OPTIMIZATION OF THE THERMAL PERFORMANCE OF MULTI-LAYER SILICON MICROCHANNEL HEAT SINKS

by

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The objective is to optimize the configuration sizes and thermal performance of a multi-layer silicon microchannel heat sink by the thermal resistance network model. The effect of structural parameter on the thermal resistance is analyzed by numercal simulation. Taking the thermal resistance as an objective function, a non-linear and multi-constrained optimization model are proposed for the silicon microchannel heat sink in electronic chips cooling. The sequential quadratic programming method is used to do the optimization design of the configuration sizes of the microchannel. For the heat sink with the size of 20 mm × 20 mm and the power of 400 W, the optimized microchannel number, layer, height, and width are 40, and 2, 2.2 mm, and 0.2 mm, respectively, and its corresponding total thermal resistance for whole microchannel heat sink is 0.0424 K/W.

Key words: *optimization, microchannel, sequential quadratic programming, thermal resistance, multi-layer*

Introduction

As the devices or systems become smaller, heat flux increases in general [1, 2]. An effective cooling strategy for the micro-devices is required especially when the cooling target is made from micro-fabrication processes with silicon substrates [3]. Therefore, heat dissipation has become one of the key design challenges. An ever-increasing interest in microchannel fluid mechanics and heat transfer has emerged because of possible cooling applications [4-6]. The silicon based microchannel heat sinks combine the attributes of high surface area per unit volume ratios, high material compatibility, and large potential heat transfer performance. These advantages make these silicon-based microchannel heat sinks extremely attractive for applications in electronic cooling. The functional designs of microchannels fabricated as an integral part of silicon wafers require that the fluid flow and heat transfer characteristics in these microchannels be known and understood.

When the heat flux of micro-electronic devices exceeds 100 W/cm^2, air cooling method is unlikely to meet the cooling needs [7]. The microchannel heat sink is a good choice for cooling of the high-power electronic device with a small volume. Single layer microchannels embedded in silicon wafer were first shown to be effective cooling solutions by Tuckerman and Pease [8] in which a maximum of 790 W/cm^2 was rejected. Although such micro heat sinks are seldom used for current electronic products, the microchannel structure

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and configuration size can significantly affect the thermal performance of heat sink. The optimization of microchannel heat sink can enhance its heat transfer performance. Vafai and Zhu [9] showed a two-layered microchannel heat sink with counter-current flow arrangement. The purpose of this microchannel structure design was to reduce the temperature gradient along the axial direction. The effectiveness of the concept was examined numerically using a finite element method and an optimization of the design parameters was also performed. Shao *et al.* [10] used sequential quadratic programming (SQP) method to optimize the shape and dimensions of the microchannel, and the effect of the dimensions of the microchannel cross section on the heat transfer performance is analyzed. An optimization study reported by Knight *et al.* [11] showed that the thermal resistance for Tuckerman and Pease [12] could be reduced by 35% by the configuration optimization, if turbulent flow is allowed. However, the required pumping power is almost five times higher.

Lorenzini and Moretti [13] optimized the geometry and flow condition of heat exchanger by a CFD code approach based on Bejan's [14] constructal theory. Sanaye and Modarrespoor [15] took the effectiveness and total cost as two objective functions which related to six design parameters (pipe diameter, pipe length, numbers of pipes per row, number of rows, fin pitch, and fin length ratio) to optimize a heat pipe heat exchanger using multiobjective optimization technique.

Thermal resistance network model is an effective analysis method for thermal resistance. Wei and Joshi [16, 17] evaluated the thermal performance of stacked high-aspect ratio microchannel heat sinks using a thermal resistance network model. The solution procedure was iterative automatically. Optimization work was performed for optimum channel aspect ratios, number of layers, conductivities, pumping power per unit area, and channel length. Skandakumaran *et al.* [18] analyzed the thermal resistance of single and multi-layer microchannel heat sinks *via* the thermal resistance network model. A single layer counter flow and a double layer counter flow microchannel heat sink with rectangular channels by employing the thermal resistance network was modeled by Chong *et al.* [19]. The prediction accuracy was verified by comparing the obtained results with those from the more comprehensive 3-D CFD conjugate heat transfer model and good agreement as obtained. Design optimization of micro heat sink for concentrating photovoltaic/thermal (CPVT) systems using a genetic algorithm was investigated by Karathanassis *et al.* [20]. Hu and Xu [21] adopted an optimization scheme based on the SQP method to optimize a single layer microchannel heat sink. The optimized microchannel heat sink was simulated by CFD method, and its total thermal resistance was compared with that from thermal resistance network model.

