

MAGNETOHYDRODYNAMIC MIXED CONVECTION IN A LID-DRIVEN RECTANGULAR ENCLOSURE PARTIALLY HEATED AT THE BOTTOM AND COOLED AT THE TOP

by

Elif Buyuk OGUT

Vocational School of Hereke, Kocaeli University, Kocaeli, Turkey

Original scientific paper
<https://doi.org/10.2298/TSCI141121053O>

In the present study, numerical simulation of magnetohydrodynamic mixed convection heat transfer and fluid flow has been analyzed in a lid-driven enclosure provided with a constant flux heater. Governing equations were solved via differential quadrature method. Moving wall of the enclosure has constant temperature and speed. The calculations were performed for different Richardson number ranging from 0.1 to 10, constant heat flux heater length from 0.2 to 0.8, location of heater center from 0.1 to 0.9, Hartmann number from 0 to 100 and aspect ratio from 0.5 to 2. Two different magnetic field directions were tested as vertical and horizontal. It was found that results of differential quadrature method show good agreement with the results of literature. The magnetic field was more effective when it applied horizontally than that of vertical way. In both direction of magnetic field, it reduced the flow strength and heat transfer. Thus, it can be used as an important control parameter for heat and fluid flow.

Key words: *magnetohydrodynamic, mixed convection, constant heat flux, differential quadrature method*

Introduction

Natural and forced convection occur simultaneously in many engineering applications such as cooling of electronic equipment, drying technology, and air solar collectors. The phenomenon is called as mixed convection. The cavity with moving lid is the most important application for this heat transfer mechanism which is seen in cooling of electronic chips, coating applications, solar energy applications, crystal growth, and food industry.

Partially heated systems are mostly encountered in cooling of electronic systems and mixed convection is occurred due to blowing air and heater. Numerical analyzis of these kinds of systems was performed for different configuration by Papanicolau and Jaluria [1]. Their results indicate that flow patterns generally include re-circulating cells due to buoyancy forces induced by the heat source. As given by Guo and Sharif [2] found out air-cooling is one of the preferred methods for cooling of electronic equipment. They have analyzed the cooling of lid driven enclosure with a flush mounted constant heat flux heater with finite length. In these systems, heat sources have constant heat flux and a blower or fan used to cool the heater. Thus, mixed convection flow is occurred. Oztop [3] made a study to investigate the mixed convection

*Author's e-mail: buyuke77@gmail.com

in porous media filled and partially heated enclosure for a square lid-driven cavity. He investigated the location of the isothermal heater on mixed convection flow and heat transfer. Ogut [4] investigated mixed convection heat transfer and fluid flow in an inclined square lid-driven enclosure numerically. Two parallel walls of the enclosure are adiabatic while one fixed wall is heated with a constant heat flux heater and moving lid is isothermal. Kahveci and Ogut [5] studied mixed convection of water-based nanofluids in a lid-driven square enclosure with a constant heat flux heater. The results show that the presence of nanoparticles in the base fluid causes a significant enhancement of heat transfer. They also show that the heat transfer rate increases considerably with a decrease in the Richardson number and the length of the heater.

Literature review shows that there has been considerable interest about the effects of magnetic fields on the convection heat transfer. But the subject mostly studied for natural convection problems due to its wide applications as crystal growth process [6], melting of silicon [7] or other related applications [8-10]. Hossain *et al.* [11] made a study to illustrate the buoyancy and thermocapillary driven convection flow of an electrically conducting fluid in an enclosure. They found that change of direction of the external magnetic force from horizontal to vertical leads to decrease in the flow rates in both the primary and the secondary cells and that causes an increase in the effect of the thermocapillary force. On the contrary, there are few studies on effects of magnetic field in mixed convection. Chamkha [12] was performed a numerical study to investigate the hydromagnetic combined convection flow in a vertical lid-driven cavity with internal heat generation or absorption. His results showed that the flow behavior and the heat transfer characteristics inside of the cavity are strongly affected by the presence of the magnetic field.

Recently, Chatterjee and Gupta [13] aim here to numerically analyze the hydromagnetic mixed convection flow and heat transfer along with entropy generation in a vertical lid-driven square enclosure involving a heat-conducting horizontal solid circular cylinder placed centrally within the enclosure. Selimefendigil and Oztop [14] a numerical study of MHD mixed convection in a nanofluid filled lid-driven enclosure with a rotating cylinder was performed. Ganji and Malvandi [15] is a theoretical investigation of natural convective heat transfer of nanofluids, inside a vertical enclosure, in the presence of a uniform magnetic field. Aminossadati *et al.* [16] the laminar natural convection in a square cavity with a thin fin is examined. The cavity is influenced by a uniform magnetic field. The side walls of the cavity are kept at different temperatures and the horizontal walls are thermally insulated. The flow and temperature fields and the heat transfer rate of the cavity are all influenced by the magnetic field, especially at higher Rayleigh numbers. As the Hartmann number increases, the magnetic field limits the convective flow circulations and, as a result, the heat transfer rate decreases.

The purpose of the present study is to analyze the mixed convection heat transfer MHD flow field in a lid-driven enclosure with constant flux heater using differential quadrature method. The effects of Richardson number, the aspect ratio of the enclosure, Hartmann number, ratio of length of heater, and ratio of location of heater are examined in detail. Thus, the study combines the effects of magnetic field on mixed convection in the presence of constant heat flux heater and the study will give information to thermal designers on control of heat and fluid flow.

Definition of physical model

The schematically configuration of the considered model is presented in fig. 1. It is a rectangular enclosure with $H = L$ which its top wall moves from left to right direction with constant velocity and temperature. Thus, an aspect ratio is defined as $A = H/L$. A constant heat flux heater with finite length, w , is mounted to the bottom wall of the enclosure. Temperature of moving wall is lower than that of heater. Remaining walls are adiabatic. Gravity acts in vertical

directions and magnetic field is effective in two different ways as vertical and horizontal. Center of location of heater is given by c .

