# MAGNETOHYDRODYNAMIC FLOW OF NANOFLUID OVER PERMEABLE STRETCHING SHEET WITH CONVECTIVE BOUNDARY CONDITIONS

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Analysis has been carried out for the magnetohydrodynamic boundary layer flow of nanofluid. The flow is caused by a permeable stretching sheet. Convective type boundary conditions are employed in modeling the heat and mass transfer process. Appropriate transformations reduce the non-linear partial differential equations to ordinary differential equations. The convergent series solutions are constructed. Graphical results of different parameters are discussed. The behaviors of Brownian motion and thermophoretic diffusion of nanoparticles have been examined. The dimensionless expressions of local Nusselt and local Sherwood numbers have been evaluated and discussed.

Key words: magnetohydrodynamic nanofluid, suction/injection, convective boundary conditions

#### Introduction

In recent times, the convective heat transfer through nanoparticles has been the topic of extensive research. The nanoparticles (nanometer sized particles) are made up of metals, carbides, oxides, or carbon nanotubes. The nanofluids are formed by adding nanoparticles into many conventional fluids like water, ethylene glycol, and engine oil. The use of additive is a process which enhances the heat transfer performance of base fluids. Choi [1] experimentally found that addition of nanoparticles in conventional/base fluid appreciably enhances the thermal conductivity of the fluid. Cooling is one of the technical challenges faced in many industries. Use of nanofluids as coolants allows for smaller size and better positioning of the radiators which eventually consumes less energy for overcoming resistance on the road. Nanoparticles in refrigerant/lubricant mixtures could enable a cost effective technology for improving the efficiency of chillers that cool buildings. Analysis of fluid flow in presence of magnetic field has promising applications in engineering, chemistry, physics, polymer industry, and metallurgy. Rate of heat cooling has key role in improving the desired characteristics of end product in such applications. An electrically conducting fluid subject to magnetic field is useful in controlling the rate of cooling. The magnetohydrodynamic (MHD) flow is also of great interest in problems associated with physiological fluids, for instance, blood plasma and blood pump machines.

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Having such salient features in mind, numerous articles have been written for the MHD flows in different flow configurations. The MHD nanofluids are further important in hyperthermia, cancer therapy, and safer surgery by cooling sink float separation, magnetic cell separation, and contrast enhancement in magnetic resonance imaging. At present, the literature on theoretical and experimental attempts about nanofluids is quite extensive. The comprehensive review on nanofluids can be found in [2-8]. Detailed review on this topic up to 2012 has been made recently in [9, 10]. Besides these, a comprehensive survey of convective transport in nanofluids is presented by Buongiorno [11]. He developed a non-homogeneous equilibrium model for convective transport to describe the heat transfer enhancement of nanofluids. Such abnormal increase in thermal conductivity occurs due to the presence of two main velocity-slip effects, namely, the Brownian diffusion and the thermophoretic diffusion of the nanoparticles. Later, Buongiorno *et al.* [12] conducted novel investigations which show no anomalous thermal conductivity enhancement in the considered fluids. Some recent research articles on the topic can be seen through the studies, [13-23] and many refs. therein.

In the polymer industry, the flow over a stretching surface has important applications. Glass blowing, continuous casting and spinning of fibres also involve the flow due to stretching surface. Crane [24] provided the classical solution for the boundary layer flow of viscous fluid over a sheet moving with velocity varying linearly with distance from a fixed point. The classical work of Crane [24] has been undertaken for the stretching flows in different configurations. Mukhopadhyay [25] investigated the slip effects on MHD boundary layer flow by an exponentially stretching sheet with suction/blowing and thermal radiation. Effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet have been investigated by Bhattacharyya et al. [26]. Exact solutions for 2-D laminar flow over a continuously stretching or shrinking sheet in an electrically conducting quiescent couple stress fluid have been derived by Turkyilmazoglu [27]. Turkyilmazoglu [28] presented a note on micropolar fluid flow and heat transfer over a porous shrinking sheet. Effect of heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium is analyzed by Sheikholeslami et al. [29]. Sheikholeslami and Ganji [30] discussed the 3-D flow of nanofluid in a rotating system. Ibrahim et al. [31] analyzed the MHD stagnation point flow of nanofluid towards a stretching sheet. Effect of slip on unsteady stagnation point flow of nanofluid over a stretching sheet is investigated by Malvandi et al. [32].