The heat transfer performance of silicon microchannel heat sink is affected by the flow state of liquid in microchannel, besides the structure and dimensions. Pressure drop is related with the configuration of microchannel cooling heat sink. Although they are reported in previous literatures, there is less study on the optimization of microchannel heat sink made from silicon by SQP method. In this study thermal resistance and pressure drop are taken as the goal function and one of multi-constraints, respectively, while the structure parameters of microchannel heat sink are independent variables. We adopt an optimization scheme based on the SQP method to optimize the single and multi-layer silicon microchannel heat sinks. Then the optimized microchannel heat sink is simulated by FLUENT 13.0 and the total thermal resistance of it is calculated and compared with that from the SQP method.

Optimization method

Physical model and computational zone

The microchannel cooling heat sink comprising of an adiabatic cover plate and a silicon substrate with many microchannels fabricated on the other side, as shown in fig. 1(a), which is used to cool an electronic chip with the size of L × W = 20 mm × 20 mm. The power is 400 W applied to the bottom of the heat sink, so the corresponding heat flux is 100 W/cm^2. The working fluid is deionized water. In the fig. 1(a), b is the width of microchannel, a is the depth of microchannel, c is the width of fin, and t is the thickness of substrate. For the symmetry of the structure of model and load, the computational zone is half of microchannel and fin. The schematic diagram of computational zone cross-section is shown in fig. 1(b).

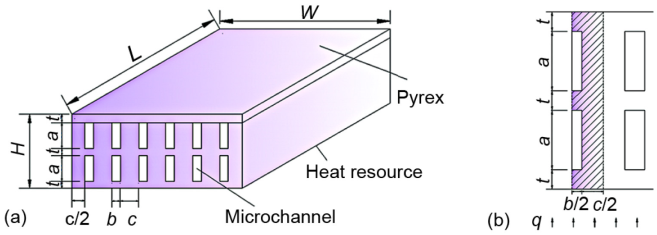

Figure 1. (a) Schematic diagram of microchannel cooling heat sink, (b) schematic diagram of computational zone cross-section

Single-phase laminar flow through each microchannel with a steady velocity is considered. The physical properties of water and silicon are constant and defined. The density, specific heat, and thermal conductivity of silicon are 2328.3 kg/m^3, 700 J/kgK, and 148 W/mK, respectively. We assume that these parameters are unchanged in this study.

Thermal resistance network

A 1-D thermal resistance network is used to model the single, and multi-layer heat sinks. Although the approach is approximate, results can still be obtained without the need for complex simulation tools.

The thermal resistance network for the single-layer with a constant temperature boundary condition on the bottom surface is shown in fig. 2.

The base thickness resistance, R_{base}, is defined as the conduction resistance from the bottom surface to the base of the solid-fluid interface:

$$R_{\text{base}} = \frac{t}{k_{\text{s}}\left(\dfrac{c}{2}+\dfrac{b}{2}\right)L} \tag{1}$$

where k_{s} [Wm^{-1}K^{-1}] is the thermal conductivity of material silicon.

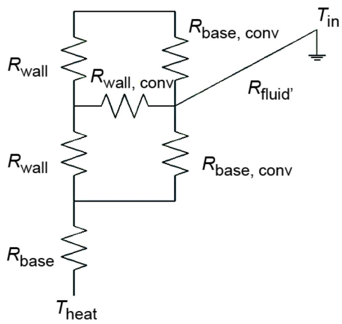

Figure 2. Thermal resistance network for single-layer heat sink

The conduction resistance through the side wall, R_{wall}, is defined:

$$R_{\text{wall}} = \frac{\frac{a}{2}}{k_{\text{s}}\left(\frac{c}{2}\right)L} \tag{2}$$

Assuming an isothermal condition at the base of the fluid, the convection resistance from the base temperature to the fluid outlet temperature node is [22]:

$$R_{\text{base,conv}} = \frac{2}{\dot{m}c_p\left[1-\exp\left(-\frac{hbL}{\dot{m}c_p}\right)\right]} \tag{3}$$

where h [Wm^{-2}K^{-1}] is the heat transfer coefficient.