Governing equations

Steady, laminar, mixed convection flow in an inclined rectangular enclosure with moving side walls heated from one side and cooled from the adjacent side and the other walls stationary and adiabatic was considered. General equations for continuity, change of linear momentum and energy equations are written:

$$\nabla \cdot \vec{V}^* = 0 \quad (1)$$

$$\rho(\vec{V}^* \cdot \nabla) \vec{V}^* = \rho \vec{g} - \nabla p^* + \mu \nabla^2 \vec{V}^* + \vec{j} \times \vec{B} \quad (2)$$

$$(\vec{V}^* \cdot \nabla) T = \alpha \nabla^2 T \quad (3)$$

where, $\vec{V}^*$ is the dimensional velocity vector, p^* – the dimensional pressure, T – the dimensional temperature, $\vec{g}$ – the gravitational acceleration, ρ – the density, μ – the viscosity, α – the thermal diffusivity of the fluid, $\vec{j}$ – the current density, and $\vec{B}$ – the magnetic field [17]. Detailed information is also given by Ece and Buyuk [8].

In the absence of an electric field, the current density can be written:

$$\vec{j} = \sigma(\vec{V}^* \times \vec{B}) \quad (4)$$

where σ is the electrical conductivity of the fluid. The magnetic Reynolds number was assumed to be small and the induced magnetic field due to the motion of the electrically conducting fluid was neglected. The Joule heating of the fluid and the effect of viscous dissipation were also considered to be negligible.

Dimensional co-ordinates with the x^* -axis measuring along the bottom wall and y^* -axis being normal to it along the left wall were used. Dimensionless variables used in the analysis were defined according to:

$$x = \frac{x^*}{L}, \quad y = \frac{y^*}{H}, \quad A = \frac{H}{L} \quad (5)$$

$$u = \frac{u^*}{U}, \quad v = \frac{v^*}{U}, \quad p = \frac{p^* - \rho_0 g y^*}{\rho_0 U^2}, \quad \theta = \frac{T - T_0}{\Delta T}, \quad \Delta T = \frac{q L}{k} \quad (6)$$

Here H and L are the dimensional height and length of the rectangular enclosure, respectively, A – the aspect ratio, U – the speed of the cold top wall of the enclosure, u^* and v^* – the dimensional velocity components in the x^* - and y^* -directions, respectively, p^* – the dimensional pressure, ρ_0 – the density of the fluid at temperature T_0 , k – the thermal conductivity of the fluid, and q'' – the heat flux at the source.

A dimensionless stream function and vorticity were defined:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (7)$$

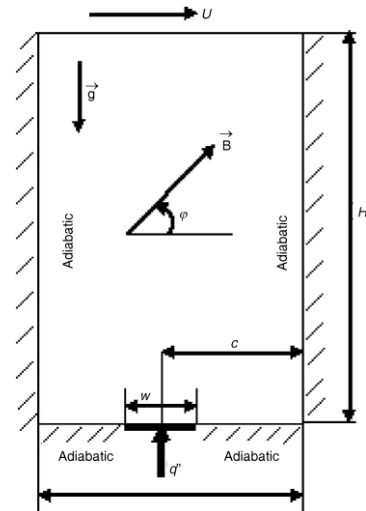


Figure 1. Configuration of physical model, co-ordinates, and boundary conditions

$$\omega = \frac{\partial v}{\partial x} - \frac{1}{A} \frac{\partial u}{\partial y} \quad (8)$$

The governing equations under Boussinesq approximation in terms of the dimensionless variables may then be written:

$$\omega = \frac{\partial^2 \psi}{\partial x^2} - \frac{1}{A} \frac{\partial^2 \psi}{\partial y^2} \quad (9)$$

$$\begin{aligned} Au \frac{\partial \omega}{\partial x} - v \frac{\partial \omega}{\partial y} - \frac{1}{\text{Re}} A \frac{\partial^2 \omega}{\partial x^2} - \frac{1}{A} \frac{\partial^2 \omega}{\partial y^2} - \frac{\text{Gr}}{\text{Re}^2} A \frac{\partial \theta}{\partial x} \\ \frac{\text{Ha}^2}{\text{Re}} A \cos \varphi \sin \varphi \frac{\partial u}{\partial x} - \cos \varphi \frac{\partial v}{\partial y} - \sin \varphi \sin \varphi \frac{\partial u}{\partial y} - \cos \varphi \frac{\partial v}{\partial y} \end{aligned} \quad (10)$$

$$u \frac{\partial \theta}{\partial x} - \frac{1}{A} v \frac{\partial \theta}{\partial y} - \frac{1}{\text{Re Pr}} \frac{\partial^2 \theta}{\partial x^2} - \frac{1}{A^2} \frac{\partial^2 \theta}{\partial y^2} \quad (11)$$

Here the Reynolds, Prandtl, Grashof, Richardson, and Hartman numbers are defined:

$$\text{Re} = \frac{UL\rho_0}{\mu}, \quad \text{Pr} = \frac{\mu}{\rho_0\alpha}, \quad \text{Gr} = \frac{\rho_0^2 g \beta L^3 \Delta T}{\mu^2}, \quad \text{Ri} = \frac{\text{Gr}}{\text{Re}^2}, \quad \text{Ha} = LB \sqrt{\frac{\sigma}{\mu}} \quad (12)$$

where μ is the viscosity, β – the coefficient of thermal expansion, and α – the thermal diffusivity of the fluid, respectively.

