ARTIFICIAL BEE COLONY ALGORITHM IN THE SOLUTION OF SELECTED INVERSE PROBLEM OF THE BINARY ALLOY SOLIDIFICATION

by

Edyta HETMANIOK *

Institute of Mathematics, Silesian University of Technology, Gliwice, Poland

Original scientific paper
DOI:10.2298/TSCI140715136H

The paper presents a procedure for reconstructing, on the basis of known measurements of temperature, the heat transfer coefficient and the distribution of temperature in given region of solidifying binary alloy in the casting mould. Solution of the considered inverse problem is found by applying the finite element method for solving the corresponding direct problem and the artificial bee colony algorithm for minimizing the functional representing the error of approximate solution.

Key words: *solidification, binary alloy, Scheil model, swarm intelligence, ABC algorithm*

Introduction

The inverse problems of thermal processes give the very useful tools for analyzing and controlling these processes [1-3]. Inverse problems are used when the causes of the described phenomenon are unknown or not completely defined and their aim is to reconstruct some missing input information which gives the possibility to determine the initial and boundary conditions such that the required run of the process could be ensured. To find the solution of an inverse problem some additional information is needed, for example, the temperature measurements in given points of the domain. However, solving such problems is difficult because the inverse problems of mathematical physics are the ill posed problems in such sense that their analytical solution may not exist and even if exists it may be neither unique nor stable [4, 5]. Therefore, each procedure successful in solving problems of that kind is interesting and desired.

Process of the alloy solidification occurs in the interval from the liquidus temperature to the solidus temperature. Model of solidification in the temperature interval is based on the heat conduction equation with the included source element in which the latent heat of fusion and the volume contribution of solid phase are taken into account. Assuming the form of function describing this contribution, the equation can be converted into the heat conduction equation with the so called substitute thermal capacity. After this transformation the considered differential equation defines the heat conduction in the entire homogeneous domain consisted of the solid phase, two-phase (mushy) zone and the liquid phase [6-9].

Solidification of the alloy is effected by the process of segregation of the alloy components, for instance, the change of concentration corresponds with the change of liquidus and solidus temperatures. The macrosegregation phenomenon and the solidification of binary alloy

* Author's e-mail: edyta.hetmaniok@polsl.pl

were considered, for example, in [10-12]. Most of the available publications are focused on direct problems. The inverse problem associated with this subject is discussed, for example, in [12, 13] as well as in the previous works of Hetmaniok [14] and Hetmaniok and Slota [15, 16] in which for describing the distribution of temperature the Stefan model was used. Novelty of the present paper lies in discussing the inverse problem for the more advanced and real model of the alloy solidification, in which the mushy zone between the liquid and solid phases is taken into consideration as well as the process of segregation of the alloy components.

For describing concentration of the alloy component the Scheil model can be used, in which the diffusion in solid phase is neglected and the convection in the liquid phase causes the equalization of concentration of the alloy component in the liquid phase [17, 18]. Another very often used model is the lever arm model, in which the immediate equalization of chemical composition of the alloy in the liquid phase and solid phase is assumed [15, 18].

In this paper we present the procedure for solving the inverse problem for the binary alloy solidification in the casting mould. Proposed approach is based on the mathematical model suitable for describing the investigated solidification process, the Scheil model describing the macrosegregation process, the finite element method supplemented by the procedure of correcting the temperature field nearby the liquidus and solidus curves [6, 7] for solving the direct problem and the Artificial Bee Colony algorithm [19-21] for minimizing the functional expressing the error of approximate solution. Goal of the discussed inverse problem is the reconstruction of heat transfer coefficient and distribution of temperature in investigated region on the basis of known measurements of temperature.

Description of the problem

The problem is considered in the region occupied by the solidifying material (Ω) and in the region taken by the casting mould (Ω_m). Region occupied by the alloy and by the mould together with the selected points of temperature measurements are illustrated in fig. 1. With respect to the thermal symmetry we discuss half of both regions. Thus, in region $\Omega = \{(x,t): x \in (0,d_1), t \in (0,t^*)\}$ the distribution of temperature of the solidifying material is expressed by means of the following heat conduction equation [6-9]:

$$c\rho \frac{\partial T(x,t)}{\partial t} = \lambda \frac{\partial^2 T(x,t)}{\partial x^2} + L\rho \frac{\partial f_s(x,t)}{\partial t}, \qquad (x,t) \in \Omega \tag{1}$$

where c, ρ, and λ are the specific heat, mass density, and thermal conductivity coefficient, respectively, L denotes the latent heat of solidification, f_S describes the volumetric solid state fraction, T is the temperature, and t and x refer to the time and spatial co-ordinates. The volumetric solid state fraction depends on the temperature, so we get:

$$\frac{\partial f_s}{\partial t} = \frac{\partial f_s}{\partial T}\frac{\partial T}{\partial t} \tag{2}$$