The purpose of present paper is to examine the nanofluid flow over a stretching surface by homotopy analysis method (HAM) [33-39]. The surface is taken permeable and exhibits convective type boundary conditions [40, 41]. In fact the nanoparticles are used to enhance the thermal conductivity of the fluid. On the other hand the convective boundary condition in dimensionless form is appeared as Biot number. Increase in Biot number give rise to the temperature. Increase in temperature corresponds to an enhancement in the thermal conductivity. So the nanofluid with convective boundary condition is more appropriate model in comparison to the constant surface temperature conditions. Convective boundary conditions represent the heat transfer rate through the surface being proportional to the local difference in temperature with the ambient conditions. Such configuration occurs in several engineering devices for instance in heat exchangers, where the conduction in the solid tube wall is greatly influenced by the condition in the fluid flowing over it.

#### Mathematical analysis

We consider the 2-D flow of nanofluid bounded by a permeable stretching sheet. The x-axis is taken along the stretching surface in the direction of motion, and y-axis is perpendicu-

lar to it. A uniform transverse magnetic field of strength,  $B_0$ , is applied parallel to the y-axis. It is assumed that the effects of induced magnetic and electric fields are negligible. Salient features of Brownian motion and thermophoresis are present. The temperature, T, and the nanoparticle fraction, C, at the surface have constant values  $T_w$  and  $C_w$ , respectively. The ambient values of T and C attained as y tends to infinity are denoted by  $T_\infty$  and  $C_\infty$ , respectively. The conservation of mass, momentum, energy, and nanoparticles equations for nanofluids are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho_f}$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_{\rm B} \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_{\rm T}}{T_{\infty}} \left( \frac{\partial T}{\partial y} \right)^2 \right]$$
(3)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_{\rm B} \frac{\partial^2 C}{\partial y^2} + \frac{D_{\rm T}}{T_{\infty}} \frac{\partial^2 T}{\partial y^2}$$
(4)

where *u* and *v* are the velocity components along the x- and y-directions, respectively,  $\rho_{\rm f}$  is the fluid density,  $\nu$  – the kinematic viscosity,  $(\rho c)_{\rm f}$  – the heat capacity of the fluid,  $\tau = (\rho c)_p / (\rho c)_{\rm f}$  – the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid,  $D_{\rm B}$  – the Brownian diffusion coefficient,  $D_{\rm T}$  – the thermophoretic diffusion coefficient,  $\alpha$  – the thermal diffusivity,  $\sigma$  – the electrical conductivity of the base fluid, and  $c_p$  – the specific heat at constant pressure.

We assume the bottom surface of the plate is heated by convection from a hot fluid at temperature,  $T_{\rm f}$ , which provides a heat transfer coefficient, *h*. The boundary conditions are prescribed:

$$u = u_w(x) = ax, \quad v = v_w, \quad -k \frac{\partial T}{\partial y} = h(T_f - T), \quad -D_m \frac{\partial C}{\partial y} = k_m(C_f - C) \quad \text{at } y = 0,$$
$$u = 0, \quad T \to T_{\infty}, \quad C \to C_{\infty} \quad \text{as } y \to \infty$$
(5)

in which  $v_w$  is the wall heat transfer velocity, k – the thermal conductivity of fluid, h – the convective heat transfer coefficient,  $D_m$  – the molecular diffusivity of the species concentration, and  $k_m$  – the wall mass transfer coefficient. Using the transformations:

$$\eta = \sqrt{\frac{a}{v}}y, \quad u = axf'(\eta), \quad v = -\sqrt{va}f(\eta), \quad \theta(\eta) = \frac{T - T_{\infty}}{T_{\rm f} - T_{\infty}}, \quad \phi(\eta) = \frac{C - C_{\infty}}{C_{\rm f} - C_{\infty}} \tag{6}$$

eq. (1) is satisfied automatically, and eqs. (2)-(5) take the forms:

$$f''' - f'^2 + ff'' - Mf' = 0$$
<sup>(7)</sup>

$$\frac{1}{\Pr}\theta'' + f\theta' + N_{\rm B}\phi'\theta' + N_{\rm T}\theta^{2} = 0$$
(8)

$$\phi'' + \operatorname{Sc} f \phi' + \frac{N_{\mathrm{T}}}{N_{\mathrm{B}}} \theta'' = 0$$
(9)