Boundary conditions

Boundary conditions are shown also in fig. 1. Velocities are taken as zero in all boundaries except top moving lid. The moving lid has isothermal and bottom wall has partial constant heat flux heater. Remaining walls are adiabatic. For these conditions, boundary conditions are written:

On top wall:

$$\theta = 0, \quad u = 1, \quad v = 0 \quad (13)$$

On bottom wall:

$$\begin{aligned} \frac{\partial \theta}{\partial y} &= 0, \quad \text{for } 0 \leq x \leq (1 - \varepsilon)/2 \\ \frac{\partial \theta}{\partial y} &= 1, \quad \text{for } (1 - \varepsilon)/2 \leq x \leq (1 + \varepsilon)/2 \\ \frac{\partial \theta}{\partial y} &= 0, \quad \text{for } (1 + \varepsilon)/2 \leq x \leq 1, \quad u = v = 0 \end{aligned} \quad (14)$$

On right and left walls:

$$\frac{\partial \theta}{\partial x} = 0, \quad u = 0, \quad v = 0 \quad (15)$$

where ε is the dimensionless length of the heater which is defined as $\varepsilon = w/L$. The boundary condition given by eqs. (13)-(15) may also be expressed in terms of the stream function.

$$\psi(x,0) = 0, \quad \psi(x,1) = 0, \quad \psi(0,y) = 0, \quad \psi(1,y) = 0 \quad (16)$$

$$\left. \frac{\partial \psi}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \psi}{\partial y} \right|_{y=1} = 1, \quad \left. \frac{\partial \psi}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \psi}{\partial x} \right|_{x=1} = 0 \quad (17)$$

Numerical approach

The polynomial-based differential quadrature (PDQ) method [18-23] was used to transform the governing equations into a set of algebraic equations. The PDQ method is an efficient discretisation technique used to obtain accurate numerical solutions using a smaller number of grid points than with low-order methods such as finite-difference, finite element and finite volume methods. The following non-uniform Chebyshev-Gauss-Lobatto grid point distribution was used in the present study to specify the discretization points:

$$x_i = \frac{1}{2} \left(1 + \cos \frac{i}{N} \pi \right), \quad i = 0, 1, 2, \dots, N \quad y_j = \frac{1}{2} \left(1 + \cos \frac{j}{M} \pi \right), \quad j = 0, 1, 2, \dots, M \quad (18)$$

The points in this grid system are more closely spaced in regions near the walls where large velocity and temperature gradients are expected to develop. The computational results were obtained by the successive over-relaxation iteration method. The convergence criteria were chosen as $|R|_{\max} = 10^{-5}$, where $|R|_{\max}$ is the maximum absolute residual value for the vorticity, stream function, and temperature equations.

Grid independency and validation

Firstly, a grid independency test to choose optimal grid for the code has been performed. Results were presented in tab. 1 for different grid dimensions as 51×51 , 81×81 , and 101×101 . The table shows that iteration number is also depended for convergence both governing parameters (Hartmann number, *etc.*) and grid dimensions. The differences in $\psi_{\max}$ values are less than 0.5%. Accordingly, as it can be observed that 81×81 grid dimensions were enough for all calculations.

To check the adequacy of the numerical scheme, results for a limiting case of mixed convection problem in rectangular cavities with moving isothermal sidewalls and constant flux heat source on the bottom wall which is done by Guo and Sharif [2]. The present results have been compared with their study using mean Nusselt numbers for different configurations of the constant heat flux heaters. A good agreement was found between the present predicted results and those of Guo and Sharif [2] as listed in tab. 2

Table 1. Grid independence study for $Ri = 1$

Grid points			51 51		81 81		101 101	
Ha	ε	φ	Iteration	$\psi_{\max}$	Iteration	$\psi_{\max}$	Iteration	$\psi_{\max}$
0	0.2		3913	-0.1075	8734	-0.1074	12670	-0.1074
100	0.2	0°	10370	-0.0284	20532	-0.0285	27648	-0.0285

Table 2. Comparison of results with those of Guo and Sharif [2] for different dimensionless length of the heat source

Ri	ε	Nu_a (present)	Nu_a [2]
0.1	0.2	8.7	8.7
0.1	0.4	7.3	7.2
0.1	0.6	7.4	7.4
0.1	0.8	8.0	8.1

Table 3. Comparison of results with those of Guo and Sharif [2] for $Ri = 1$

Ri	ε	c/w	Nu_a (present)	Nu_a [2]
1	0.2	0.2	10.9	12.5
1	0.2	0.3	9.7	10.0
1	0.2	0.4	9.0	9.0
1	0.2	0.5	8.7	8.7

Table 4. Comparison of results with those of Rudraiah *et al.* [24] for different Ha and $Pr = 0.733$ and $A = 1$

Gr	Ha	Nu_a (present)	Nu_a (ref. [24])
$2 \cdot 10^4$	0	2.53	2.52
$2 \cdot 10^4$	10	2.23	2.22
$2 \cdot 10^4$	50	1.08	1.08
$2 \cdot 10^4$	100	1.01	1.01

($Ri = 0.1$) and tab. 3 ($Ri = 1$). These results showed strong support of the use of the present numerical method. These values are taken from their graphic. A comparison with study of Rudraiah *et al.* [24] has been performed and results are presented in tab. 4. As it can be observed, there is an acceptable agreement between the results.

Evaluation of heat transfer

The local heat transfer coefficient is defined:

$$h_x = \frac{q}{T_x(x) - T_0} \quad (19)$$

The local heat transfer coefficient i at a given point on the heat source surface where $T_s(x)$ is the local temperature on the surface.

Local Nusselt number:

$$Nu = \frac{h_x L}{k} = \frac{1}{\theta_s(x)} \quad (20)$$

Average Nusselt number:

$$Nu_a = \frac{\bar{h} L}{k} = \frac{1}{\varepsilon_0} \frac{1}{\theta_s(x)} dx \quad (21)$$

where $\theta_s(x)$, is the local dimensionless temperature.