Substituting relation (2) to (1) and transforming the obtained equation we have:

$$\left(c - L\frac{\partial f_s}{\partial T}\right)\rho \frac{\partial T(x,t)}{\partial t} = \lambda \frac{\partial^2 T(x,t)}{\partial x^2} \tag{3}$$

Let us introduce the so called substitute thermal capacity:

$$C = c - L\frac{\partial f_s}{\partial T} \tag{4}$$

By using it the previous equation can be written in the form:

$$C\rho\frac{\partial T(x,t)}{\partial t}=\lambda\frac{\partial^2 T(x,t)}{\partial x^2},\qquad (x,t)\in\Omega \tag{5}$$

In region $\Omega_m=\{(x,t): x\in(d_1,d_2), t\in(0,t^*)\}$ of the casting mould the distribution of temperature is described by equation:

$$c_m\rho_m\frac{\partial T_m(x,t)}{\partial t}=\lambda_m\frac{\partial^2 T_m(x,t)}{\partial x^2},\qquad (x,t)\in\Omega_m \tag{6}$$

where c_m, ρ_m, and λ_m are the specific heat, mass density, and thermal conductivity coefficient, respectively, of the material the casting mould is made of, and T_m denotes the temperature of casting mould.

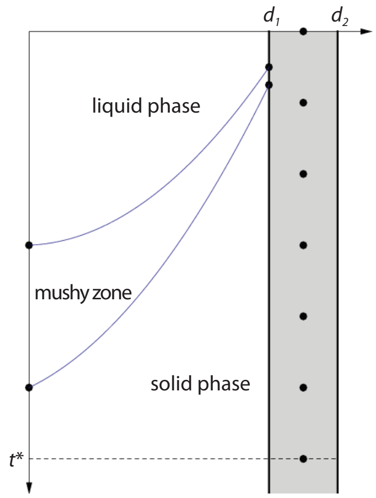

Figure 1. Region of the problem

To complete the previos equations let us define the following initial conditions:

$$T(x,0)=T_0(x),\qquad x\in[0,d_1] \tag{7}$$

$$T_m(x,0)=T_{m,0}(x),\qquad x\in(d_1,d_2] \tag{8}$$

and the homogeneous boundary condition of the second kind resulting from the assumed heat symmetry of the region:

$$-\lambda\frac{\partial T(0,t)}{\partial x}=0,\qquad t\in(0,t^*) \tag{9}$$

On the cast-mould interface the boundary condition of the fourth kind is defined:

$$T(d_1,t)=T_m(d_1,t),\qquad t\in(0,t^*) \tag{10}$$

$$-\lambda\frac{\partial T(d_1,t)}{\partial x}=-\lambda_m\frac{\partial T_m(d_1,t)}{\partial x},\qquad t\in(0,t^*) \tag{11}$$

Outside of the mould the third kind boundary condition is given:

$$-\lambda_m\frac{\partial T_m(d_2,t)}{\partial x}=\alpha\left(T_m(d_2,t)-T_\infty\right),\qquad t\in(0,t^*) \tag{12}$$

where α denotes the heat transfer coefficient and T_∞ means the ambient temperature.

By assuming the linear variation of function f_S with respect to temperature, then since the following equalities must be satisfied:

$$f_s(T_L)=0\quad\text{and}\quad f_s(T_S)=1 \tag{13}$$

where T_L and T_S denote the liquidus and solidus temperature, respectively, we obtain:

$$f_s(T)=\frac{T_L-T}{T_L-T_S}\quad\text{for}\quad T\in[T_S,T_L] \tag{14}$$

A number of various hypothesis concerning the form of function f_S, describing the volumetric solid state fraction, can be found in literature. One of possibilities is to assume the linear dependence of the solid phase on the temperature in the mushy zone [6-8] which, together with the conditions described, results in the form of relation (14). Differentiating this function we get:

$$\frac{\mathrm{d} f_s(T)}{\mathrm{d} T} = \frac{-1}{T_L - T_S} \quad \text{for} \quad T \in [T_S, T_L] \tag{15}$$

hence, the substitute thermal capacity is equal to:

$$C = \begin{cases} c_l & T > T_L, \\ c_{mz} + \dfrac{L}{T_L - T_S} & T \in [T_S, T_L], \\ c_s & T < T_S \end{cases} \tag{16}$$

where c_l, c_m, and c_S denote the specific heat of the liquid phase, mushy zone, and the solid phase, respectively. In eq. (5) the values of density and thermal conductivity coefficient vary as well in dependence on the temperature:

$$\rho = \begin{cases} \rho_l & T > T_L, \\ \rho_{mz} & T \in [T_S, T_L], \\ \rho_s & T < T_S, \end{cases} \qquad \lambda = \begin{cases} \lambda_l & T > T_L, \\ \lambda_{mz} & T \in [T_S, T_L], \\ \lambda_s & T < T_S \end{cases} \tag{17}$$

Thus, the substitute thermal capacity, density and thermal conductivity coefficient, described in eqs. (16) and (17), depend on temperature, whereas the liquidus and solidus temperatures vary with regard to the concentration of the alloy component. Other parameters of the governing equations are constant.

