $\delta = 2r\lambda_0(2 - \sigma_r)\text{Ra}_x^{1/4} / [\sigma_r x \text{Pr}(r + 1)]$ – the temperature jump parameter, $\text{Pr} = \mu/\alpha$ – the Prandtl number, $N = 16\sigma_q T_\infty^3 / 3\mu\rho c_p k$ – the thermal radiation parameter, $Nr = (\rho_p - \rho_{f\infty})\Delta C_w / \rho_{f\infty}\beta(1 - C_\infty)\Delta T_w$ – the buoyancy ratio parameter for $\Delta C_w = C_w - C_\infty$, $\Delta T_w = T_w - T_\infty$, $Rb = \gamma(\rho_{m\infty} - \rho_f)\Delta n_w / \rho_{f\infty}\beta\Delta T_w(1 - C_\infty)$ – the bioconvection Rayleigh number ($\Delta n_w = n_w - n_\infty$), $A = x^2 \text{Ra}_x^{-1/2} / k$ – the permeability parameter, $Nb = \tau D_B \Delta C_w \text{Pr} / \mu$ and $Nt = \tau D_T \Delta T_w \text{Pr} / T_\infty \mu$ the Brownian motion parameter and the thermophoresis parameter, respectively, $\text{Le} = \mu / \text{Pr} D_B$ – the Lewis number, $Lb = \mu / \text{Pr} D_n$ – the bioconvection Lewis number, $\text{Pe} = bW_c / D_n$ – the bioconvection Peclet number, and $\sigma = n_\infty / \Delta n_w$ – the bioconvection constant.

The reduced local Nusselt number, local Sherwood number, and local density number of the motile microorganisms may be found in terms of the dimensionless temperature at the sheet surface, concentration of nanoparticle, and microorganisms at the sheet surface, respectively.

$$Nur = Ra_x^{1/4} Nu_x = -\theta'(0) \quad (14)$$

$$Shr = Ra_x^{1/4} Sh_x = -\phi'(0) \quad (15)$$

$$Nnr = Ra_x^{1/4} Nn_x = -\chi'(0) \quad (16)$$

The HAM solution

The coupled non-linear boundary value problems eqs. (8)-(13) are solved by using the HAM [23-27]. The initial approximations are selected:

$$f_0(\eta) = \frac{\lambda_1 [1 - \exp(-\eta)]}{1 + \lambda_2}, \quad \theta_0(\eta) = \frac{\exp(-\eta)}{1 + \delta} \quad (17)$$

$$\phi_0(\eta) = \exp(-\eta), \quad \chi_0(\eta) = \exp(-\eta) \quad (18)$$

The auxiliary linear operators are chosen as follows, respectively:

$$\ell_1[f] = f''' + f'', \quad \ell_2[\theta] = \theta'' + \theta', \quad \ell_3[\phi] = \phi'' + \phi', \quad \ell_4[\chi] = \chi'' + \chi' \quad (19)$$

The non-linear operators are given by:

$$N_1[f(\eta; q), \theta(\eta; q), \phi(\eta; q), \chi(\eta; q)] = \frac{\partial^3 f(\eta; q)}{\partial \eta^3} + \frac{3}{4} \left(\frac{1}{Pr} + N \right) f(\eta; q) \frac{\partial^2 f(\eta; q)}{\partial \eta^2} + \theta(\eta; q) - Nr\phi(\eta; q) - Rb\chi(\eta; q) - A \frac{\partial f(\eta; q)}{\partial \eta} \quad (20)$$

$$N_2[f(\eta; q), \theta(\eta; q), \phi(\eta; q), \chi(\eta; q)] = \frac{\partial^2 \theta(\eta; q)}{\partial \eta^2} + \frac{3}{4} f(\eta; q) \frac{\partial \theta(\eta; q)}{\partial \eta} + Nb \frac{\partial \theta(\eta; q)}{\partial \eta} \frac{\partial \phi(\eta; q)}{\partial \eta} + Nt \left[\frac{\partial \theta(\eta; q)}{\partial \eta} \right]^2 \quad (21)$$