The convection resistance from the solid sidewall temperature to the fluid outlet node is [22]:

$$R_{\text{wall,conv}} = \frac{2}{\dot{m}c_p\left[1-\exp\left(-\frac{haL}{\frac{\dot{m}c_p}{2}}\right)\right]} \tag{4}$$

The advection resistance from the fluid outlet temperature to the fluid inlet temperature is:

$$R_{\text{fluid'}} = \frac{1}{\dot{m}c_p} \tag{5}$$

where $\dot{m}$ [kgs^{-1}] is mass flow rate, and c_p [Jkg^{-1}K^{-1}] – the specific heat of water.

The total thermal resistance of the liquid flow of microchannel, R_{fluid}, can be expressed:

$$R_{\text{fluid}} = \frac{R_{\text{fluid'}}}{2Nn} \tag{6}$$

where N is the channel number per layer, and n – the layer number.

The heat transfer coefficient from the heat sink to the water is calculated:

$$h = \frac{\text{Nu}\, k_f}{D_{\text{h}}} \tag{7}$$

where D_h [m] is the hydraulic diameter of cross-section, $D_h = 2ab/(a + b)$, k_f [Wm^{-1}K^{-1}] – the thermal conductivity of the liquid, and Nu – the average Nusselt number caculated [23]:

$$\text{Nu} = 2.253 + 8.164\left(\frac{\alpha}{1+\alpha}\right)^{1.5}, \qquad \text{for Re} < 1000 \tag{8}$$

where Re is the Reynolds number:

$$\text{Re} = \frac{\rho_f v_{\text{ave}} D_{\text{h}}}{\mu} \tag{9}$$

where ρ_f [kgm^{-3}] is the density of liquid, v_{ave} [ms^{-1}] – the mean velocity of liquid, and μ [Pa·s] – the dynamic viscosity.

It should be noted that the Reynolds number in the optimization process is always lower than 1000 for the reason that the inlet velocity is fixed at 1 m/s and the hydraulic diameter is far smaller than 1 m. Thus, eq. (8) is suitable for this study.

An 1-D thermal resistance network is studied by Lei *et al.* [22] to analyze multi-layered microchannel heat sinks. We also use simulation methods to analyze the thermal resistance of multi-layer silicon microchannel network. The equivalent resistance, R_1, for the single-layer case is shown in fig. 2, is solved below using a combination of series and parallel resistances:

$$R_1 = \frac{R_{\text{base}} + \left(R_{\text{base,conv}} \parallel \{R_{\text{wall}} + [R_{\text{wall,conv}} \parallel (R_{\text{base,conv}} + R_{\text{wall}})]\} \right)}{2N} \tag{10}$$

where the parallel symbol "||" is defined:

$$x \parallel y = \frac{xy}{x+y} \tag{11}$$

The resistance network for the single-layer structure is repeated or *stacked* to obtain the resistance network for the multi-layer structure, fig. 3. The equivalent resistance, R_n, for the multi-layer one shown in fig. 3, is solved using a combination of series and parallel resistances:

$$R_n = \frac{R_{\text{base}} + \left[R_{\text{base,conv}} \parallel \left(R_{\text{wall}} + \{R_{\text{wall,conv}} \parallel [R_{\text{wall}} + (R_{n-1} \parallel R_{\text{base,conv}})]\} \right) \right]}{2N} \tag{12}$$