Results and discussion

A complicated numerical study has been performed to see the effects of Richardson number, length and location of constant heat flux heater, aspect ratio, direction of magnetic field and Hartmann number on mixed convection flow, and thermal behavior in a lid-driven enclosure with a constant heat flux heater. Prandtl number of fluid is taken as 0.71 for whole study. All calculations were performed for $Gr = 10^4$ and Reynolds number was changed to get various Richardson numbers.

Figure 2 presents the streamlines (on the left) and isotherms (on the right) for square enclosure ($A = 1$) at different Richardson number, which changes between 0.1 and 100, in the case of without magnetic field ($Ha = 0$) and $\varepsilon = 0.2$. As well-known from the literature, the value of the Richardson number is a measure of the importance of natural convection to forced convection. In this case, heater is located to the middle of the bottom wall ($c/L = 0.5$). Single main rotating cell was formed inside the enclosure and it rotates in clockwise rotating directions. There are two minor cells in the corner of the enclosure but they were not shown in the figure. These cells are disappeared with increasing of Richardson number, namely effects of buoyancy. Flow strength also decreases and oval shaped cell turns to circle shape with increasing of Richardson number. Isotherms plot shows that thermal boundary layer over the heater becomes thicker with increasing

of Richardson number. Isotherms are almost parallel to each other for $Ri = 100$. This indicates that the conduction plays the dominant role for heat transfer process. However, isotherms are gradually distorted toward to moving wall with increasing of y -direction.

The magnetic force in two directions as horizontal ($\varphi = 0^\circ$) and vertical ($\varphi = 90^\circ$) at different Richardson number has been applied and results of thermal and flow fields are given in fig. 3. Comparison of obtained results is plotted in fig. 3 at $A = 1$, $\varepsilon = 0.2$, $Ha = 100$, and $c/L = 0.5$ for different Richardson numbers. These figures can be compared with those of fig. 2 which are given for the absence of the magnetic field. In fig. 3(a), streamlines (on the left) and isotherms (on the right) are given for $Ri = 0.1$. When magnetic field is applied to the system in horizontal direction, multiple cells were formed which are elongates parallel to the moving lid. The top one is extremely thin. The dimensions of cells in y -direction increase with decreasing through to the bottom of the enclosure. The fluid at the bottom of the enclosure is motionless. When magnetic field is applied horizontally, single cell was formed near the lid as seen from fig. 3(b). Isotherms are distributed almost diagonally from left top corner to the right wall. It means that the magnetic field strongly affects the flow and temperature field. For value of $Ri = 10$, namely natural convection is dominant in the system. The center of bot-

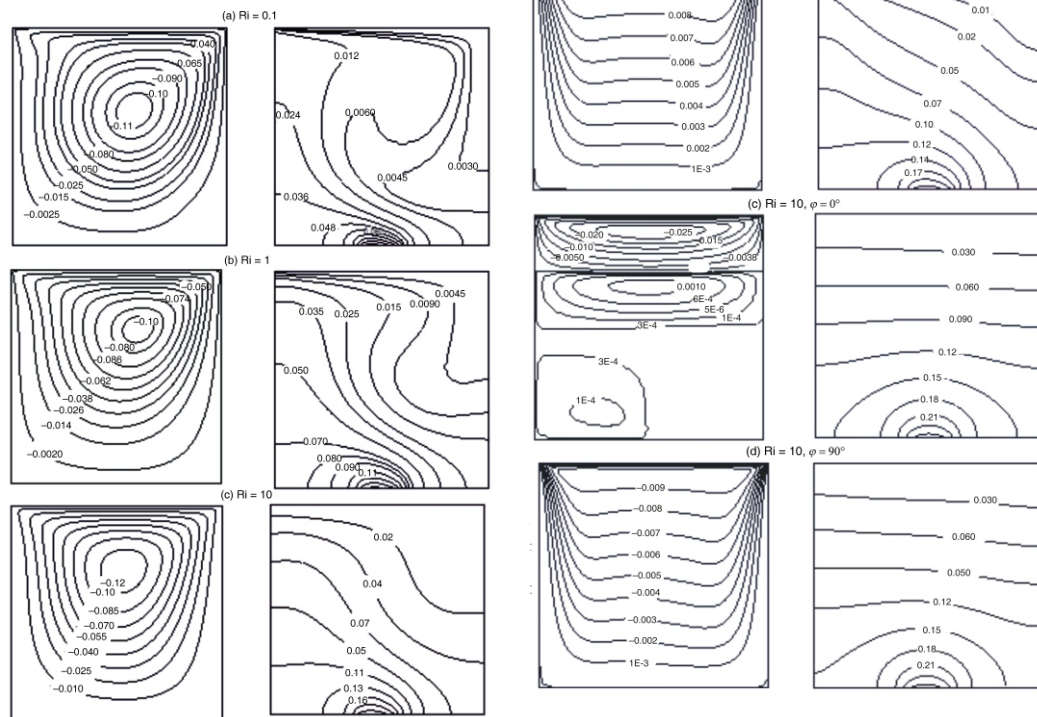


Figure 2. Streamlines (on the left) and isotherms (on the right) for $A = 1$, $\varepsilon = 0.2$, $Ha = 0$, $c/L = 0.5$; (a) $Ri = 0.1$, (b) $Ri = 1$, (c) $Ri = 10$

Figure 3. Streamlines (on the left) and isotherms (on the right) for $A = 1$, $\varepsilon = 0.2$, $Ha = 100$, and $c/L = 0.5$; (a) $Ri = 0.1$, $\varphi = 0^\circ$, (b) $Ri = 0.1$, $\varphi = 90^\circ$, (c) $Ri = 10$, $\varphi = 0^\circ$, (d) $Ri = 10$, and $\varphi = 90^\circ$