$$f(0) = S, \qquad f'(0) = 1, \qquad \theta'(0) = -\gamma_1 [1 - \theta(0)], \qquad \phi'(0) = -\gamma_2 [1 - \phi(0)], \qquad (10)$$
$$f'(\infty) = 0, \qquad \theta(\infty) = 0, \qquad \phi(\infty) = 0$$

where prime indicates the differentiation with respect to  $\eta$ . Moreover, Prandtl number, thermal Biot number,  $\gamma_1$ , concentration Biot number,  $\gamma_2$ , mass transfer parameter, S with S > 0 for suction, and S < 0 for injection, Schmidt number, Hartman number, Brownian motion parameter,  $N_{\rm B}$ , and thermophoresis parameter,  $N_{\rm T}$ , are defined by the definitions:

$$\Pr = \frac{\nu}{\alpha}, \qquad \gamma_1 = \frac{h}{k} \sqrt{\frac{\nu}{a}}, \qquad \gamma_2 = \frac{k_{\rm m}}{D_{\rm m}} \sqrt{\frac{\nu}{a}}, \qquad S = -\frac{\nu_{\rm w}}{\sqrt{\nu a}}, \qquad S = -\frac{\nu}{D_{\rm B}}, \qquad M = \frac{\sigma B_0^2}{\rho_{\rm f} a}, \qquad N_{\rm B} = \frac{(\rho c)_p D_{\rm B} (C_{\rm w} - C_{\infty})}{(\rho c)_{\rm f} \nu}, \qquad N_{\rm T} = \frac{(\rho c)_p D_{\rm T} (T_{\rm w} - T_{\infty})}{(\rho c)_{\rm f} T_{\infty} \nu}$$
(11)

The local Nusselt number, Nu, and Sherwood number, Sh, are:

$$Nu = \frac{xq_{w}}{k(T_{f} - T_{\infty})}, \qquad Sh = \frac{xq_{m}}{D_{B}(C_{f} - C_{\infty})}$$
(12)

where  $q_w$  and  $q_m$  denote the wall heat and mass fluxes, respectively. In dimensionless form:

NuRe<sub>x</sub><sup>-1/2</sup> = 
$$-\theta'(0)$$
, ShRe<sub>x</sub><sup>-1/2</sup> =  $-\phi'(0)$  (13)

where  $\operatorname{Re}_{x} = u_{w}(x)x/\nu$  is the local Reynolds number.

We choose the initial guesses  $f_0(\eta)$ ,  $\theta_0(\eta)$  and  $\phi_0(\eta)$  and the linear operators  $\mathbf{L}_f$ ,  $\mathbf{L}_{\theta}$  and  $\mathbf{L}_{\phi}$  in the forms:

$$f_0(\eta) = S + 1 - \exp(-\eta)$$
 (14)

$$\theta_0(\eta) = \frac{\gamma_1}{1 + \gamma_1} \exp(-\eta) \tag{15}$$

$$\phi_0(\eta) = \frac{\gamma_2}{1 + \gamma_2} \exp(-\eta) \tag{16}$$

$$\mathbf{L}_{\mathrm{f}}(f) = f''' - f' \tag{17}$$

$$\mathbf{L}_{\theta}(\theta) = \theta'' - \theta \tag{18}$$

$$\mathbf{L}_{\phi}(\phi) = \phi'' - \phi \tag{19}$$

where the convergence depends upon the non-zero auxiliary parameters  $\hbar_{f}$ ,  $\hbar_{\theta}$ , and  $\hbar_{\phi}$ .

### Analysis of the results

### Convergence of the derived series solutions

The solution of problem contained in eqs. (7)-(10) is computed employing HAM. The convergence region and rate of approximations for the functions f,  $\theta$ , and  $\phi$  can be controlled and adjusted through the auxiliary parameters  $\hbar_f$ ,  $\hbar_{\theta}$ , and  $\hbar_{\phi}$  (tab. 1). For the admissible values of  $\hbar_f$ ,  $\hbar_{\theta}$ , and  $\hbar_{\phi}$ , the  $\hbar$ -curves of  $f''(\eta)$ ,  $\theta'(\eta)$ , and  $\phi'(\eta)$  for 14<sup>th</sup> order of approximations are displayed. Figure 1 shows that the admissible values of  $\hbar_f$  is

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 $-1.5 \le h_{\rm f} \le -0.4$ . Figure 2 displays that the range for  $\theta$  is  $-1.7 \le h_{\theta} \le -0.6$ , and similarly for  $\phi$  is  $-1.6 \le h_{\phi} \le -0.7$ . Further, the series solutions converge in the whole region of  $\eta$  when  $h_{\rm f} = h_{\theta} = h_{\phi} = -1.2$ .