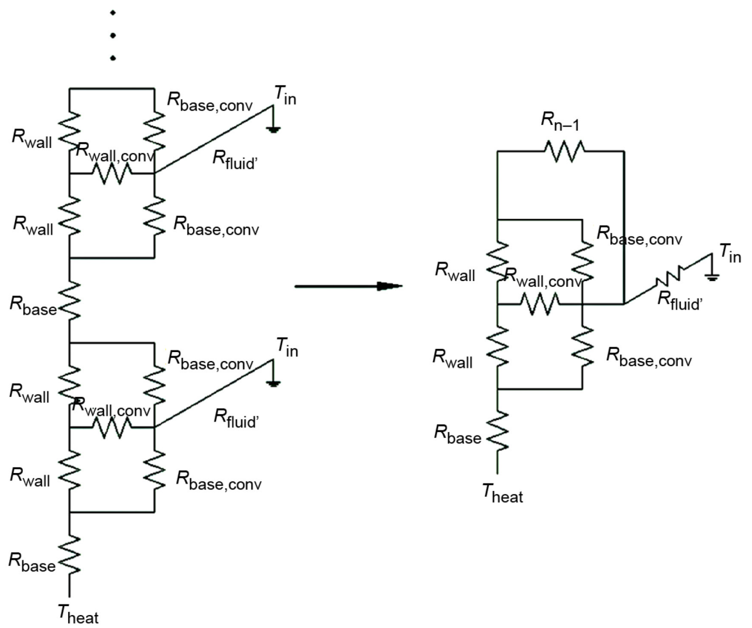

Figure 3. Thermal resistance network *n* layer heat sink

The total thermal resistance of whole microchannel heat sink $R_{n,\text{total}}$ is the sum of the thermal resistance of the solid parts and liquid parts, which can be expressed:

$$R_{n,\text{total}} = R_n + R_{\text{fluid}} \tag{13}$$

Besides thermal resistance, the pressure drop affects the heat transfer performance of microchannel heat sink [24]:

$$\Delta P = 2 f \rho_f v_{\text{ave}}^2 \frac{L}{D_h} \tag{14}$$

Where f is the friction factor for rectangular channels and suggested by Biber and Belady [25]. It is shown in eq. (15):

$$f = \frac{\left[\frac{3.44}{\sqrt{x^+}} + \frac{24 + \frac{0.674}{x^+} - \frac{3.44}{\sqrt{x^+}}}{1 + 0.000029\sqrt{x^+}} \right] G(\alpha)}{\text{Re}} \tag{15}$$

where x^+ is the non-dimensional hydraulic length:

$$x^+ = \frac{L}{D_h \text{ Re}} \tag{16}$$

and $G(\alpha)$ is defined as:

$$G(\alpha) = 1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5 \tag{17}$$

The thermal resistance per unit area, R'', is defined as:

$$R'' = \frac{T_{\text{s}} - T_{\text{in}}}{\frac{q}{A}} \tag{18}$$

where T_{s} [K] is the maximum surface temperature in the heat sink, T_{in} [K] – the inlet water temperature, q [W] – the heated power, and A [m^2] – the heated surface area of heat sink.

The pumping power to drive the working fluid through the channels is calculated by:

$$P = Q \, \Delta P \tag{19}$$

where Q [mlmin^{-1}] is the volumetric flow rate.

Optimization design and results

The base idea to solve the optimization problem is to construct a merit function and several constraints. Then the computing program is used to solve the merit function with the goal attainment programming. The problem formulation allows the objectives to be under- or over-achieved, which is slightly imprecise relative to the initial design goals.

The number of microchannel, N, the width of microchannel, b, and fin, c, the height of microchannel, a, substrate thickness, t, and the number of layers, n, are selected as design variables. They are expressed as x_1, x_2, x_3, x_4, x_5, and x_6, respectively, and written in vector $\vec{x} = [x_1, x_2, x_3, x_4, x_5, x_6]$. The eqs. (11) and (12) are goal function and one of multi-constraints, and can be written as a function of $\vec{x}$, $f_1(x)$, and $f_2(x)$. The width of microchannel heat sink is constraint and the boundary of variables is defined. The optimization problem is:

$$\begin{cases} \text{find} & x \\ \text{min} & F(x) = \min \ f_1(x) \\ \text{s.t.} & x_1x_2 + x_1x_3 - 0.02 = 0 \\ & x_4x_6 + (x_6+1)x_5 - 0.005 = 0 \\ & f_2(x) \le 5000 \\ & 5 \le x_1 \le 100 \\ & 2 \cdot 10^{-4} \le x_2 \le 4 \cdot 10^{-3} \\ & 2 \cdot 10^{-4} \le x_3 \le 4 \cdot 10^{-3} \\ & 2 \cdot 10^{-4} \le x_4 \le 5 \cdot 10^{-3} \\ & 2 \cdot 10^{-4} \le x_5 \le 5 \cdot 10^{-4} \\ & 1 \le x_6 \le 10 \end{cases} \tag{19}$$

