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WATER-BASED SQUEEZING FLOW IN THE PRESENCE OF CARBON NANOTUBES BETWEEN TWO PARALLEL DISKS

by

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Present study is dedicated to investigate the water functionalized carbon nanotubes squeezing flow between two parallel discs. Moreover, we have considered MHD effects normal to the disks. In addition we have considered two kind of carbon nanotubes named: single wall carbon nanotubes and multiple wall carbon nanotubes with in the base fluid. Under this squeezing flow mechanism model has been constructed in the form of partial differential equation. Transformed ordinary differential equations are solved numerically with the help of Runge-Kutta-Fehlberg method. Results for velocity and temperature are constructed against all the emerging parameters. Comparison among the single wall carbon nanotubes and multiple wall carbon nanotubes are drawn for skin friction coefficient and local Nusselt number. Conclusion remarks are drawn under the observation of whole analysis.

Key words: squeezing flow, carbon nanotubes, MHD, numerical, parallel disks

Introduction

The squeezing flow of a viscous incompressible fluid between two parallel disks moving normal to their own planes has very useful application in hydro-dynamical machines, particularly in turbo machinery and polymer processing, compression and injection mounding. At the end of 19th century Iijima [1] discovered few potential applications in the field of carbone nanotubes (CNT) for semiconductor devices, solar cells, composites, atomic force microscope tips, ultra-capacitors, radar-absorbing coating, technical textiles, gas storage, *etc.* [2]. Their unique quasi 1-D structure gives them extraordinary electronic (high current carrying capability) as well as mechanical properties [3-5]. Other applications under investigation are their use as chemical carriers for pharmaceutical applications. There are many different types of CNT, but they are normally categorized as either single-walled (SWNT) or multi-walled nanotubes (MWNT). A single-walled CNT is just like a regular straw. It has only one layer, or wall. Multi-walled CNT are a collection of nested tubes of continuously increasing diameters. They can range from one outer and one inner tube (a double-walled nanotube) to as many as 100 tubes (walls) or more. So the main characteristic of these CNT is to provide su-

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perb thermal conductivity as compare to other metallic particles. According to Murshed *et al.* [6], CNT provide roundabout six times better thermal conductivity as compared to other materials at the room temperature.

The study of heat and mass transfer unsteady squeezing viscous flow is an interesting topic of research due to its wide spectrum of scientific and engineering applications such polymer processing, compression, injection modeling, lubricant system, transient loading of mechanical components, and the squeezed films in power transmission, food processing, and cooling water [7]. The earlier work on the squeezing flow under lubrication approximation was reported by Stefan [8]. After that, Mahmood *et al.* [9] investigated the heat transfer characteristics in the squeezed flow over a porous surface. The MHD squeezing flow of a viscous fluid between parallel disks was analysed by Domairry and Aziz [10]. Joneidi *et al.* [11] presented the analytic and numerical solutions for MHD squeezing flow between parallel disks.

So far the previous literature described the examination of nanofluid past a vertical plate or stretching surface when the nanoparticle at the surface is actively controlled. However, very recently, Kuznetsov and Nield [12] revisited their pervious study model of natural convective bounday layer flow a nanofluid past a vertical plate and they assumed that the nanoparticle fraction at the boundary is passively controlled rather than actively and the nanoparticle flux at the wall is zero. In such condition the previous model is more physically realistic one. Furthermore, Khan *et al.* [13] applied the passive controlled model and examined triple diffusive free convection along a horizontal plate. Numerous authors discuss the study of fluid flow and heat transfer in the presence of various nanoparticles [14-20].

This work looks at the water functionalized CNT squeezing flow between two parallel disks in presence of a time-dependent magnetic field. The innovative points of present study are considering two kind of CNT named: SWCNT and MWCNT within the base fluid, also the effect of parameters appeared in the mathematical formulation such as the squeezing parameter, the MHD Hartmann number and the solid volume fraction of nanoparticles on velocity and temperature. The dependency of the skin friction and the local Nusselt numbers on the previous mentioned parameters is numerically investigated.

