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APPROXIMATE EXPRESSIONS FOR THE LOGARITHMIC MEAN VOID FRACTION

by

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Short paper
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The logarithmic mean void fraction was introduced in literature by El Hajal et al. In the present study, approximate expressions for the logarithmic mean void fraction will be presented because the original formula for the computation of the logarithmic mean void fraction in finite precision floating-point arithmetic may suffer from serious round-off problems when both differences, ε_h and ε_{ra}, are very close to each other. This situation corresponds to very low values or very high values of mass quality. The analogy between the logarithmic mean temperature difference in heat exchangers and the logarithmic mean void fraction in two-phase flow will be used. These approximations can be applied in the computational studies.

Key words: *approximate expressions, logarithmic mean void fraction, analogy*

The void fraction (ε) is defined as the ratio of the pipe cross-sectional area occupied by the gas phase to the pipe cross-sectional area. In two-phase flow systems, the void fraction is a necessary parameter in design and operation. There are many methods to determine the void fraction. In 2003, El Hajal *et al*. [1] introduced a newly defined logarithmic mean void fraction ($LM\varepsilon$) method for calculation of vapor void fractions spanning from low pressures up to pressures near the critical point. The researchers defined $LM\varepsilon$ as follows:

$$LM\varepsilon = \frac{\varepsilon_{\mathrm{h}} - \varepsilon_{\mathrm{ra}}}{\ln\left(\dfrac{\varepsilon_{\mathrm{h}}}{\varepsilon_{\mathrm{ra}}}\right)} \tag{1}$$

where the void fraction based on homogeneous model (ε_{h}) is calculated as follows:

$$\varepsilon_{\mathrm{h}} = \frac{1}{1 + \left(\dfrac{1-x}{x}\right)\left(\dfrac{\rho_g}{\rho_l}\right)} \tag{2}$$

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and the Steiner [2] horizontal tube version of the vertical tube expression of Rouhani-Axelsson [3] gives the void fraction ($\varepsilon_{\rm ra}$) as follows:

$$\varepsilon_{\rm ra} = \frac{x}{\rho_g}\left[[1+0.12(1-x)]\left(\frac{x}{\rho_x}+\frac{1-x}{\rho_l}\right)+\frac{1.18(1-x)[g\sigma(\rho_l-\rho_g)]^{0.25}}{G\rho_l^{0.5}}\right]^{-1} \tag{3}$$

Thome *et al.* [4] used the above definition of $LM\varepsilon$, eq. (1), in developing a new general flow pattern/flow structure based heat transfer model for condensation inside horizontal, plain tubes based on simplified flow structures of the flow regimes to cover the range from low to high reduced pressures.

It is well known that the computation of the logarithmic mean in finite precision floating-point arithmetic may suffer from serious round-off problems when both differences are very close to each other [5]. To avoid this problem, the current study presents approximate expressions for the $LM\varepsilon$ using the analogy method. The analogy between heat transfer and two-phase flow was used before in literature. For example, Awad and Muzychka [6] proposed new definitions of two-phase viscosity using an analogy between thermal conductivity of porous media and viscosity in two-phase flow.

In the current study, the analogy between the logarithmic mean temperature difference ($\Delta T_{\rm LM}$ or LMTD) in heat exchangers and the $LM\varepsilon$ in two-phase flow will be used to propose approximate expressions for the $LM\varepsilon$. In heat exchangers, the definition of the logarithmic mean temperature difference ($\Delta T_{\rm LM}$ or *LMTD*) is:

$$\Delta T_{\rm LM} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} \tag{4}$$

where ΔT_1 and ΔT_2 are the temperature differences between the two fluids at the inlet and the outlet of heat exchanger respectively. It is clear that eq. (1) is analogous to eq. (4).

In literature, there are many substitution/approximations/replacements for $\Delta T_{\rm LM}$ or *LMTD* in heat exchangers since 1970 instead of the direct use of the form in eq. (4). For instance, Underwood [7] presented simple formula to calculate the $\Delta T_{\rm LM}$ or *LMTD* in heat exchangers. He used older development, recent refinement, and direct use of ΔT_1 and ΔT_2 to propose an average of powered ΔT_1 and ΔT_2 to approximate $\Delta T_{\rm LM}$. The proposed average was:

$$\Delta T_{\rm LM} = \left(\frac{\Delta T_1^{1/3} + \Delta T_2^{1/3}}{2}\right)^{\frac{1}{1/3}} \tag{5}$$

It should be noted that Underwood [7] proposed his original approximation in the year 1933 [8]. Then, he pursued his results in 1970 and reported the approximation stated in eq. (5). However, Ram [9] reported the same approximation 18 years later in 1988. Therefore, credit should be given to Underwood for his 1970 contribution not to Ram [9].