The goal attainment SQP method is used to optimize the previous problem. It is an iterative method which solves a quadratic programming (QP) problem at each major iteration. An approximation is from the Hessian matrix of the Lagrangian function by a quasi-Newton updating method. A QP sub-problem whose solution is used to form a search direction for a line search procedure is generated. Then the optimized results of configuration sizes of microchannel cooling heat sink listed in tab. 1 are obtained. While the inlet velocity is 1 m/s, the total and component thermal resistances are shown in tab. 2. Besides, the pressure drop of liquid in microchannel is 4.775 kPa.

Table 1. Structural parameters of optimal microchannel cooling heat sink

N	b [mm]	c [mm]	a [mm]	t [mm]	n
40	0.2	0.3	2.2	0.2	2

Table 2. Optimization results of total and component thermal resistance

$R_{2,\text{total}}$ [KW^{-1}]	R_base [KW^{-1}]	R_wall [KW^{-1}]	$R_\text{base,conv}$ [KW^{-1}]	$R_\text{wall,conv}$ [KW^{-1}]	$R_\text{fluid'}$ [KW^{-1}]
0.0424	0.2703	2.4775	16.5681	0.7531	1.0878

In the case of other conditions unchanged, the length of the whole microchannel, L, is increased to 40 mm, 60 mm, and 80 mm, respectively. The results of unit thermal resistance and component thermal resistance are shown in tab. 3.

Table 3. Results of unit thermal resistance and component thermal resistance

L [mm]	R'' [KW^{-1}cm^{-2}]	R_base [KW^{-1}]	R_wall [KW^{-1}]	$R_\text{base,conv}$ [KW^{-1}]	$R_\text{wall,conv}$ [KW^{-1}]
40	0.1968	0.1351	1.2387	8.2841	0.3766
60	0.2244	0.0901	0.8258	5.5227	0.2510
80	0.2512	0.0676	0.6194	4.1420	0.1883

Numerical simulation and discussion

The numerical simulation is used to verify the results by SQP method. The cooling performance of optimized microchannel heat sink is compared with it before optimization. The non-opitimized heat sink with the same surface area of microchannels is also studied by CFD simulation. It includes ten microchannels with the width and height of 1 mm and 1.76 mm, respectively. Other configuration and sizes are the same as the optimal heat sink. The commercial software FLUENT 13.0 is used in this study, which is based on finite volume method. The inlet velocity and density of deionized water are 1 m/s and 998.2 kg/m^3, respectively. The specific heat and thermal conductivity of deionized water are, respectively, 4186 J/kgK and 0.613 W/mK. The Reynolds number is approximately 365, which is less than 1400. Thus, the flow in the channel is laminar.

The temperature distribution of microchannel heat sink are shown in figs. 4(a) and (b). The highest temperature of heat sink before optimization is 357.9 K, while the highest temperature of optimal microchannel heat sink is 326.7 K. Although they have the same surface area to diffuse the excessive heat of electronic chips, the structural distinction leads to their temperature difference as high as 31.2 K. The total thermal resistance $R_{2,\text{total}}$ is 0.053 K/W, which shows that the results of numerical simulation agree with the analysis results.