Mathematical model

Consider MHD incompressible water based nanofluid flow between two parallel infinite disks in such a way there is a finite distance between them. We incorporate CNT nanoparticles: SWCNT and MWCNT within the base fluid (water). Magnetic field, $B_0(1 - at)^{-1/2}$, effects are applied normal to the disks and on the bases of the flow assumption low Reynolds



Figure 1. Geometry of the problem

number, the induced magnetic field is neglected. Uniform temperatures T_w and T_h are defined at lower surface of the lower disk at z = 0, and upper surface of the upper disk z = h(t). Furthermore, geometry of the problem is settled in such a way that the upper disk is moving with the velocity $aH(1 - at)^{-1/2}/2$ in both directions (towards and away) from the stationary lower plate at z = 0. Physical interpretation of the model is presented in fig. 1. The cylindrical co-ordinate system (r, α , z) is considered and due to the rotational symmetry of the flow $(\partial/\partial \alpha = 0)$, the azimuthal component v of the velocity V = (u, v, w) vanishes identically. Thus the governing and energy equations for the unsteady 2-D flow of a viscous fluid take the following form:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho_{\rm nf}} \frac{\partial p}{\partial r} + \frac{\mu_{\rm nf}}{\rho_{\rm nf}} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma}{\rho_{\rm nf}} B^2(t)u \tag{2}$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho_{\rm nf}} \frac{\partial p}{\partial z} + \frac{\mu_{\rm nf}}{\rho_{\rm nf}} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right)$$
(3)

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha_{\rm nf} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right)$$
(4)

In eqs. (1)-(4), u and w are the velocity components along the r- and z-directions, respectively, p – the pressure, T – the temperature, ρ_{nf} – the density of nanofluid, μ_{nf} – the dynamic viscosity of nanofluid, and k_{nf} – the thermal conductivity of the nanofluid which are defined:

$$\mu_{\rm nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \qquad \rho_{\rm nf} = (1-\phi)\rho_f + \phi\rho_{\rm CNT}, (\rho c_p)_{\rm nf} = (1-\phi)(\rho c_p)_{\rm f} + \phi(\rho c_p)_{\rm CNT}, \qquad \nu_{\rm nf} = \frac{\mu_{\rm nf}}{\rho_{\rm nf}}, \frac{k_{\rm nf}}{k_{\rm f}} = \frac{1-\phi + 2\phi\left(\frac{k_{\rm CNT}}{k_{\rm CNT} - k_{\rm f}}\right)\ln\left(\frac{k_{\rm CNT} + k_{\rm f}}{2k_f}\right)}{1-\phi + 2\phi\left(\frac{k_{\rm f}}{k_{\rm CNT} - k_{\rm f}}\right)\ln\left(\frac{k_{\rm CNT} + k_{\rm f}}{2k_{\rm f}}\right)}, \qquad \alpha_{\rm nf} = \frac{k_{\rm nf}}{(\rho c_p)_{\rm nf}}$$
(5)

with k_{CNT} is the thermal conductivity of the CNT fraction, k_{f} – the thermal conductivity of base fluid, ρc_p – the specific heat capacity, and ϕ – the solid volume fraction of nanoparticles. The respective boundary conditions are written:

$$u = 0, \qquad w = \frac{dh}{dt}, \qquad \text{at} \qquad z = h(t)$$

$$u = 0, \qquad w = 0, \qquad \text{as} \qquad z \to 0$$

$$T = T_w \qquad \text{at} \qquad z = 0,$$

$$T = T_h \qquad \text{at} \qquad z = h(t).$$
(6)

Here T_w is the temperature of the lower disk at z = 0, and T_h – the temperature of the upper disk at z = h(t). Introducing the following transformations:

$$u = \frac{ar}{2(1-at)} f'(\eta), \qquad w = \frac{aH}{(1-at)^{1/2}} f(\eta),$$

$$B(t) = \frac{B_0}{\sqrt{1-at}}, \qquad \eta = \frac{z}{H\sqrt{1-at}}, \qquad \theta = \frac{T-T_h}{T_w - T_h}.$$
(7)

Through eqs. (2)-(4), eliminating the pressure gradients from the resulting equations we finally obtain:

$$\frac{1}{(1-\phi)^{2.5}} f''' - S\left[(1-\phi) + \phi \frac{\rho_{\rm CNT}}{\rho_{\rm f}}\right] (\eta f'' + 3f'' - ff''') - Mf'' = 0$$
(8)

$$\frac{k_{\rm nf}}{k_f} \theta "+ S \Pr\left[(1-\phi) + \phi \frac{(\rho c_p)_{\rm CNT}}{(\rho c_p)_{\rm f}} \right] (2f ' \theta - \eta \theta ') = 0$$
(9)

with the boundary conditions defined as:

$$f(0) = 0,$$
 $f'(0) = 0,$ $\theta(0) = 1,$ $f(1) = 1/2,$ $f'(1) = 0,$ $\theta(1) = 0$ (10)

where prime denotes derivative with respect to η , while S – the squeeze number, M – the Hartman number, and Pr – the Prandtl number, respectively, defined:

$$S = \frac{aH^2}{2\nu_{\rm f}}, \qquad \mathbf{M} = \sqrt{\frac{\sigma B_0^2 H^2}{a\rho_{\rm f}}}, \qquad \mathbf{Pr} = \frac{\mu_{\rm f}(\rho c_p)_{\rm f}}{\rho_{\rm f} k_{\rm f}}$$
(11)