$$N_3[f(\eta; q), \theta(\eta; q), \phi(\eta; q), \chi(\eta; q)] = \frac{\partial^2 \phi(\eta; q)}{\partial \eta^2} + \frac{3}{4} Le f(\eta; q) \frac{\partial \phi(\eta; q)}{\partial \eta} + \frac{Nt}{Nb} \frac{\partial^2 \theta(\eta; q)}{\partial \eta^2} \quad (22)$$

$$N_4[f(\eta; q), \theta(\eta; q), \phi(\eta; q), \chi(\eta; q)] = \frac{\partial^2 \chi(\eta; q)}{\partial \eta^2} + \frac{3}{4} Lbf(\eta; q) \frac{\partial \chi(\eta; q)}{\partial \eta} - Pe\sigma \frac{\partial^2 \phi(\eta; q)}{\partial \eta^2} - Pe \left[\frac{\partial \phi(\eta; q)}{\partial \eta} \frac{\partial \chi(\eta; q)}{\partial \eta} + \frac{\partial^2 \phi(\eta; q)}{\partial \eta^2} \chi(\eta; q) \right] \quad (23)$$

with the boundary conditions:

$$\left. \frac{\partial f(\eta; q)}{\partial \eta} \right|_{\eta=0} = \lambda_1 + \lambda_2 \left. \frac{\partial f^2(\eta; q)}{\partial \eta^2} \right|_{\eta=0}, \quad f(\eta; q)|_{\eta=0} = 0$$

$$\theta(\eta; q)|_{\eta=0} = 1 + \delta \left. \frac{\partial \theta(\eta; q)}{\partial \eta} \right|_{\eta=0}, \quad \phi(\eta; q)|_{\eta=0} = 1, \quad \chi(\eta; q)|_{\eta=0} = 1 \quad (24)$$

$$\left. \frac{\partial f(\eta; q)}{\partial \eta} \right|_{\eta=\infty} = 0, \quad \theta(\eta; q)|_{\eta=\infty} = 0, \quad \phi(\eta; q)|_{\eta=\infty} = 0, \quad \chi(\eta; q)|_{\eta=\infty} = 0 \quad (25)$$

where $q \in [0, 1]$ is the embedding parameter. The auxiliary functions are chosen:

$$H_f(\eta) = H_\theta(\eta) = H_\phi(\eta) = H_\chi(\eta) = 1 \quad (26)$$

Results and discussion

Liao [23-27] pointed out that the auxiliary parameter, h , played a vital role in the convergence of the HAM solutions. By means of h -curves of $f''(0)$, $\theta'(0)$, $\phi'(0)$, and $\chi'(0)$ in fig. 2 obtained by the 12th approximation for $Pr = 6$, $Le = 5$, $N = 2$, $\lambda_1 = 1$, $Pe = Nb = Nt = 0.5$, $Nr = Lb = \delta = 0.5$, $\lambda_2 = \sigma = 0.2$, $Rb = 0.1$, and $A = 0.01$, it is straightforward to choose a proper value of h to ensure the convergence of the solution series.

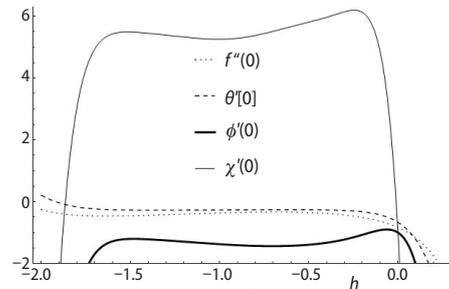


Figure 2. The h -curves of $f''(0)$, $\theta'(0)$, $\phi'(0)$, and $\chi'(0)$ of the HAM solution

Unless special indicated, the values of the parameters in this paper are used as the previous values. The reliability of analytical results is verified with numerical solutions obtained by finite difference method using MAPLE 14.0. The asymptotic boundary conditions at $\eta \rightarrow \infty$ were replaced by those at $\eta = 6$.