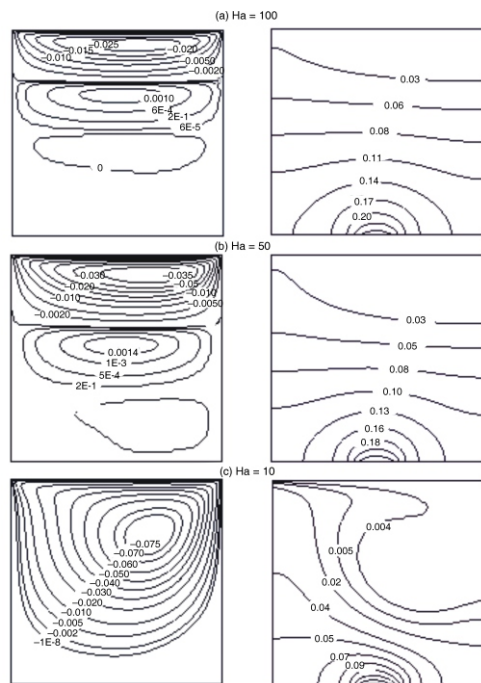


Figure 4. Streamlines (on the left) and Isotherms (on the right) for $A = 1$, $\varepsilon = 0.2$, $c/L = 0.5$, $Ri = 0.1$, $\varphi = 0^\circ$; (a) $Ha = 100$, (b) $Ha = 50$, and (c) $Ha = 10$

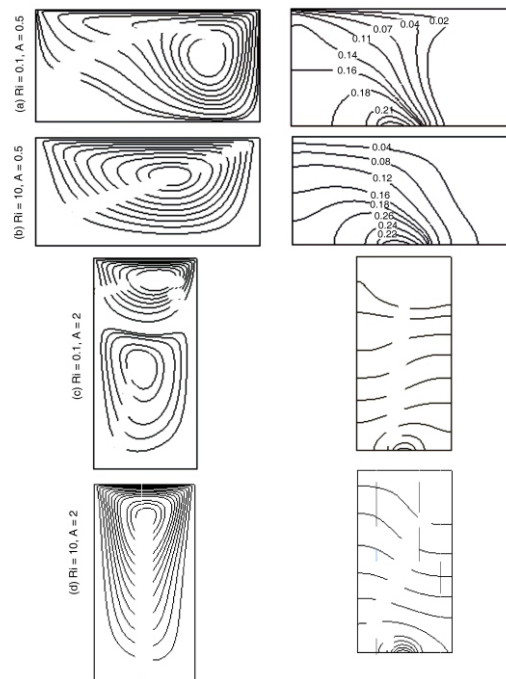


Figure 5. Streamlines (on the left) and isotherms (on the right) for $c/L = 0.5$, $\varepsilon = 0.2$, $Ha = 0$; (a) $Ri = 0.1$, $A = 0.5$, (b) $Ri = 10$, $A = 0.5$, and (c) $Ri = 0.1$, $A = 2$, (d) $Ri = 10$, $A = 2$

tom cell sits near the left bottom corner. Due to domination of the conduction mode of heat transfer, isotherms are parallel to each other for horizontally effective magnetic field as seen from the fig. 3(c). As illustrated from fig. 3(d), streamlines show same trend but isotherms are affected with the application of direction of magnetic field.

Power of Hartmann number on flow and temperature field is shown in fig. 4 via streamlines and isotherms for $A = 1$, $\varepsilon = 0.2$, $c/L = 0.5$, $Ri = 0.1$ and $\varphi = 0^\circ$. It was tested in horizontal direction since magnetic field is more effective in that way as indicated in earlier figures. As shown from the fig. 4 that the multiple cells were formed for higher values of Hartmann number and flow strength is reduced. Primary cell tends to move closer to the moving lid as the strength of the magnetic field increases. This results also supported by Chamkha [12]. The magnetic field suppresses the heat transfer and the isotherms become more parallel to the bottom wall as shown from isotherms patterns in fig. 4 (on the right column).

An aspect ratio is defined as height to bottom length of the enclosure, *e. i.* $A = H/L$. It changes between 0.5 and 2 for this study. Figure 5 gives the streamlines and isotherms for different values of aspect ratios at different Richardson numbers. The moving fluid due to lid-driven top wall impinges to right side and second impingement of fluid occurs to the bottom wall due to shallow cavity. Thus, main cells sits near the right wall and other minor cell forms at the left bottom corner due to domination of forced convection as seen from fig. 5(a). In this case, streamlines move towards to the left vertical wall and the fluid temperature is same almost at the right half of the enclosure. When natural convection becomes dominant, namely $Ri = 10$, again main cell locates at the middle of the cavity. Isotherms are almost parallel to each other with the ef-

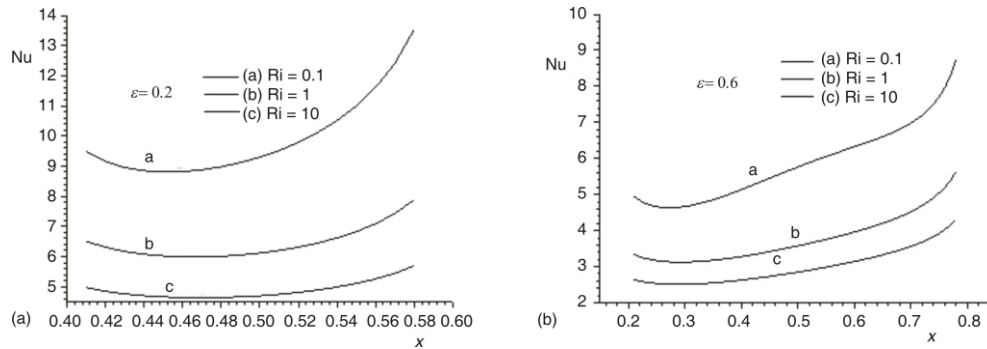


Figure 6. Local Nusselt number for $A = 1$, $Ha = 0$, and $c/L = 0.5$; (a) $\varepsilon = 0.2$, (b) $\varepsilon = 0.6$