Physical quantities of interest are the skin friction coefficient and Nusselt number which are defined:

$$C_{\rm f} = \frac{\mu_{\rm nf} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)_{z=h(t)}}{\rho_{\rm nf} \left(\frac{-aH}{2\sqrt{1-at}}\right)^2}, \qquad {\rm Nu} = \frac{k_{\rm nf} \left(\frac{\partial T}{\partial z}\right)_{z=h(t)}}{k_{\rm f} \left(T_{\rm w} - T_{h}\right)}$$
(12)

Making use of eq. (7) in eq. (12), we get:

$$\frac{H^2}{r^2} \operatorname{Re}_{\mathrm{r}} C_{\mathrm{fr}} = \frac{1}{(1-\phi)^{2.5} \left[(1-\phi) + \phi \frac{\rho_{\mathrm{CNT}}}{\rho_{\mathrm{f}}} \right]} f''(1), \qquad \sqrt{1-at} \operatorname{Nu} = -\frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} \theta'(0) \quad (13)$$

where $\operatorname{Re}_{r} = [\rho_{f} r a H (1 - at)^{1/2}]/a \mu_{f}$ is the local squeezed Reynolds number.

Results and discussion

The mentioned coupled differential eqs. (8) and (9) along with the boundary conditions defined in eq. (10) are solved numerically. Since the present mathematical model contains the two point boundary value problem, so we apply Runge-Kutta-Fehlberg (RKF) method to solve this system. The step size is taken as $\Delta \eta = 0.01$ and the procedure for RKF method is repeated until we get the asymptotically convergent results within a tolerance level of 10^{-6} .

The MHD incompressible flow of water based CNT between two parallel infinite disks is studied numerically. Both SWCNT and MWCNT with different solid volume fraction of nanotubes are considered. The variation of axial velocity SWCNT with squeezing and MHD parameters is depicted in figs. 2(a) and (b), respectively. During squeezing, the velocity increases in the axial direction from minimum to maximum. It can be seen that the axial velocity is almost independent of both parameters for different values of volume fraction



Figure 2. Variation of axial velocity for SWCNT with (a) squeezing parameter S and (b) with MHD parameter M

of nanotubes. No appreciable effects of nanotubes on the axial velocity could be observed in any case. The effects of squeezing and magnetic parameters on the radial velocity of SWCNT for different values of volume fraction of nanotubes are reported in figs. 3(a) and (b). It is observed that the radial velocity increases to maximum and then decreases to zero for each value of volume fraction of nanotubes. It is important to note that S = 0 shows no squeeze, S > 0shows that both discs move apart, and S < 0 shows that both discs move towards each other. Consequently, the maximum velocity increases when S < 0 and decreases when S > 0, as shown in fig. 3(a). The effects of magnetic field on the radial velocity are presented in fig.



Figure 3. Variation of radial velocity for SWCNT with (a) squeezing parameter S and (b) MHD parameter M

3(b) when the discs move apart. In the absence of magnetic field, the radial velocity is highest at the center of the discs. As the magnetic field increases, it creates a Lorentz force which opposes the motion and as a result the maximum velocity at the center decreases.

The effects of squeeze and magnetic parameters on the dimensionless temperature of SWCNT between two parallel discs are illustrated in figs 4(a) and (b), respectively. In the presence of magnetic field, the dimensionless temperature of conventional fluid ($\phi = 0$) is found to be maximum between two discs moving apart (S > 0) and it decreases when the volume fraction of nanotubes increases. This is due to increase in thermal conductivity of nanotubes with an increase in the solid volume fraction. In the other case, when both discs move toward each other and the nanofluids is squeezed more, the behavior of dimensionless temperature is just opposite. Figure 4(b) reveals that, for conventional fluid $\phi = 0$, the effects of magnetic field on the dimensionless temperature are negligible and these effects are noticeable as the volume fraction of nanotubes increases. The net effect of magnetic field is to decrease the dimensionless temperature due to Lorentz force.