Figure 1. The *h*-curve for *f* 

# **Results and discussion**

The effects of various parameters on the velocity, temperature, and concentration profiles are discussed. Figures 3 and 4 are plotted to show the effects of Hartman number and mass transfer parameter, S, on the velocity profile, f'. Figure 3 shows the effects of Hartman number on f'. Application of magnetic field has the tendency to slow down the movement of the fluid particles and consequently the velocity decreases. Figure 4 displays the effect of S on f'. In this figure the velocity field f' decreases when S increases. In fact applying suction leads to draw the amount of fluid particles into the wall and hence the velocity boundary layer decreases.

Effects of the Brownian motion parameter,  $N_{\rm B}$ , thermophoresis parameter,  $N_{\rm T}$ , Schmidt number, Prandtl number, Hartman number, mass transfer parameter, S, thermal Biot number,  $\gamma_1$ , and concentration Biot number,  $\gamma_2$ , on the temperature profile,  $\theta$ , and the concentration profile,  $\phi$ , are shown in figs. 5-9. It is noted that an increase in the Brownian motion parameter,  $N_{\rm B}$ , thermophoresis parameter,  $N_{\rm T}$ , and Schmidt number, increase the temperature profile,  $\theta$ , as shown in figs. 5-7. The effects of Prandtl number, on the temperature profile are depicted in fig. 8. This graph shows that the temperature profile  $\theta$ , decreases





Table 1. Convergence of HAM solutions for different order of approximations when  $N_{\rm B} = 0.4$ ,  $N_{\rm T} = 0.3$ , Sc = 1, Pr = 1, M = 0.4, S = 0.5,  $\gamma_1 = 1$ , and

$\gamma_2 = 0.9$			
Order of approximations	-f''(0)	$-\theta'(0)$	- <b>¢</b> ′(0)
1	1.54000	0.441053	0.344668
5	1.45883	0.431349	0.319776
10	1.45934	0.430920	0.318765
15	1.45934	0.430912	0.318750
20	1.45934	0.430912	0.318750
25	1.45934	0.430912	0.318750
30	1.45934	0.430912	0.318750
35	1.45934	0.430912	0.318750
40	1.45934	0.430912	0.318750
50	1.45934	0.430912	0.318750
60	1.45934	0.430912	0.318750

when Prandtl number increases. In fact, the thermal diffusivity decreases by increasing Prandtl number and thus the heat diffused away slowly from the heated surface. Figure 9 illustrates the effects of Hartman number on temperature profile  $\theta$ . The Lorentz force is a resistive force which opposes the fluid motion. As a sequence the heat is produced and thus thermal boundary layer thickness increases. Further, the temperature profile  $\theta$  decreases when S is increased, fig. 10. The temperature profile  $\theta$  also increases when the thermal Biot number,  $\gamma_1$ , increases (fig. 11). Figure 12 illustrates the effects of  $N_{\rm B}$  on  $\phi$ . The concentration profile,  $\phi$ , decreases by increasing the Brownian motion parameter,  $N_{\rm B}$ . Influence of  $N_{\rm T}$  on  $\phi$  can be seen in fig. 13. There is an increase in  $\phi$  when  $N_{\rm T}$  is increased. Figures 14-17 display the effects of Schmidt number, Prandtl number, Hartman number, and S on the concentration profile  $\phi$ . It is observed that concentration profile,  $\phi$ , decreases by increasing these parameters.





Figure 16. Influence of M on  $\phi$ 

Figure 17. Influence of S on  $\phi$ 

It is observed from fig. 18 that the mass fraction field  $\phi$ , increases when thermal Biot number,  $\gamma_1$ , is increased. Also the concentration profile,  $\phi$ , increases by increasing concentration Biot number,  $\gamma_2$ , as depicted in fig. 19.