For illustrations of the results, solutions are plotted for special parameters. In order to validate the present results, a comparison of numerical solutions with the analytical results obtained by HAM is presented in figs. 3 and 4. The profiles indicate that the two results are in good agreement.

The effects of velocity slip λ_2 on the dimensionless velocity and the profiles of temperature for different values of temperature jump parameter δ are shown. The velocity value decreases with the velocity slip parameter and the rising in temperature jump parameter leads to the decrease of the surface temperature and thickness of the thermal boundary-layer.

In fig. 5, the effects of stretching velocity parameter λ_1 on the dimensionless velocity of nanofluids are shown. As the stretching velocity increases, the velocity value decreases. The profiles of temperature distribution for different values of thermal radiation parameter N for $Le = 1$, $Nb = 0.2$ and $Nt = 0.1$ are shown in fig. 6. It indicates that the rising in thermal radiation parameter leads to the increase of the surface temperature and thickness of the thermal boundary-layer.

The variation of volume fraction of nanoparticles with transverse distance in the concentration boundary-layer is shown in fig. 7 for different values of Lewis number. It is found that an increase in Lewis number results in reduction of the volume fraction of nanoparticles and concentration boundary-layer thickness. This is because Brownian motion coefficient decreases with increasing transverse distance and as a result the rescaled nanoparticle volume fraction decreases rapidly for large Lewis number.

Figure 8 illustrates the effects of the bioconvection Peclet number and the bioconvection constant, σ , on the density of motile microorganisms in nanofluids. It can be seen that the motile microorganism boundary-layer thickness decreases with the increasing in σ and Peclet number which is like bioconvection Lewis number.

The effects of bioconvection Lewis number, Lb , on the density of motile microorganisms in nanofluids are showed in fig. 9. The density of motile microorganisms decreases as Lb

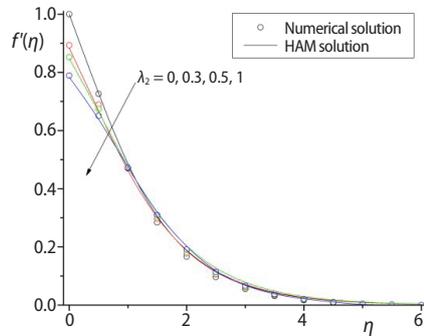


Figure 3. Effects of velocity slip parameter, λ_2 , on velocity

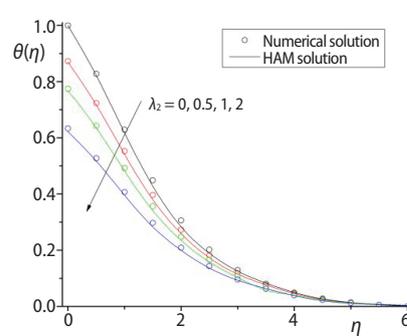


Figure 4. Effects of temperature jump parameter, δ , on temperature

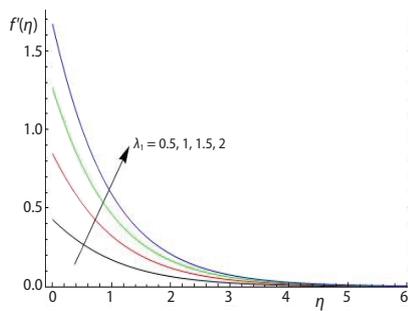


Figure 5. Effects of the stretching velocity parameter, λ_1 , on velocity

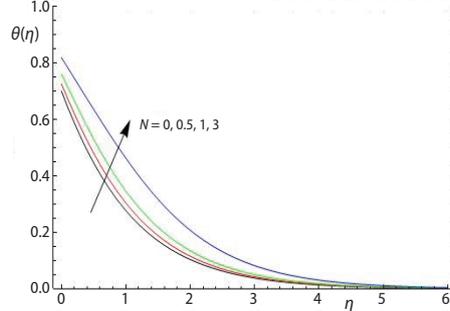


Figure 6. Effects of the thermal radiation parameter, N , on temperature

increases. Simultaneously, the motile microorganism boundary-layer thickness decreases. Like the bioconvection Peclet number, the bioconvection Lewis number plays the same role as regular Lewis number.