The Scheil model [14, 16-18] is one of the models describing the macrosegregation phenomenon occurring in the solidifying cast. In this model, with respect to the fact that the diffusion coefficient in the solid phase is significantly lower than in the liquid phase, we assume that $D_S = 0$, which means that the diffusion in the solid phase does not occur. From the other side, the convection in the liquid phase causes the equalization of concentration of the alloy component in the liquid phase, thus, in this connection, it is assumed in the model that $D_l \to \infty$.

Let us discretize interval $[0, t^*]$ with nodes t_i, $i = 0, 1, ..., p^*$ and assume that the values of concentration in moments t_i, for $i = 0, 1, ..., p$, are known. Then, by applying the mass balance of the alloy component in the region of cast for the moment of time t_{p+1}, we receive the equation:

$$m_0 Z_0 = m_L(t_{p+1}) Z_L(t_{p+1}) + \sum_{i=1}^{p+1} m_S(t_i) Z_S(t_i) \tag{18}$$

where m_0 means mass of the alloy, Z_0 denotes the initial concentration of the alloy component, $Z_L(t_{p+1})$ and $Z_S(t_{p+1})$ are the concentrations of the alloy in liquid phase and solid phase in moment t_{p+1}, respectively, as well as $m_L(t_{p+1})$ and $m_S(t_{p+1})$ describe mass of the alloy in liquid state and solid state in moment t_{p+1}. By introducing the partition coefficient $k = Z_S(t)/Z_L(t)$, we get:

$$Z_L(t_{p+1}) = \frac{m_0 Z_0 - \sum_{i=1}^{p} m_S(t_i) Z_S(t_i)}{k\, m_S(t_{p+1}) + m_L(t_{p+1})} \tag{19}$$

Region of the alloy under consideration is divided into the control volumes V_j of length Δx_j, $j = 1,...,n$. If contribution of the solid phase in volume V_j in moment t will be denoted by $f_j(t)$, then the masses of metal in the solid state and in the liquid state contained in volume V_j in moment t are represented by relations:

$$m_{s,j}(t) = V_j \rho_s f_j(t) \quad \text{and} \quad m_{L,j}(t) = V_j \rho_l \left[1 - f_j(t)\right] \tag{20}$$

By using the above relations eq. (19) is transformed to the form:

$$Z_L(t_{p+1}) = \frac{d_1 \rho_l Z_0 - \rho_s \sum\limits_{i=1}^{p} Z_S(t_i) \left(\sum\limits_{j=0}^{n} \left\{ \Delta x_j \left[f_j(t_i) - f_j(t_{i-1}) \right] \right\} \right)}{k\, \rho_S \sum\limits_{j=0}^{n} \left\{ \Delta x_j \left[f_j(t_{p+1}) - f_j(t_p) \right] \right\} + \rho_l \sum\limits_{j=0}^{n} \left\{ \Delta x_j \left[1 - f_j(t_{p+1}) \right] \right\}} \tag{21}$$

Aim of the discussed inverse problem is to determine the temperature distribution in considered region and to identify the heat transfer coefficient on boundary of the casting mould in case when the values of temperature in selected points of the casting mould $[(x_i,t_j) \in \Omega_m]$ are given:

$$T(x_i, t_j) = U_{ij}, \quad i = 1, 2, ..., N_1, \quad j = 1, 2, ..., N_2 \tag{22}$$

where N_1 refers to the number of sensors and N_2 denotes the number of measurements read from each sensor.

If we assume the form of heat transfer coefficient α as fixed, the investigated problem turns into the direct problem which can be, respectively, easily solved. In this way we are able to find the values of temperature $T_{ij} = T(x_i, t_j)$ corresponding to the given form of heat transfer coefficient α. By using the calculated temperatures T_{ij} and the known temperatures U_{ij} we may construct a functional describing the error of approximate solution and expressed by the following formula:

$$J(\alpha) = \left[\sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (T_{ij} - U_{ij})^2 \right]^{1/2} \tag{23}$$

Minimization of this functional, executed with the aid of ABC algorithm, enables to find the values of sought heat transfer coefficient such that the reconstructed values of temperature will be as closed as possible to the measured values. Physical analysis of the process ensures the existence of some value of parameter α on the boundary, which gives the current measurements of temperature. And although we cannot be sure about the uniqueness of solution, in case of the perturbed measurements read in some periods of time, from the technical point of view, our goal is to determine any value of α (physically reasonable) which ensures the given distribution of temperature.

Solution of the direct problem

For solving the direct problem we use the finite element method completed with the procedure of correcting the field of temperature in the neighbourhood of liquidus and solidus curves. Let us present below the short description of this method. More detailed version, as well as its generalization, can be found in literature [6, 7, 22].

