EXACT SOLUTION OF FRACTIONAL NIZHNIK-NOVIKOV-VESELOV EQUATION

by

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The fractional Nizhnik-Novikov-Veselov equation is converted to its differential partner, and its exact solutions are successfully established by the exp-function method.

Key words: *fractional complex transform, exp-function method, Nizhnik-Novikov-Veselov equation*

Introduction

In this paper we consider the following Nizhnik-Novikov-Veselov (NNV) equation [1]:

$$\begin{cases} aD_x^{3\beta}u + bD_y^{3\beta}u - 3auD_x^{\beta}v - 3avD_x^{\beta}u - 3buD_y^{\beta}w - 3bwD_y^{\beta}u - D_t^{\alpha}u = 0 \\ D_x^{\beta}u = D_y^{\beta}v, \quad D_y^{\beta}u = D_x^{\beta}w, \quad 0 < \alpha, \quad \beta \le 1 \end{cases} \tag{1}$$

where a and b are arbitrary constants. Equation (1) can describe a general heat and fluid flow in a porous media.

Using the fractional complex transform [2, 3]:

$$u(x,y,t) = u(\xi), \quad v(x,y,t) = v(\xi), \quad w(x,y,t) = w(\xi)$$
$$\xi = \frac{\tau x^{\beta}}{\Gamma(1+\beta)} + \frac{\lambda y^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+t)} \tag{2}$$

where τ, λ, and c are non-zero arbitrary constants, we can convert eq. (1) into its ordinary partner, which reads:

$$\begin{cases} (a\tau^3 + b\lambda^3)u''' - 3a\tau uv' - 3a\tau vu' - 3\lambda buw' - 3bw\lambda u' + cu' = 0 \\ \tau u' = \lambda v', \quad \lambda u' = \tau w' \end{cases} \tag{3}$$

Integrating (3) with respect to ξ yields:

$$(a\tau^3 + b\lambda^3)u'' - \left(\frac{3a\tau^2}{\lambda} + \frac{3b\lambda^2}{\tau}\right)u^2 + cu + \xi_0 = 0 \tag{4}$$

where ξ_0 is a constant of integration.

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Using the exp-function method [4], we suppose that the solution of eq. (4) can be expressed as:

$$u(\xi)=\frac{a_1\exp(\xi)+a_0+a_{-1}\exp(-\xi)}{\exp(\xi)+b_0+b_{-1}\exp(-\xi)} \tag{5}$$

Substituting eq. (5) into eq. (4), we have:

$$\frac{1}{A}[E_3\exp(3\xi)+E_2\exp(2\xi)+E_1\exp(\xi)+E_0+E_{-1}\exp(-\xi)+E_{-2}\exp(-2\xi)+E_{-3}\exp(-3\xi)]=0$$

where $A=[b_0+\exp(\xi)+b_{-1}\exp(-\xi)]^2$.

Equating the coefficients of exp(iξ), i = 0, ±1, ±2, ±3 to be zero, we can obtain a series of exact solutions. Hereby we write down only one for simplicity:

$$u(\xi)=\frac{\dfrac{c\lambda\tau+\sqrt{c^2\lambda^2\tau^2+12bd\tau\lambda^4+12ad\lambda\tau^4}}{6b\lambda^3+6a\tau^3}\exp(\xi)+a_0}{\exp(\xi)-\dfrac{a_0b\lambda^3+aa_0\tau^3+a_0c}{2d}+\dfrac{a_2\sqrt{c^2\lambda^2\tau^2+12bd\lambda^4\tau+12ad\lambda\tau^4}-a_2c\lambda\tau}{2d\lambda\tau}\exp(-\xi)}$$

where a_0 and a_2 are parameters.

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References

[1] Yang, X. J., Baleanu, D., Fractal Heat Conduction Problem Solved by Local Fractional Variation Iteration Method, *Thermal Science, 17* (2013), 2, pp. 625-628
[2] He, J.-H., Li, Z. B., Converting Fractional Differential Equations into Partial Differential Equations, *Thermal Science, 16* (2012), 2, pp. 331-334
[3] Li, Z.-B., *et al.*, Exact Solutions of Time-Fractional Heat Conduction Equation by the Fractional Complex Transform, *Thermal Science, 16* (2012), 2, pp. 335-338
[4] He, J.-H., Exp-Function Method for Fractional Differential Equations, *International Journal of Nonlinear Sciences and Numerical Simulation, 14* (2013), 6, pp. 363-366

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