The variation of skin friction of water-based SWCNT between two parallel discs with volume fraction of nanotubes is presented in fig. 5(a) for different values of squeeze parameter, and in fig. 5(b) for different values of magnetic parameter. As the volume fraction of nanotubes increases, the density of CNT increases and as a result the skin friction also increases in both cases. In fact, it increases the pumping power to flow CNT between parallel discs. It can be seen that the skin friction is higher when both discs move apart (S > 0) and lower when both discs move towards each other, as shown in fig. 5(a). On the other hand,



Figure 4. Variation of dimensionless temperature for SWCNT with (a) squeezing parameter *S* and (b) MHD parameter *M*

Figure 5. Variation of skin friction for SWCNT with (a) squeezing parameter *S* and (b) MHD parameter *M* skin friction is lower in the absence of magnetic field and increases with increasing magnetic field. The reason is again the Lorentz force which opposes the flow. This is shown in fig. 5(b) where the discs are moving apart (S > 0). The comparison of skin friction and Nusselt numbers for SWCNT and MWCNT between parallel discs is presented in figs. 6(a) and (b), respectively. For conventional fluid ($\phi = 0$), the skin friction and Nusselt numbers are lowest and increase with an increase in the volume fraction of nanotubes. This is due to increase in density and thermal conductivity of CNT with an increase in the volume fraction of nanotubes. It is important to note that, due to higher thermal conductivity, SWCNT have higher Nusselt numbers than MWCNT, as shown in fig. 6(b). However, when the discs move apart, the skin friction of SWCNT shows opposite behavior, fig. 6(a). The variation of Nusselt numbers with volume fraction of SWCNT is depicted in figs. 7(a) and (b) for different values of squeezing and magnetic parameters, respectively. As expected, the Nusselt numbers increase with volume fraction of CNT in each case. When the discs move apart, the Nusselt numbers are found to be lower and as the discs come closer the Nusselt numbers increase due to squeezing effect. This is shown in fig. 7(a) in the presence of magnetic field. The effects of magnetic field on the Nusselt numbers are negligible when the discs move towards each other, as shown in fig. 7(b). Although behavior of the isotherms remains same but slight difference will appear because of the variation of squeezing parameter S. So, figs. 8(a)-(c), are demonstrating the effects of squeezing parameter S on isotherms plots when Hartmann number, M = 1, and nanoparticle volume fraction is $\phi = 0.2$.



Figure 6. Comparison among water--based CNT; (a) for skin friction, (b) for

Variation of Nusselt number for SWCNT with (a) squeezing parameter S and (b) MHD parameter M



Figure 8. Isotherms plots for various values of S (for color image see journal web site)

Concluding remarks

All this studies carried out the squeezing flow phenomena for water-based CNT fluid. To sustain this phenomena more effective, magnetic field is applied normal to the discs. Initially we compare the results of water-based SWCNT and MWCNT. After this compare results are constructed for water-based SWCNT for velocity, temperature skin friction coefficient and Nusselt number. Here are following key points for whole analysis.

- Water-based SWCNT has low skin friction as compare to water-based MWCNT,
- Higher heat transfer rate is found for water-based SWCNT for Nusselt number as compare to water-based MWCNT,
- For every increasing value of squeezing parameter, S, and Hartmann number, M, velocity profile $f'(\eta)$ is gradually decreasing,
- Increase in the nanoparticle volume fraction, ϕ , has increasing effect on velocity profile $f'(\eta)$ for each value of S and M,
- Temperature profile has opposite variation with increasing values of nanoparticle volume fraction for S > 0 and S < 0, and
- Temperature profile is rising with increasing values of nanoparticle volume fraction, ϕ , when M = 0 and M = 10.

Nomenclature

- magnetic field strength, $[kgs^{-2}A^{-1}]$ B_0
- specific heat, $[Jkg^{-1}K^{-1}]$ $\begin{array}{c} c_p \\ C_{\mathrm{f}} \end{array}$
- friction coefficient, $[Jkg^{-1}K^{-1}]$
- f - dimensionless stream function
- thermal conductivity, $[WK^{-1}m^{-1}]$ k
- magnetic parameter М
- squeezing parameter S
- Nu_x local Nusselt number
- Prandtl number of base fluid Pr
- Re_x local Reynolds number T
- local fluid temperature, [K]
- -x-component of velocity, [ms⁻¹] и -z-component of velocity, $[ms^{-1}]$ w
- radius of the disk, [m] r
- distance between the disks, [m] \overline{z}

Greek symbols

- thermal diffusivity, [m²s⁻¹] α
- volume fraction of CNT ф
- similarity variable η
- dynamic viscosity, [Nsm⁻² μ
- kinematic viscosity, [m²s⁻ v
- electric conductivity, [Sm⁻¹] σ
- density, [kgm⁻³] ρ
- heat capacity of CNT, $[kgm^{-3}K^{-1}]$ ρC
- dimensionless temperature θ

Subscripts

- nanofluid nf
- base fluid f
- CNT - carbon nanotube

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