For determining distribution of temperature in moment t_{p+1} we use the finite element method with the use of the implicit scheme by obtaining the following system of linear equations:

$$\left(\frac{1}{\Delta t}\mathbf{C}+\mathbf{K}\right)T^{p+1}=\frac{1}{\Delta t}\mathbf{C}T^{p}+\mathbf{g} \tag{24}$$

where **C** is the thermal capacity matrix, **K** denotes the conductivity matrix and **g** is a vector resulting from the assumed boundary conditions. Solving the obtained system of linear equations we get the distribution of temperature in moment t_{p+1} which will be now corrected.

Let us consider the node x_i occurring in moment t_p in the liquid phase. Thus, its temperature T_i^p is greater than the liquidus temperature T_L. So, in the next step of calculations the values of parameters corresponding to the liquid phase will be used for this node. If temperature T_i^{p+1} in this node, determined in the next step of calculations for moment t_{p+1}, is still greater than the liquidus temperature T_L, then the node still remains in the liquid phase. In this case we accept that temperature T_i^{p+1} is properly determined and does not require any more corrections.

In case when temperature T_i^{p+1} will be lower than the liquidus temperature T_L, the nodes passes to another phase (mushy zone or solid phase) and values of the thermal parameters significantly change. Let us assume that the node passes to the mushy zone, it means $T_i^{p+1}\in(T_S,T_L]$. Passing of the node from the liquid phase to another phase occurred in time $\Delta t = t_{p+1} - t_p$ and for part of this time the values of parameters should correspond to another phase. In this case the temperature in node x_i should be corrected. For this purpose we use the energy balance for the control element V_i with central node x_i. Change of enthalpy of this element connected with its cooling from temperature T_i^p to temperature T_i^{p+1} is equal to:

$$\Delta H_i = C_l\rho_l(T_i^p - T_i^{p+1})\Delta V_i$$

where C_l denotes the substantial thermal capacity of the liquid phase and ρ_l means density of the liquid phase. In real one can distinguish two stages of releasing this heat. The first one is connected with the cooling to the liquidus temperature T_L and the second one with the cooling from the liquidus temperature T_L. Thus we have:

$$\Delta H_{i1} = C_l\rho_l(T_i^p - T_L)\Delta V_i,$$

$$\Delta H_{i2} = C_{mz}\rho_{mz}(T_L - \overline{T}_i^{p+1})\Delta V_i$$

where C_{mz} describes the substantial thermal capacity of the mushy zone and ρ_{mZ} denotes density of the mushy zone, whereas $\overline{T}_i^{p+1}$ refers to the corrected value of temperature in node x_i. From the balance:

$$\Delta H_i = \Delta H_{i1} + \Delta H_{i2}$$

we obtain the formula for the corrected value of temperature in node x_i:

$$\overline{T}_i^{p+1} = T_L - \frac{C_l\rho_l}{C_{mz}\rho_{mz}}(T_L - T_i^{p+1})$$

In the similar way we can derive the formula for the corrected value of temperature in case when node x_i passes from the mushy zone to the solid phase by obtaining:

$$\overline{T}_i^{p+1} = T_S - \frac{C_{mz}\rho_{mz}}{C_s\rho_s}(T_S - T_i^{p+1})$$

where C_s denotes the substantial thermal capacity of the solid phase and ρ_s describes density of the solid phase.

It is also possible to deduce the formula for the correction of temperature in case when node x_i passes immediately from the liquid phase to the solid phase, excluding the mushy zone. Anyway it is better to select the length of time step Δt so that such situation will not happen.

In the next step we determine contribution of the solid phase in volume V_j in moment t_{p+1} by using relation (14). Next, on the basis of formula (21) the value $Z_L(t_{p+1})$ of the alloy component concentration in moment t_{p+1} is calculated, which determines the new values of the liquidus and solidus temperatures T_L and T_S, respectively.

Artificial bee colony algorithm

Functional (23), defined in the previous section, will be minimized by applying the Artificial Bee Colony Algorithm, used already by the author of current paper and her co-workers for solving the inverse problems connected with heat conduction [23, 24]. The ABC algorithm was invented by Karaboga [19], Karaboga and Basturk [20], and Karaboga and Akay [21] by inspirations taken from the swarms of bees searching for the nectar around the hive and sharing the information about locations of the promising sources of food with the aid of a special kind of dance taking place in the hive and called the waggle dance. When a bee finds some good sources of nectar, it takes the sample and returns to the hive for informing the other swarm members about the discovery. Next, the bees waiting in the hive leave the hive and fly in pointed direction for further exploration of the most attractive from among the localized sources of nectar. Thus, the region is double searched by the bees – at first more generally in order to localize the best sources of nectar and then more precisely around the best selected locations. This way of behavior is imitated by the artificial algorithm in the following way (see also [20, 21]).

Initialization of the algorithm

(1) Initial data:
$J(\mathbf{y})$ – minimized function;
SN – number of the explored sources of nectar (number of bees-scouts and bees-viewers);
lim – number of "corrections" of the source location $\mathbf{y}^i$;
MCM – maximal number of cycles.