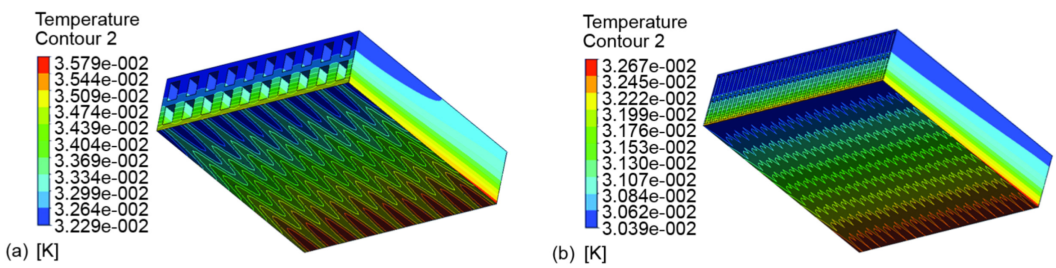

Figure 4. (a) Temperature distribution of microchannel heat sink before optimization (b) temperature distribution of optimal microchannel heat sink *(for color image see jounal web site)*

Analytical and numerical results of the unit thermal resistance for the different channel length are shown in fig. 5. The unit thermal resistances increase with the increase of channel length. Thus, a shorter channel will lead to a smaller thermal resistance. It may be due to the extension of the fluid retention period in the longer channel under the condition of invariable velocity. When the overall length of the channel is 20 mm, the unit thermal resistance of the whole microchannel heat sink R'' is 0.153 K/W/cm^2, which shows that the result of numerical simulation agrees with the analysis result (0.1696 K/W/cm^2). The relative error between them is only 9.787%, which is less than 10% of the maximum error ranges. Therefore, the results show that short channels instead of

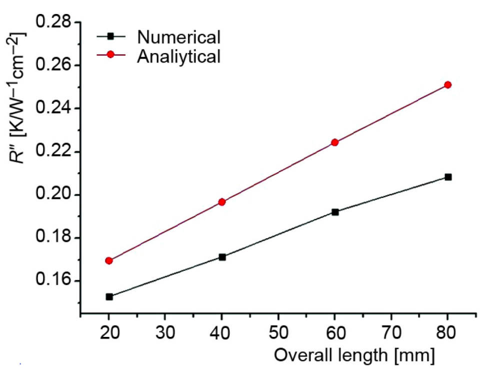

Figure 5. Effect of channel length on unit thermal resistance: analytical and numerical

long channels should be used. If the surface to be cooled is relatively large, several microchannel cooling heat sinks with short channels should be integrated instead of single microchannel cooling heat sink with long channels.

Figure 6 demonstrates the Nusselt number as a function of Reynolds number that is associated with the inlet velocity for the optimal microchannel heat sink. The inlet velocity is from 1 m/s to 3 m/s while the structural parameter of heat sink is not changed. The heat transfer performance of heat sink is enhanced with the increasing inlet velocity. This means that a high Reynolds number will improve the cooling performance of microchannel heat sinks, although it leads to the unnecessary penalty of pressure drop and pumping power.

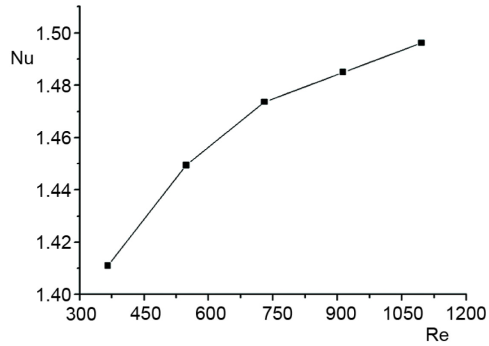

Figure 6. Variations in Nusselt number with Reynolds number

The optimal geometric parameters are limited under a specified pumping power condition. Figure 7 shows the influence of pumping power on unit thermal resistance and the corresponding Nusselt numbers based on a fixed optimal-design parameters condition. Figure 7(a) reveals that the unit thermal resistance, R'', decreases rapidly with the pumping power. But the reduction rate of the unit thermal resistance at a high pumping power is much less than that at a low one. Figure 7(b) shows that a relationship between pumping power and Nusselt numbers. As the pumping power increases from 0.077 W to 1.090 W, the unit thermal resistance decreases from 0.153-0.138 K/W/cm^2, as well as the Nusselt number changes from 1.411 to 1.508. So a good heat transfer property of heat sink is at the expense of a higher pumping power.