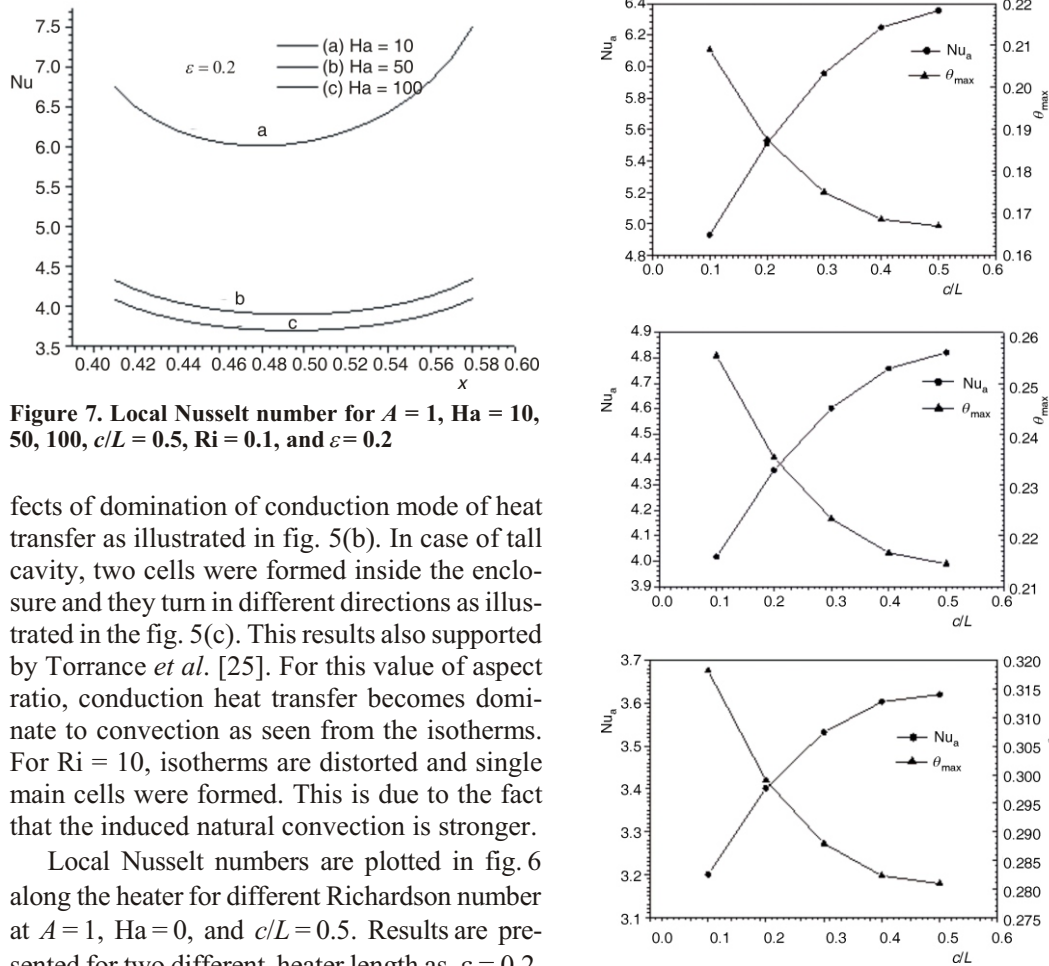


Figure 8. Variation of location of heater on heat transfer and maximum source temperatures, and Nusselt number for $Ri = 1$, $A = 1$, $\varepsilon = 0.2$, and $\varphi = 0^\circ$; (a) $Ha = 0$, (b) $Ha = 10$, (c) $Ha = 100$

fects of domination of conduction mode of heat transfer as illustrated in fig. 5(b). In case of tall cavity, two cells were formed inside the enclosure and they turn in different directions as illustrated in the fig. 5(c). This results also supported by Torrance *et al.* [25]. For this value of aspect ratio, conduction heat transfer becomes dominate to convection as seen from the isotherms. For $Ri = 10$, isotherms are distorted and single main cells were formed. This is due to the fact that the induced natural convection is stronger.

Local Nusselt numbers are plotted in fig. 6 along the heater for different Richardson number at $A = 1$, $Ha = 0$, and $c/L = 0.5$. Results are presented for two different heater length as, $\varepsilon = 0.2$, (fig. 6(a)) and $\varepsilon = 0.6$, fig. 6(b). Local Nusselt number decreases at the left side of the heater and it increases along the heater for $\varepsilon = 0.2$. There is a

Table 5. Average Nusselt number for different governing parameters

ε	Ha	c/L	A	φ	Ri	Nu_a	ε	Ha	c/L	A	φ	Ri	Nu_a
0.2	0	0.5	1	—	0.1	9.82	0.2	0	0.3	1	—	1	5.96
0.2	0	0.5	1	—	1	6.35	0.2	0	0.2	1	—	1	5.51
0.2	0	0.5	1	—	10	4.82	0.2	0	0.1	1	—	1	4.93
0.6	0	0.5	1	—	0.1	5.83	0.2	0	0.5	0.5	—	0.1	3.85
0.6	0	0.5	1	—	1	3.75	0.2	0	0.5	0.5	—	10	3.02
0.6	0	0.5	1	—	10	2.97	0.2	0	0.5	2	—	0.1	5.77
0.2	100	0.5	1	0°	0.1	3.76	0.2	0	0.5	2	—	10	5.18
0.2	100	0.5	1	90°	0.1	4.52	0.6	0	0.5	0.5	—	0.1	3.17
0.2	100	0.5	1	0°	10	3.59	0.6	0	0.5	0.5	—	10	1.97
0.2	100	0.5	1	90°	10	3.60	0.6	0	0.5	2	—	0.1	2.92
0.2	10	0.5	1	0°	0.1	6.31	0.6	0	0.5	2	—	10	3.21
0.2	50	0.5	1	0°	0.1	3.97	0.2	0	0.5	1	—	1	6.35
0.2	100	0.5	1	0°	0.1	3.76	0.4	0	0.5	1	—	1	4.55
0.2	0	0.5	1	—	1	6.35	0.6	0	0.5	1	—	1	3.75
0.2	0	0.4	1	—	1	6.25	0.8	0	0.5	1	—	1	3.18

minimum value around $x = 0.46$ due to lower flow strength at that point. Its value decreases with increasing of Richardson number. However, there is a linear increasing for the case of $\varepsilon = 0.6$. Again, higher values are observed at the edge of the heater due to high temperature difference.