Later, Paterson [10] presented a replacement for the $\Delta T_{\rm LM}$ or *LMTD* in heat exchangers. He used the geometric mean temperature difference (*GMTD*) and arithmetic mean temperature difference (*AMTD*) enclosure to build a log mean approximation as:

$$\Delta T_{\rm LM} = \frac{2}{3}GMTD + \frac{1}{3}AMTD \tag{6}$$

where *GMTD* and *AMTD* are defined by:

$$GMTD = \sqrt{\Delta T_1 \Delta T_2} \tag{7}$$

$$AMTD = \frac{\Delta T_1 + \Delta T_2}{2} \tag{8}$$

The selection of 2/3 and 1/3 in eq. (6) was made so that $(\Delta T_{LM}/\Delta T_2) \to 1$ as $(\Delta T_1/\Delta T_2) \to 1$.

Afterward, Chen [11] presented comments on improvements on a replacement for the ΔT_{LM} or *LMTD* in heat exchangers. He suggested the use of powered *GMTD* and *AMTD* product to build a log mean approximation rather than the linear combination used by Paterson [10], as:

$$\Delta T_{LM} = \sqrt[3]{GMTD^2}\sqrt[3]{AMTD} \tag{9}$$

where *GMTD* and *AMTD* were defined by eq. (7) and eq. (8), respectively. The selection of 2/3 and 1/3 in eq. (9) was made so that $(\Delta T_{LM}/\Delta T_2) \to 1$ as $(\Delta T_1/\Delta T_2) \to 1$.

Also, Chen [11] compared a few ΔT_{LM} replacements and concluded in favor of the form presented in eq. (10) that appeared to have some accuracy advantage:

$$\Delta T_{LM} \cong \left(\frac{\Delta T_1^{0.3275} + \Delta T_2^{0.3275}}{2} \right)^{\frac{1}{0.3275}} \tag{10}$$

It can be seen that Chen [11] used eq. (5) and a selected range of temperature differences to obtain eq. (10) where the approximation parameter 0.3275 replaced 1/3 in Underwood approximation [7].

Recently, Salama [12] presented a note on approximations for the ΔT_{LM} or *LMTD* in heat exchangers. He assumed that the approximation parameter in Underwood [7] was allowed to change as a fitting parameter in non-linear regression problem over the temperature difference ratio range $0 \le (\Delta T_1/\Delta T_2) \le 1$. His expression was:

$$\Delta T_{LM} \cong \left(\frac{\Delta T_1^{0.3241} + \Delta T_2^{0.3241}}{2} \right)^{\frac{1}{0.3241}} \tag{11}$$

Salama [12] emphasized that the fitting parameter in eq. (9) was obtained over the range $0 \le (\Delta T_1/\Delta T_2) \le 1$ while in eq. (10) was obtained over the range $0 \le (\Delta T_1/\Delta T_2) \le 0.67$. From a practical point of view that the difference between 0.3275 (Chen's) and 0.3241 (Salama's) was very small.

Also, Salama [12] mentioned that if both the approximation parameters and the constant 2 were allowed to change and the results obtained were:

$$\Delta T_{LM} \cong \left(\frac{\Delta T_1^{0.3241} + \Delta T_2^{0.3241}}{1.99996} \right)^{\frac{1}{0.3241}} \tag{12}$$

Salama [12] mentioned that the Underwood class had the advantage of direct use of ΔT_1 and ΔT_2 and the accuracy of the approximations (*i. e.*, less than 1% error) in the temperature difference ratio range $0.05 \le (\Delta T_1/\Delta T_2) \le 1$. His two approximations, eqs. (11) and (12), showed slight improvement due to the use of non-linear regression over the limited temperature difference ratio range $0 \le (\Delta T_1/\Delta T_2) \le 1$.