(2) Initial population – random selection of the initial sources localizations represented by the $\overline{d}$-dimensional vectors $\mathbf{y}^i$, $i = 1, \ldots, SN$

(3) Calculation of the values $J(\mathbf{y}^i)$, $i = 1, \ldots, SN$, for the initial population.

Main algorithm

(1) Modification of the sources localizations by the bees-scouts.
(a) Every bee-scout modifies the position $\mathbf{y}^i$ according to the formula:

$$v_j^i = y_j^i + \phi_{ij}(y_j^i - y_j^k), \quad j \in \{1, \ldots, \overline{d}\}$$

where

$$\left.\begin{array}{l} k \in \{1, \ldots, SN\},\, k \neq i, \\ \phi \in [-1, 1] \end{array}\right\} \text{ – randomly selected numbers.}$$

(b) If $J(\mathbf{v}^i) \leq J(\mathbf{y}^i)$, then position $\mathbf{v}^i$ replaces $\mathbf{y}^i$. Otherwise, position $\mathbf{y}^i$ remains unchanged. Steps (a) and (b) are repeated lim number of times. We take: lim = $SN \cdot \overline{d}$.

(2) Calculation of the probabilities P_i for the positions $\mathbf{y}^i$ selected in step 1. We use formula

$$P_i = \frac{fit_i}{\sum_{j=1}^{SN} fit_j}, \quad i = 1,...,SN$$

where

$$fit_i = \begin{cases} \dfrac{1}{1+J(\mathbf{y}^i)}, & \text{if} \quad J(\mathbf{y}^i) \geq 0, \\ 1+|J(\mathbf{y}^i)|, & \text{if} \quad J(\mathbf{y}^i) < 0 \end{cases}$$

(3) Every bee-viewer chooses one of the sources $\mathbf{y}^i$, $i = 1, ... , SN$, with probability P_i. Of course, one source can be chosen by a group of bees.
(4) Every bee-viewer explores the chosen source and modifies its position according to the procedure described in step 1.
(5) Selection of $\mathbf{y}^{best}$ for the current cycle – the best source among the sources determined by the bees-viewers. If the current $\mathbf{y}^{best}$ is better that the one from the previous cycle, it is accepted as $\mathbf{y}^{best}$ for the entire algorithm.
(6) If in step 1, the bee-scout did not improve the position $\mathbf{y}^i$ ($\mathbf{y}^i$ did not change), it leaves source $\mathbf{y}^i$ and moves to the new one, according to formula:

$$y_j^i = y_j^{\min} + \varphi_{ij}(y_j^{\max} - y_j^{\min}), \quad j = 1,...,\overline{d}$$

where $\phi_{ij} \in [0,1]$, $y_j^{\min}$ and $y_j^{\max}$ denote the minimal and maximal, respectively, value of the j-th co-ordinate in the current population.
Steps 1-6 are repeated MCN times.

Numerical example

Let us consider a numerical example for verifying the elaborated procedure. We solve the problem of solidifying cast of length d_1 = 0.4 m within the mould of length d_2 = 0.2 m described by parameters: c_l = 1275 J/kgK, c_S = 1077 J/kgK, c_m = 620 J/kgK, ρ_l = 2498 kg/m^3, ρ_S = 2824 kg/m^3, ρ_m = 7500 kg/m^3, λ_l = 183 W/mK, λ_l = 183 W/mK, λ_m = 40 W/mK, L = 390000 J/kg, ambient temperature T_∞ = 300 K, initial temperature of the solidifying cast T_0 = 960 K, initial temperature of the casting mould $T_{m,0}$ = 590 K, initial concentration of the alloy component Z_0 = 0.02 and partition coefficient k = 125.

Goal of the considered problem is to reconstruct the heat transfer coefficient α on boundary of the solidifying cast and the distribution of temperature inside the investigated region, basing on the known measurements of temperature in selected points of the mould. The measurements of temperature were read from one thermocouple (N_1 = 1) located in the middle of the mould (in point x = 0.5). The measurements were taken in four cycles – at every 1, 2, 5 and 10 seconds, respectively.

In the performed experiment we have known the exact value of the sought heat transfer coefficient, which was equal to 250 W/m^{2}K. For constructing functional (23) we used the exact values of temperature, calculated for the known exact value of α, and values burdened by the 1% and 2% random error of normal distribution, for each frequency of measurements. Thus, twelve experiments have been executed altogether, thanks to which the procedure could be examined with respect to the exactness and stability of obtained results.

For solving the appropriate direct problem, connected with the inverse problem under consideration, we used the finite element method completed with the procedure of correcting the temperature field in the neighbourhood of the liquidus and solidus curves. Region of the cast was divided into 200 and region of the mould into 100 equal elements. Step for the time interval was taken as 0.1 second. With respect to the time variable the implicit difference scheme was applied. To avoid the inverse crime the input data were generated for much more dense grid.