Figures 20 and 21 describe the variations of the Nusselt number Nu(Re<sub>x</sub>)<sup>-1/2</sup> for  $N_{\rm B}$ ,  $N_{\rm T}$ , and Scmidt number. It is noticed that heat transfer rate decreases as  $N_{\rm B}$  and  $N_{\rm T}$  increase for Scmidt number. Figure 22 shows the effects of  $\gamma_1$  and S on the Nusselt number. In this figure, heat transfer rate increases as  $\gamma_1$  increases for S. Figures 23 and 24 illustrate the variation in dimensionless mass transfer rate Sh(Re<sub>x</sub>)<sup>-1/2</sup> vs.  $N_{\rm B}$  and  $N_{\rm T}$ . Here, the mass transfer rate increases with an increase in  $N_{\rm B}$  and decreases with an increase in  $N_{\rm T}$ . Effects of  $\gamma_2$  and mass transfer parameter S on the Scmidt numberare displayed in fig. 25. It is noted that mass transfer rate increases for higher  $\gamma_2$ .







0.6

0.4

Figure 23. Effects of  $N_{\rm B}$  and Sc and on  $-\phi'(0)$ 



Figure 24. Effects of  $N_{\rm T}$  and Sc on  $-\phi'(0)$ 

Figure 25. Effects of  $\gamma_2$  and S and on  $-\phi'(0)$ 

### Conclusions

The flow of nanofluid generated by a permeable stretching sheet is studied. Effects of different parameters on the velocity, temperature, and concentration distributions are explored. The following observations are worth mentioning.

- The effects of Hartman number and S are similar on the velocity profile f'.
- Increase in  $N_{\rm B}$ ,  $N_{\rm T}$ , Schmidt number, M, and  $\gamma_1$  enhances the temperature profile while increase in Prandtl number and S decreases the temperature profiles.
- There is enhancement of concentration for increasing  $N_{\rm T}$ ,  $\gamma_1$ , and  $\gamma_2$ .
- Local Nusselt number increases by larger  $\gamma_1$ .
- Local Sherwood number increases by increasing  $N_{\rm B}$  and  $\gamma_2$ .

#### Nomenclature

- а - stretching constant,  $[s^{-1}]$
- $B_0$  uniform magnetic field strength, [kgs<sup>-1</sup>A<sup>-1</sup>]
- concentration C
- $C_{\infty}$  ambient fluid concentration
- $C_{\rm f}~$  convective fluid concentration
- specific heat,  $[m^2 s^{-2}]$
- $\dot{D}_{\rm B}$  Brownian diffusion coefficient, [kgm<sup>-1</sup>s<sup>-1</sup>]
- $D_{\rm m}$  molecular diffusivity of the species concentration
- $D_{\mathrm{T}}$ thermophoretic diffusion coefficient,  $[kgm^{-1}s^{-1}K^{-1}]$
- convective heat transfer coefficient,  $[WK^{-1}m^{-2}]$ h
- thermal conductivity, [WK<sup>-1</sup>m<sup>-1</sup>] k
- $k_{\rm m}$  wall mass transfer coefficient M Hartman number k<sub>m</sub>
- $N_{\rm B}$  Brownian motion parameter
- $N_{\rm T}$  thermophoresis parameter
- Nu local Nusselt number
- Prandtl number Pr
- surface mass flux, [Wm<sup>-2</sup>]  $q_{\mathrm{m}}$

- $q_w$  surface heat flux, [Wm<sup>-2</sup>]
- $Re_x$  local Reynolds number
- S mass transfer parameter
- Sc Schmidt number
- Sh local Sherwood number
- T temperature, [K]
- $T_{\infty}$  ambient fluid temperature, [K]
- $T_{\rm f}$  convective fluid temperature, [K]
- u, v, -x- and y-direction velocity
- components, [ms<sup>-1</sup>] – wall heat transfer velocity, [ms<sup>-1</sup>]  $v_{\rm w}$

#### Greek symbols

- $\alpha$  thermal diffusivity, [m<sup>2</sup>s<sup>-1</sup>]
- density, [kgm<sup>3</sup>] D
- electrical conductivity, [s<sup>3</sup>A<sup>2</sup>kg<sup>-1</sup>m<sup>-3</sup>] σ
- ratio between the effective heat capacity of the τ nanoparticles and fluid
- kinematic viscosity, [m<sup>2</sup>s<sup>-1</sup>]

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