These approximations of the ΔT_{LM} have been applied in a number of computational studies. For example, Yee and Grossmann [13] presented a mixed integer non-linear programming (MINLP) model that could generate networks where utility cost, exchanger areas and selection of matches were optimized simultaneously. They approximated *LMTD* terms in the objective function using the first Chen approximation, eq. (9). Later, Zamora and Grossmann [14] presented a global optimization algorithm to rigorously solve the MINLP model by Yee and Grossmann [13] for the synthesis of heat exchanger networks under the simplifying assumptions of linear area cost, arithmetic mean temperature difference driving forces and no stream splitting. Also, Zhu and Nie [15] investigated pressure drop considerations for heat exchanger network (HEN) grassroots design. The researchers approximated LMTD terms using the Paterson approximation eqs. (6)-(8) in their study. On the other hand, Salama [12] did not consider the Paterson approximation , and the first Chen equation, eq. (9), due to their inferior approximation in the subsequent development.

Using the analogy between the logarithmic mean temperature difference (ΔT_{LM} or *LMTD*) in heat exchangers and the *LMε* in two-phase flow, 6 approximate expressions for the *LMε* can be obtained:

$$LM\varepsilon \cong \left(\frac{\varepsilon_h^{1/3} + \varepsilon_{ra}^{1/3}}{2} \right)^{\frac{1}{1/3}} \tag{13}$$

$$LM\varepsilon \equiv \frac{2}{3}\sqrt{\varepsilon_{\mathrm{h}}\varepsilon_{\mathrm{ra}}} + \frac{1}{3}\frac{\varepsilon_{\mathrm{h}} + \varepsilon_{\mathrm{ra}}}{2} \tag{14}$$

$$LM\varepsilon \equiv (\sqrt{\varepsilon_{\mathrm{h}}\varepsilon_{\mathrm{ra}}})^{2/3} \left(\frac{\varepsilon_{\mathrm{h}} + \varepsilon_{\mathrm{ra}}}{2} \right)^{1/3} \tag{15}$$

$$LM\varepsilon \cong \left(\frac{\varepsilon_{\mathrm{h}}^{0.3275} + \varepsilon_{\mathrm{ra}}^{0.3275}}{2} \right)^{\frac{1}{0.3275}} \tag{16}$$

$$LM\varepsilon \cong \left(\frac{\varepsilon_{\mathrm{h}}^{0.3241} + \varepsilon_{\mathrm{ra}}^{0.3241}}{2} \right)^{\frac{1}{0.3241}} \tag{17}$$

$$LM\varepsilon \cong \left(\frac{\varepsilon_{\mathrm{h}}^{0.3241} + \varepsilon_{\mathrm{ra}}^{0.3241}}{1.9996} \right)^{\frac{1}{0.3241}} \tag{18}$$

Similar to using the approximations of the ΔT_{LM} in the computational studies to avoid the problem of the original formula for the computation of the logarithmic mean temperature difference in finite precision floating-point arithmetic might suffer from serious round-off problems when both temperature differences (ΔT_1 and ΔT_2) were very close to each other [5], these new approximations of the *LMε*, eqs. (13-18), can be also applied in the computational studies in many applications.

Based on this study, this paper suggests the transposition of approximate expressions developed by other authors for the logarithmic mean temperature in heat exchangers to the logarithmic mean void fraction in two-phase flows. These approximations are sought to be numerically more robust and do not suffer round-off errors when ε_{h} and $\varepsilon_{\mathrm{ra}}$, defined in eqs. (1) and (2) are very close to each other. These 6 approximations of the *LMε* will be useful in the field of two-phase flows. These 6 approximations of the *LMε* can be applied in the computational studies because the original formula for the computation of the logarithmic mean void fraction in finite precision floating-point arithmetic may suffer from serious round-off problems when both differences ε_h and ε_{ra}, are very close to each other either at very low values or very high values of mass quality (x).

Nomenclature

G – total mass flux, [kgm^{-2}s^{-1}]
g – acceleration of gravity, [ms^{-2}]
T – temperature, [K]
x – mass quality, [–]

Greek symbols

Δ – difference
ε – void fraction, [–]
ρ – density, [kgm^{-3}]
σ – surface tension, [Nm^{-1}]

Subscripts

1 – at the inlet
2 – at the outlet
g – gas
h – homogeneous
l – liquid
ra – Rouhani-Axelsson

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