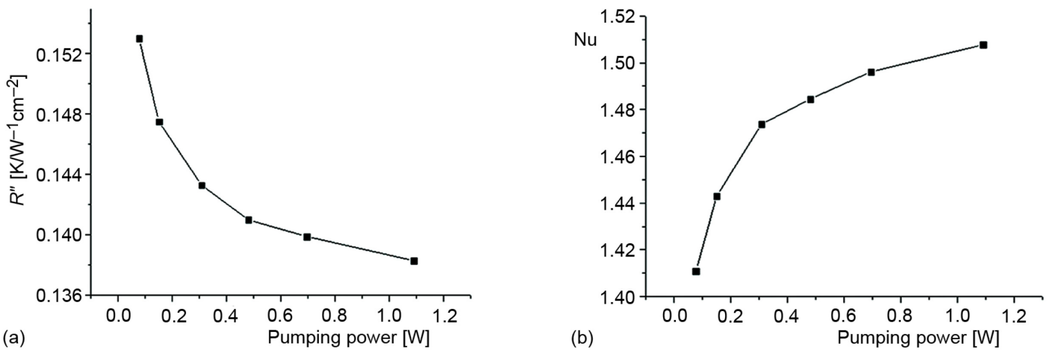

Figure 7. Relationship of the: (a) optimal unit thermal resistance with the (b) Nusselt numbers under various pumping power

In order to study the effect of the aspect ratio on the flow in rectangular microchannels, the channel size of the optimized microchannel heat sink is used. Figure 8 shows the variations of aspect ratio and number of the channel on the temperatures in the

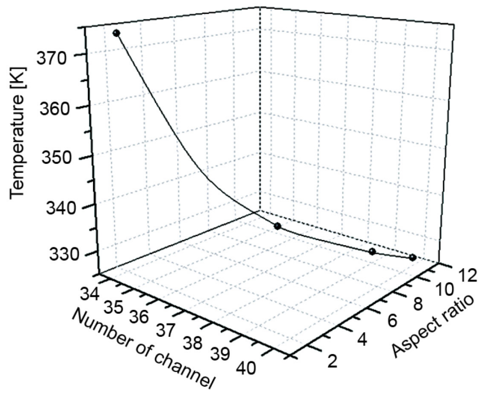

Figure 8. Effect of aspect ratio and number of channel on temperature

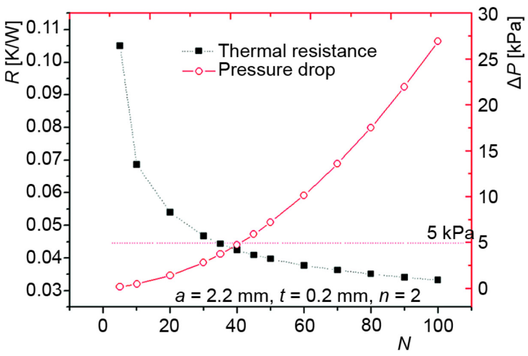

Figure 9. Number of channel *vs.* maximum temperature and pressure drop

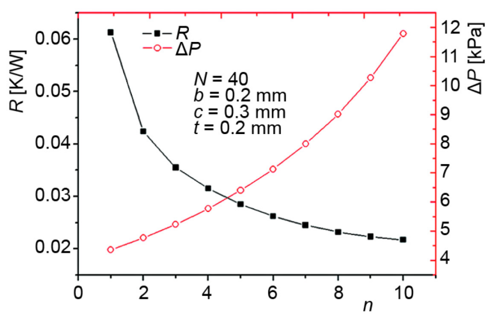

Figure 10. Variations in both the overall thermal resistance and pressure drop with the number of layer

microchannel heat sink in the case of other conditions unchanged. The temperature ranges from 373.7-326.7 K while the aspect ratio and the number of channel vary from 2-11 and from 34-40, respectively. The maximum temperature of the microchannel decreases asymptotically as the aspect ratio and number of channel increases. It decreases slowly at the high aspect ratio and the large channel number. The combined effect of increasing channel number and aspect ratio is helpful for improving the overall thermal performance of the microchannel heat sinks.

Figure 9 shows the thermal resistance and pressure drop for different channel number. For number of channel $34 < N < 40$, it is obvious that thermal resistance decreases rapidly as well as the pressure drop. When $N > 40$, both thermal resistance and pressure drop change slowly. For a fixed inlet velocity, increasing channel number increases the convective heat transfer area, and diminishes the thermal resistance. The pressure drop is also increased due to a drop in cross-sectional area of each micro channel.