The variation of local Nusselt number with different velocity slip parameters and bioconvection Rayleigh numbers for $Nb = Lb = 0.1$ is shown in fig. 10. It shows that Nusselt number decreases with an increase in the thermophoresis parameter, Nt . This is due to that thermophoresis increases the temperature in boundary-layer, leading to a rise in the thermal boundary-layer thickness.

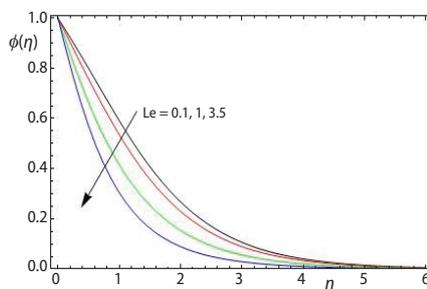


Figure 7. Effects of Lewis number on volume fraction of nanoparticles profiles

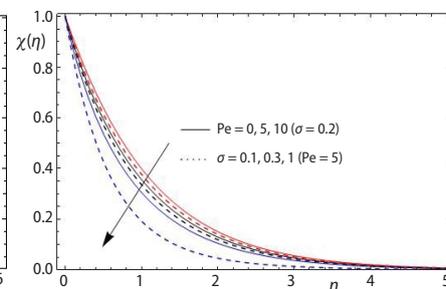


Figure 8. Effects of Peclet number and σ on volume fraction of gyrotactic microorganism

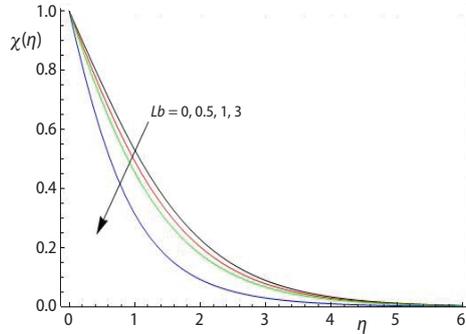


Figure 9. Effect of Lb on volume fraction of gyrotactic microorganism

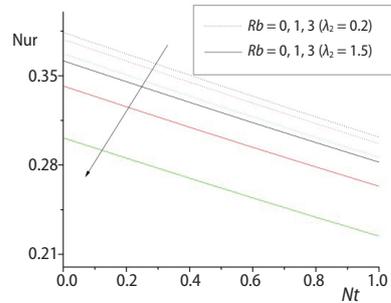


Figure 10. Effects of λ_2 and Rb on Nusselt number

Figure 11 displays the variation of the reduced Sherwood numbers for different values of Lewis number, thermophoresis parameter, and bioconvection Peclet number for $Rb = \lambda_2 = 0.5$ and $Nb = Lb = 0.1$. As expected, the Sherwood number increases with Lewis number while decreases with increasing thermophoresis parameter, Nt . This is due to the fact that the nanoparticle volume fraction increases with this parameter including the concentration boundary-layer, which can be attributed to the fact that when the Lewis number and bioconvection Peclet number are high, the nanoparticle concentration is low. Thus mass transfer from the plate to the fluid since the concentration at the plate surface is higher than that of the fluid.

It can be seen from fig. 12 that the gyrotactic microorganisms density number, Nnr , increases with the bioconvection Peclet number, bioconvection constant, σ , parameter and bioconvection Lewis number Lb for $\lambda_2 = 0.5$.