For minimizing functional (23) we applied the ABC algorithm executed for $SN = 5$ bees and $MCN = 10$ cycles, recognized as the best values on the basis of some previous testing calculations [23, 24]. To initiate the procedure the input locations of sources were randomly selected from interval [0, 1000]. Additionally, with regard to the heuristic nature of ABC algorithm, which means that each execution of the procedure can give slightly different results, the calculations were evaluated for 10 times and the best of received results were accepted as the reconstructed elements. Thanks to this we could avoid the suspicion that the obtained results are accidental.

The received results are collected in tab. 1. The table presents the mean values of the reconstructed heat transfer coefficient α and relative errors (δ_α) of these reconstructions as well as the relative (δ_T) and maximal absolute (Δ_T) errors of temperature reconstruction, obtained for the unnoised input data and input data burdened by 1% and 2% error, for the successive cycles of measurements taken at every 1, 2, 5 and 10 s. Analyzing the errors of the heat transfer coefficient reconstruction we observe that in each of twelve executed experiments the reconstruction errors are comparable with the input data errors. In three first cycles of experiments the errors are even significantly lower. The situation is getting worse in the last cycle of measurements read in every 10 s for 2% perturbation of input data, when the output error is at the level of 5%. But even in this case the resulting error can be considered as acceptable. Frequency of taken measurements and value of the input error influences the results, which is understandable, however the resulting errors keep the level of input error which indicates stability of the procedure. Moreover, results obtained in all ten executions of the procedure, concerning each considered case, were very similar, standard deviations of these results in each case were small which confirms stability of the discussed procedure. It can be also noticed that in the second cycle of measurements the resulting error for the case of 2% input error is lower than for the case of 1% input error, which is surprising. It may be explained by the fact that the minimized functional is constructed on the basis of burdened input data, therefore it may accidentally happen that the reconstructed values adopt better to the exact values in the case of measurements of worse quality.

Table 1. Reconstructed mean values of the heat transfer coefficient α and relative errors (δ_α) of these reconstructions, together with the relative (δ_T) and maximal absolute (Δ_T) errors of temperature reconstruction, obtained for the unnoised input data and input data burdened by 1% and 2% error, for the successive cycles of measurements

Measurements	Noise	$\overline{\alpha}$	δ_α [%]	δ_T [%]	Δ_T [K]
Every 1 s	0%	249.999	$1.20 \cdot 10^{-4}$	$9.37 \cdot 10^{-8}$	$6.12 \cdot 10^{-7}$
	1%	247.687	0.925	0.022	0.142
	2%	254.241	1.684	0.040	0.259
Every 2 s	0%	249.999	$1.20 \cdot 10^{-4}$	$9.37 \cdot 10^{-8}$	$6.12 \cdot 10^{-7}$
	1%	248.015	0.749	0.019	0.122
	2%	248.364	0.654	0.015	0.100
Every 5 s	0%	250.820	0.328	0.008	0.050
	1%	250.823	0.329	0.008	0.050
	2%	255.075	2.030	0.047	0.309
Every 10 s	0%	249.474	0.210	0.005	0.032
	1%	253.867	1.547	0.036	0.236
	2%	236.428	5.429	0.129	0.841

Last two columns of tab. 1 show the relative and maximal absolute errors of temperature reconstruction obtained for the unnoised input data and input data burdened by 1% and 2% error for the successive cycles of measurements. Values of these errors confirm the above observations, but the most important is that all of them are marginal which indicates a very good reconstruction of the distribution of temperature in investigated region. Distributions of relative errors of the reconstructed temperature in control point obtained for the unnoised input data and the input data burdened by 2% error, for the cycles of measurements taken at every 5 s and at every 10 s are displayed in figs. 2 and 3, respectively. We may observe in the figures that the errors increase in time, however in the entire time interval they remain at the very low level.

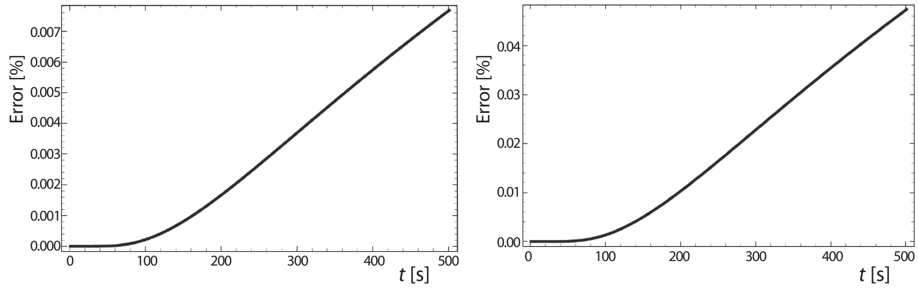

Figure 2. Relative error of the reconstructed temperature distribution in control point obtained for the cycle of measurements taken at every 5 s, for the unnoised input data (left figure) and for 2% noise of input data (right figure)