For the fixed channel numbers, N, channel width, b, fin width, c, and substrate thickness, t, the results of the total thermal resistance and pressure drop of heat sink for different layer n is shown in fig. 10. With the increase of layer numbers, n, the total thermal resistance decreases and the pressure drop increases. The thermal resistance declines from 0.061-0.02 K/W (approximately 70%) when the layer number is from 1 to 10. In general, it is agreement with the work of Lei *et al.* [22]. Meanwhile, the pressure drop increases from 4.3-12.0 kPa for a fixed velocity 1 m/s. Then, it decreases moderately as the number of layer increases (from 24-10 Kcm^3/W).

Conclusions

Thermal resistance network model is established to analyze the thermal resistance of microchannel cooling heat sink. Based on the thermal resistance network model and pressure drop formula, the SQP method is presented to solve this non-linear, multi-constrained problem. Then the optimal heat sink dimension is obtained. The optimized results of design variable are x = [40, 0.2, 0.3, 2.2, 0.2, 2]. The thermal resistance, $R_{2,\text{total}}$, of whole heat sink is 0.0424 K/W.

The unit thermal resistances increases with the increase of overall channel length. The combined effect of the aspect ratio and number of channel on the enhancement of thermal performance is also significant. A high aspect ratio and number of channel enhance the heat transfer performance. The numerical results which are related to show that the total thermal resistance decreases as the channel number and layer number increase. But the pressure drop increase for a fixed inlet velocity. The simulated conclusions agree well with the results by SQP method.

Nomenclature

A – heated surface area of heat sink, [m^2]
a, b – depth and width of microchannel, [mm]
c – width of fin, [mm]
c_p – specific heat of water, [$\text{Jkg}^{-1}\text{K}^{-1}$]
D_h – hydraulic diameter of microchannel, [m]
f – friction factor for rectangular channels, [–]
h – heat transfer coefficient, [$\text{Wm}^{-2}\text{K}^{-1}$]
k_f – thermal conductivity of working fluid, [$\text{Wm}^{-1}\text{K}^{-1}$]
k_s – thermal conductivity of material silicon, [$\text{Wm}^{-1}\text{K}^{-1}$]
L, W, H – length, width, and height of heat sink, [m]
$\dot{m}$ – mass flow rate, [kgs^{-1}]
N – number of channel per layer, [–]
Nu – Nusselt number, [–]
n – number of layer, [–]
P – pumping power, [W]
ΔP – pressure drop, [Pa]
Q – volumetric flow rate, [mlmin^{-1}]
q – heated power, [W]
R'' – thermal resistance per unit area, [$\text{KW}^{-1}\text{cm}^{-2}$]
R_1 – equivalent resistance for the single-layer case, [KW^{-1}]
R_base – conduction resistance from the bottom surface to the base of the solid-fluid interface, [KW^{-1}]
$R_\text{base,conv}$ – convection resistance from the base temperature to the fluid outlet temperature node, [KW^{-1}]
Re – Reynolds number, [–]
R_fluid – total thermal resistance of the liquid flow of microchannel, [KW^{-1}]
$R_\text{fluid'}$ – advection resistance from the fluid outlet temperature to the fluid inlet temperature, [KW^{-1}]
R_n – equivalent resistance for the multi-layer case, [KW^{-1}]
$R_{n,\text{total}}$ – total thermal resistance of whole microchannel heat sink, [KW^{-1}]
R_wall – conduction resistance through the side wall, [KW^{-1}]
$R_\text{wall,conv}$ – convection resistance from the solid sidewall temperature to the fluid outlet node, [KW^{-1}]
T_in – temperature of inlet water, [K]
T_s – maximum surface temperature in the heat sink, [K]
t – thickness of substrate, [mm]
v_ave – mean velocity of liquid, [ms^{-1}]
x^+ – non-dimensional hydraulic length, [–]

Greek symbols

α – aspect ratio, [–]
ρ_f – density of working fluid, [kgm^{-3}]
μ – dynamic viscosity, [Nm^{-2}s]

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