The effects of Hartmann number on variation of local Nusselt number is discussed via fig. 7 for $c/L = 0.5$, $Ri = 0.1$, and $\varepsilon = 0.2$. The values of Hartmann number has been used as $Ha = 0, 10, 50$ and 100 . The figure can also be compared with fig. 6 in the presence of no-magnetic fields ($Ha = 0$). General observation from this figure indicates that heat transfer is reduced with the increasing of Hartmann number. For huge Rayleigh number, local Nusselt number becomes almost constant. It means that magnetic field retards the convection. Thus, conduction mode of heat transfer becomes dominant. The figure also shows that local Nusselt number closer to each other with the increasing of Hartmann number. It means that Hartmann number becomes insignificant for $Ha \gg 100$. Thus, our limit for Hartmann number was chosen less than 100.

Variation of location of heater on mean Nusselt number and maximum source temperatures is plotted at different Hartmann number in fig. 8(a)-(c) for $Ri = 1$, $A = 1$, $\varepsilon = 0.2$. Globally, fig. 8 shows that mean Nusselt number increases as heater closer to the mid-section of the bottom wall due to contacting with more fluid. However, minor cells were formed at the corners and they decrease the heat transfer. Maximum fluid temperature decreases with increasing of ratio of c/L . Figures also indicate that heat transfer is reduced with the increasing of power of magnetic field. On the contrary, maximum heat transfer increases with increasing of Hartmann number. Similar trends are obtained with Guo and Sharif [2] but higher heat transfer is observed in their case. It means that, both direction and location of moving wall is strongly effective on system.

Finally, tab. 5 lists the average variation of average Nusselt numbers for different parameters. As seen from the table that average Nusselt number decreases with increasing of Rich-

ardson numbers. Magnetic field is more effective in the case of $Ri < 0.1$. Heat transfer also decreases with increasing of Hartmann number. It increases with increasing of aspect ratio of the enclosure for all values of heater length.

Conclusions

Mixed convection in a lid-driven enclosure with a constant heat flux heater in the presence of magnetic field was numerically investigated using differential quadrature method in the present study. The effects of Richardson number, the aspect ratio of the enclosure, Hartmann number, ratio of length of heater, and ratio of location of heater are examined in detail. The differential quadrature method gives results which are compared favorably with the literature. Using of smaller grid and lower computational time are the advantages of the method. The method is also more useful for complicated enclosure.

Results from the considered model showed that heat transfer is higher in deep cavities than that of shallow cavities. It decreases with increasing of Richardson numbers and Hartmann numbers. It is found that magnetic field affects the flow and temperature field and it retards the heat transfer. Thus, magnetic field can be a control parameter from the heat transfer and flow field point of view. Both location and length of heater are effective parameters on heat transfer and flow field that heat transfer decreases with increasing of dimensionless length of heater.

The results also show that, with an increase in the Richardson number, average Nusselt number takes lower values due to the weaker forced convection. The magnetic field reduces the circulation in the cavity. When the magnetic field becomes stronger, it causes the convection heat transfer to reduce and, subsequently, conduction heat transfer becomes dominate. The average heat transfer rate increases with increasing of aspect ratio and with increasing of ratio of (c/L) location of heater on bottom surface. The average heat transfer rate decreases the x -directional magnetic field is more effective in damping convection than the y -directional magnetic field.

Nomenclature

A – aspect ratio, [–]
 $\vec{B}$ – magnetic field strength, [T]
 c – distance of the midpoint of the source plate from the right wall, [m]
 Gr – Grashof number, [–]
 $\vec{g}$ – gravitational acceleration, [ms^{-2}]
 H – height of enclosure, [m]
 Ha – Hartmann number, [–]
 $\vec{j}$ – current density [Am^{-2}]
 k – thermal conductivity, [$Wm^{-1}K^{-1}$]
 L – length of enclosure, [m]
 Nu – Nusselt number, [–]
 Pr – prandtl number, [–]
 p – pressure, [Pa]
 q – heat flux at the source, [Wm^{-2}]
 Re – Reynolds number, [–]
 Ri – Richardson number, [–]
 T – temperature, [K]
 U – lid speed, [ms^{-1}]
 u, v – x - y velocity components, [ms^{-1}]

w – length of heat source, [m]
 x, y – cartesian co-ordinates, [m]

Greek symbols

α – thermal diffusivity of the fluid, [m^2s^{-1}]
 β – coefficient of thermal expansion of the fluid, [$1/K$]
 ε – dimensionless length of the heat source, [–]
 θ – dimensionless temperature, [–]
 μ – viscosity [$kgm^{-1}s^{-1}$]
 ρ – density of the fluid, [kgm^{-3}]
 σ – electrical conductivity of the fluid, [sm^{-1}]
 φ – angle of direction of the magnetic field with respect to the coordinate system, [rad]
 ψ – stream function, [m^2s^{-1}]
 ω – vorticity, [s^{-1}]