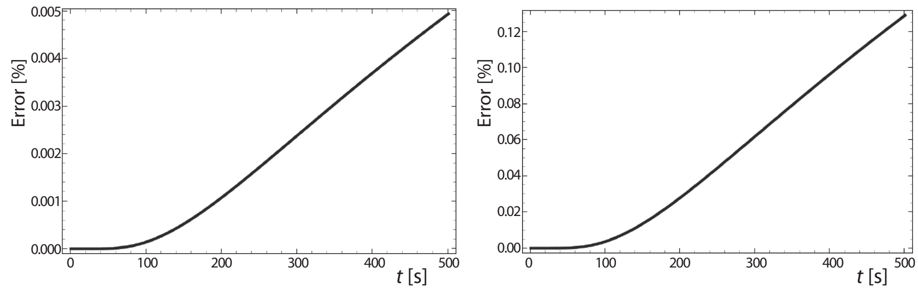

Figure 3. Relative error of the reconstructed temperature distribution in control point obtained for the cycle of measurements taken at every 10 s, for the unnoised input data (left figure) and for 2% noise of input data (right figure)

Conclusions

In this paper the procedure for solving the inverse problem of the binary alloy solidification in the casting mould consisted in determination of the heat transfer coefficient, on the basis of known measurements of temperature, was presented. Proposed approach was based on the mathematical model suitable for describing investigated solidification process, the Scheil model describing the macrosegregation process, the finite element method supplemented by the appropriate procedure of correcting the temperature field for solving the direct problem and the Artificial Bee Colony algorithm for minimizing the functional expressing the error of approximate solution.

Presented examples of calculations show a very good approximation of the exact solution and stability of the algorithm in terms of the input data errors. In each investigated case of input data and frequency of taken measurements the errors of heat transfer coefficient identification were smaller, or at least comparable, with the errors of input data and the errors of temperature reconstruction were insignificant. Moreover, the results obtained in multiple execution of the procedure were very similar. Summing up, the proposed procedure constitutes the effective tool for solving the inverse problem of considered kind. The conclusion is confirmed by the computations made on the basis of experimental data obtained in the solidification process of alloy Al-Cu (5% Cu). The experiment was executed by applying the equipment UMSA (Universal Metallurgical Simulator and Analyzer) [25]. The obtained results are very promising and they will be an object of a separate paper. Discussion of experimental research of similar kind can be found in [26, 27].

Acknowledgement

This project has been financed from the funds of the National Science Centre granted on the basis of decision DEC-2011/03/B/ST8/06004.

The Author would like to thank the Reviewers for their valuable comments and suggestions that greatly help to improve the paper.

Nomenclature

C – substitute thermal capacity, [Jkg^{-1}K^{-1}]
$\mathbf{C}$ – thermal capacity matrix, [–]
c – specific heat, [Jkg^{-1}K^{-1}]
D – diffusion coefficient, [m^{2}s^{-1}]
$\bar{d}$ – dimension of minimized problem, [m]
d – length, [m]
f_j – contribution of solid phase in volume, [–]
f_S – volumetric solid state fraction, [–]
$\mathbf{g}$ – vector of boundary conditions, [–]
ΔH_i – change of enthalpy, [J]
J – minimized functional, [–]
$\mathbf{K}$ – conductivity matrix, [–]
k – partition coefficient, [–]
L – latent heat of solidification, [Jkg^{-1}]
m – mass of the alloy, [kg]
N_1 – number of sensors, [–]
N_2 – number of measurements, [–]
p^* – number of time nodes, [–]
SN – number of bees, [–]
T – temperature of the cast, [K]
T_{ij} – computed temperature, [K]
T_∞ – ambient temperature, [K]
t – time, [s]
t^* – length of the time interval, [s]
U_{ij} – measured temperature, [K]
V_j – control volume, [m^3]
x – spatial variable, [–]
Δx_j – length of the control volume, [m]
$\mathbf{y}$ – location of bee, [–]
Z_0 – concentration of the alloy, [–]

Greek symbols

α – heat transfer coefficient, [Wm^{-2}K^{-1}]
δ – relative percentage error, [%]
Δ – absolute error, [–]
λ – thermal conductivity, [Wm^{-1}K^{-1}]
ρ – mass density, [kgm^{-3}]
Ω – region of the cast, [–]

Subscripts

l – liquid phase
m – mould
mz – mushy zone
S – solid phase
0 – initial
1 – cast region
2 – mould region

References

[1] Okamoto, K., Li, B.Q., A Regularization Method for the Inverse Design of Solidification Processes with Natural Convection, *Int. J. Heat Mass Transfer*, *50* (2007), 21-22, pp. 4409-4423
[2] Szeliga, D., *et al*., M., Parameters Identification of Material Models Based on the Inverse Analysis, *Int. J. Appl. Math. Comput. Sci.*, 14 (2004), 4, pp. 549-556
[3] Hristov, J., An Inverse Stefan Problem Relevant to Boilover: Heat Balance Integral Solutions and Analysis, *Thermal Science*, 11 (2007), 2, pp. 141-160
[4] Beck, J.V., *et al*., *Inverse Heat Conduction: Ill Posed Problems,* Wiley Intersc., New York, USA, 1985