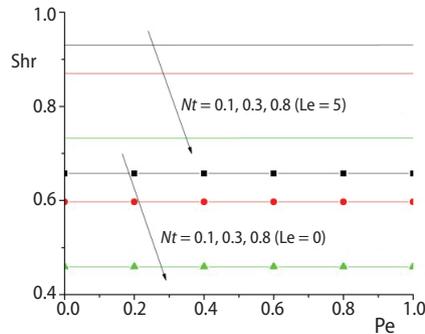


Figure 11. Effects of Lewis number and Nt of Sherwood number

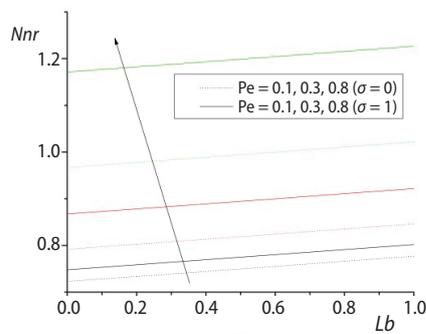


Figure 12. Effects of Peclet number and σ on density number of the motile microorganisms

Conclusions

This paper presents an investigation for bioconvection heat transfer of a nanofluid containing gyrotactic microorganisms over a stretching sheet, in which the effects of radiation, velocity slip and temperature jump are taken into account. The main results can be classified as follows.

- Velocity profiles decrease with the velocity slip parameter and the stretching velocity parameter.
- Temperature decreases with the temperature jump parameter and increases with the thermal radiation parameter inside thermal boundary-layer.

- Density of nanoparticles decreases with increasing Lewis number inside the boundary-layer.
- Density of motile microorganisms decreases with increasing bioconvection Peclet number, bioconvection constant and bioconvection Lewis number inside the boundary-layer.
- Nusselt number decreases with an increase in slip parameter, bioconvection Rayleigh number, and thermophoresis parameter. Sherwood number increases with Lewis number, whereas decreases with thermophoresis parameter. The gyrotactic microorganisms density number increases with bioconvection Peclet number, bioconvection constant parameter and bioconvection Lewis number.

Acknowledgment

The work is supported by the National Natural Science Foundations of China (Nos.51276014, 11772046).

Nomenclature

A	– permeability parameter, [–]	x, y	– streamwise and cross-stream co-ordinate, [m]
a	– positive constant, [s ⁻¹]	W_c	– maximum cell swim-ming speed, [ms ⁻¹]
b	– chemotax constant, [m]	<i>Greek symbols</i>	
C	– nanoparticle fraction, [–]	α	– effective thermal diffusivity, [m ² s ⁻¹]
C_w, C_∞	– nanoparticle volume fraction at stretching surface and at y infinity, [–]	β	– volumetric expansion coefficient, [–]
D_B	– Brownian diffusion coefficient, [–]	γ	– average volume of a microorganism, [m ³]
D_n	– diffusivity of microorganisms, [–]	δ	– temperature jump parameter, [–]
D_T	– thermophoretic diffusion coefficient, [–]	η	– similarity variable, [–]
g	– gravitational field, [ms ⁻²]	λ_1	– stretching velocity parameter, [–]
k	– absorption coefficient, [–]	λ_2	– velocity slip parameter, [–]
Lb	– bioconvection Lewis number, [–]	μ	– dynamic viscosity, [Pas]
Le	– Lewis number, [–]	ρ	– density, [kgm ⁻³]
N	– thermal radiation parameter, [–]	ρc_p	– heat capacity, [Jm ⁻³ K ⁻¹]
Nb	– Brownian motion parameter, [–]	σ	– bioconvection constant, [–]
Nr	– buoyancy ratio parameter, [–]	σ_q	– Stefan–Boltzmann constant, [–]
Nt	– thermophoresis parameter, [–]	σ_T	– thermal accommodation coefficient, [–]
n	– microorganisms fraction, [–]	σ_v	– tangential momentum accommodation coefficient, [–]
n_w, n_∞	– microorganisms fraction at stretching surface and at y infinity, [–]	ψ	– stream function, [–]
Pe	– bioconvection Peclet number, [–]	<i>Subscripts</i>	
Pr	– Prandtl number, [–]	w	– surface
Ra _x	– local Rayleigh number, [–]	∞	– large value of y
T	– temperature, [K]	f	– nanofluid
T_w, T_∞	– temperature at stretching surface and at y infinity, [K]	p	– nanoparticle
u, v	– fluid velocity component in x- and y-direction, [ms ⁻¹]		

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