Superscripts

* – dimensional variable

References

- [1] Papanicolau, E., Jaluria, Y., Mixed Convection from a Localized Heat Source in a Cavity with Conducting Walls, a Numerical Study, *Numerical Heat Transfer, Part A*, 23 (1993), 4, pp. 463-484
- [2] Guo, G., Sharif, M. A. R., Mixed Convection in Rectangular Cavities at Various Aspect Ratios with Moving Isothermal Sidewalls and Constant Flux Heat Source on the Bottom Wall, *International Journal of Thermal Sciences*, 43 (2004), 5, pp. 465-475
- [3] Oztop, H. F., Combined Convection Heat Transfer in a Porous Lid-Driven Enclosure due to Heater with Finite Length, *Int. Comm. Heat Mass Transfer*, 33 (2006), 6, pp. 772-779
- [4] Ogut, E. B., Mixed Convection in an Inclined Lid-Driven Enclosure with a Constant Flux Heater Using Differential Quadrature (DQ) Method, *International Journal of the Physical Sciences*, 5 (2010), 15, pp. 2287-2303
- [5] Kahveci, K., Ogut, E. B., Mixed Convection of Water-Based Nanofluids in a Lid-Driven Square Enclosure with a Heat Source, *Heat Transfer Research*, 42 (2011), 8, pp. 711-735
- [6] Oreper, G. M., Szekely, J., The Effect of an Externally Imposed Magnetic Field on Buoyancy Driven Flow in a Rectangular Cavity, *Journal of Crystal Growth*, 64 (1983), 3, pp. 505-515
- [7] Ozoe, H., Maruo, M., Magnetic and Gravitational Natural Convection of Melted Silicon-Two Dimensional Numerical Computations for the Rate of Heat Transfer, *JSME*, 30 (1987), 263, pp. 774-784
- [8] Ece, M. C., Buyuk, E. Natural Convection Flow under a Magnetic Field in an Inclined Rectangular Enclosure Heated and Cooled on Adjacent Walls, *Fluid Dyn. Res.*, 38 (2006), 8, pp. 564-590
- [9] Oztop, H. F., et al., Numerical Simulation of Magnetohydrodynamic Buoyancy-Induced Flow in a Non-Isothermally Heated Square Enclosure, *Comm Nonlinear Science Numerical Simulation*, 14 (2009), 3, pp. 770-778
- [10] Ogut, E. B., Magnetohydrodynamic Natural Convection Flow in an Enclosure with a Finite Length Heater Using the Differential Quadrature (DQ) Method, *Numerical Heat Transfer Part A-Applications*, 58 (2010), 11, pp. 900-921
- [11] Hossain, M. A., et al., Buoyancy and Thermocapillary Driven Convection Flow of an Electrically Conducting Fluid in an Enclosure with Heat Generation, *Int. J. Thermal Sci.*, 44 (2005), 7, pp. 676-684
- [12] Chamkha, A., Hydromagnetic Combined Convection Flow in a Vertical Lid-Driven Cavity with Internal Heat Generation or Absorption, *Numerical Heat Transfer, Part A*, 41 (2002), 5, pp. 529-546
- [13] Chatterjee, D., Gupta, S. K., Hydromagnetic Mixed Convective Transport in a Nonisothermally Heated Lid-Driven Square Enclosure Including a Heat-Conducting Circular Cylinder, *Industrial & Engineering Chemistry Research*, 53 (2014), 51, pp. 19775-19787
- [14] Selimefendigil, F., Oztop, H. F., Numerical study of MHD Mixed Convection in a Nanofluid Filled Lid Driven Square Enclosure with a Rotating Cylinder, *International Journal of Heat and Mass Transfer*, 78 (2014), Nov., pp. 741-754
- [15] Ganji, D. D., Malvandi, A., Natural Convection of Nanofluids inside a Vertical Enclosure in the Presence of a Uniform Magnetic Field, *Powder Technology*, 263 (2014), Sep., pp. 50-57
- [16] Aminossadati, S. M. et al., Computational Analysis of Magnetohydrodynamic Natural Convection in a Square Cavity with a Thin Fin, *European Journal of Mechanics B/Fluids*, 46 (2014), July-Aug., pp. 154-163
- [17] Sutton, G. W., Sherman, A., Engineering Magnetohydrodynamics, McGraw-Hill, New York, USA, 1965
- [18] Bellman, R. E., Casti, J., Differential Quadrature and Long Term Integration, *J. Math. Anal. Appl.*, 34 (1971), 2, pp. 235-238
- [19] Shu, C., Richards, B. E., Application of Generalized Differential Quadrature to Solve Two-Dimension Incompressible Navier-Stokes Equations, *Int J Numer Methods Fluids*, 15 (1992), 7, pp. 791-798
- [20] Shu, C., Richards, B. E., Parallel Simulation of Incompressible Viscous Flows by Generalized Differential Quadrature, *Comput System Eng.*, 3 (1992), 1-4, pp. 271-281
- [21] Kahveci, K., Oztuna S., MHD Natural Convection Flow and Heat Transfer in a Laterally Heated Partitioned Enclosure, *European Journal of Mechanics B/Fluids*, 28 (2009), 6, pp. 744-752
- [22] Ece, M. C. ve E. Buyuk, Natural Convection Flow under a Magnetic Field in an Inclined Rectangular Enclosure Heated and Cooled on Adjacent Walls, *Fluid Dynamics Research*, 38 (2006), 8, pp. 564-590
- [23] Ece, M. C. ve E. Buyuk, The Effect of an External Magnetic Field on Natural Convection in an Inclined Rectangular Enclosure, Proceedings of the Institution of Mechanical Engineers, Part C, *Journal of Mechanical Engineering Science*, 221 (2007), 12, pp. 1609-1622
- [24] Rudraiah, N., et al., Effect of a Magnetic Field on Free Convection in a Rectangular Enclosure, *Int. J. Engng. Sci.*, 33 (1995), 8, pp. 1075-1084
- [25] Torrance, K., et al., Cavity Flows Driven by Buoyancy and Shear, *J Fluid Mec.*, 2 (1972), 51, pp. 221-231