[5] Beck, J. V., Blackwell, B., Inverse Problems. in: *Handbook of Numerical Heat Transfer*, Wiley Intersc., New York, USA, 1988
[6] Mochnacki, B., Suchy, J. S., *Numerical Methods in Computations of Foundry Processes*, PFTA, Cracow, Poland, 1995
[7] Majchrzak, E., Mochnacki, B., Application of the Bem in the Thermal Theory of Foundry, *Eng. Anal. Bound. Elem., 16* (1995), 2, pp. 99-121
[8] Mochnacki, B., Numerical Modeling of Solidification Process, in: *Computational Simulations and Applications* (Ed. J. Zhu), InTech, Rijeka, Croatia, 2011, pp. 513-542
[9] Santos, C. A., *et al*., Determination of Transient Interfacial Heat Transfer Coefficients in Chill Mold Castings, *J. Alloys and Compounds*, *319* (2001), 1-2, pp. 174-186
[10] Meyer, G.H., A Numerical Method for the Solidification of a Binary Alloy, *Int. J. Heat Mass Transfer*, *24* (1981), pp. 778-781
[11] Mochnacki, B., *et al*., Modelling of Segregation in the Process of Al-Si Alloy Solidification, *Soldification of Metals and Alloys*, *2* (2000), 44, pp. 229-234
[12] Samanta, D., Zabaras, N., Control of Macrosegregation During the Solidification of Alloys Using Magnetic Fields, *Int. J. Heat Mass Transfer*, *49* (2006), 25-26, pp. 4850-4866
[13] Yang, G. Z., Zabaras, N., The Adjoint Method for an Inverse Design Problem in the Directional Solidification of Binary Alloys, *J. Comput. Phys.*, *140* (1998), 2, pp. 432-452
[14] Slota, D., Reconstruction of the Boundary Condition in the Problem of the Binary Alloy Solidification, *Arch. Metall. Mater.*, *56* (2011), 2, pp. 279-285
[15] Hetmaniok, E., Slota, D., Numerical Procedure of Solving Some Inverse Problem in Solidification of the Binary Alloy, *Comput. Assisted Meth. Eng. Sci., 19* (2012), 4, pp. 393-402
[16] Hetmaniok, E., Slota, D., Experimental Verification of the Procedure of Reconstructing the Boundary Condition in the Problem of Binary Alloy Solidification, *Steel Res. Int.*, *Special edition Metal Forming* (2012), pp. 1043-1046
[17] Mochnacki, B., Suchy, J. S., Simplified Models of Macrosegregation, *J. Theor. Appl. Mech.*, *44* (2006), 2, pp. 367-379
[18] Mochnacki, B., *et al*., Simulation of Heat and Mass Transfer in Domain of Casting Made From Binary Alloy, *Arch. Foundry Eng*., *8* (2008), 4, pp. 121-126
[19] Karaboga, D., An Idea Based on Honey Bee Swarm for Numerical Optimization, Technical Report, Computer Engineering Department, Engineering Faculty, Erciyes University, Kayseri, Turkey, 2005
[20] Karaboga, D., Basturk, B., On the Performance of Artificial Bee Colony (ABC) Algorithm, *Appl. Soft Computing*, *8* (2007), 1, pp. 687-697
[21] Karaboga, D., Akay, B., A Comparative Study of Artificial Bee Colony Algorithm, *Appl. Math. Comput.*, *214* (2009), 1, pp. 108-132
[22] Szargut, J., Mochnacki, B., Difference Mathematical Model of Ingot Solidification, *Arch. Hutn*., *16* (1971), 3, pp. 269-289
[23] Hetmaniok, E., *et al*., Solution of the Inverse Heat Conduction Problem by Using the ABC Algorithm, *Lect. Notes Comput. Sc., 6086* (2010), pp. 659-668
[24] Hetmaniok, E., *et al.*, Comparison of ABC and ACO Algorithms Applied for Solving the Inverse Heat Conduction Problem, *Lect. Notes Comput. Sc., 7269* (2012), pp. 249-257
[25] Kasprzak, M., *et al.*, Method and Apparatus for Universal Metallurgical Simulation and Analysis, US Patent No. US 7, 354, 491 B2, 2008
[26] Ohura, K., *et al.*, Warpage Simulation and the Experimental Verification of an L-Plate Sand Mold Casting by Using the Thermoelastoplastic FEM Code, *Proceedings*, 10th Internacional Conference on Numerical Methods in Industrial Forming Processes Dedicated to Prof. O. C. Zeinkiewicz, Pohang, Republic of Korea, VOl. 1252, pp. 523-530, 2010
[27] Susac, F., *et al.*, Experimental Study of the Heat Transfer and Air Gap Evolution during Casting of an AC4CH Aluminum Alloy, *Proceedings*, RIKEN International Symposium, Kobe, Japan, *2008*, pp. 49-52

Paper submitted: July 15, 2014
Paper revised: November 28, 2014
Paper accepted: November 28, 2014