# TEMPERATURE DISTRIBUTION OF AN INFINITE SLAB UNDER POINT HEAT SOURCE

by

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*The temperature field in an infinite slab under an instantaneous or continuous point heat source is studied numerically. The numerical results reveal the temperature distribution and its change regularity, which are significant for the temperature control encountered in many practical manufacturing processes, such as the laser treatment processes on the surface of films, welding and cutting, and even the design of measuring devices for thermal properties of material.*

Key words: *point heat source, numerical simulation, infinite slab*

## Introduction

The heat source method has been used to solve heat conduction problems encountered in many practical manufacturing processes over the years [1-4], such as the laser treatment processes on the surface of films, welding and cutting, and even the design of measuring devices for thermal properties of material. In all of these applications, the analysis is mainly based on the solution of the instantaneous point heat source in an infinite space [5], but for most of the practical problems, object space is finite and the problems themselves are with different boundary conditions. So, in order to obtain the solution of the practical problems, alternatives [6, 7] are suggested for improving the heat source method, which, still is the approximate solution to some extent. To exhibit the characteristics of the temperature response under different types of point heat source, the temperature field in an infinite slab under an instantaneous or continuous point heat source is studied numerically in this paper.

## Governing equations

Assume the thickness of the slab is $L$, the initial temperature of the body is uniform and equal to the ambient temperature $\theta_f$. Further, assume the temperature distribution in the body is with axial symmetry along the slab plane. Then, the problem, with constant properties, is governed by the following equation in the cylindrical coordinate system, fig. 1:

$$\frac{\partial T}{\partial t} = a\nabla^2 T = a\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right], \quad 0 \le z \le L; \quad 0 \le r \le \infty \tag{1}$$

Initial condition:

$$t = 0, \quad T = \theta - \theta_f \tag{2}$$

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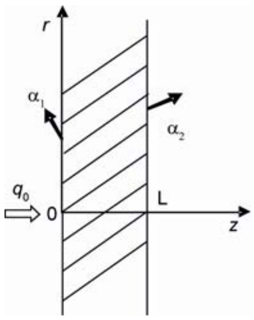


**Figure 1. The scheme of heat conduction in the infinite slab**

where $\theta$ is the temperature at any point in the body, $T$ – the temperature rise, $a$ – the thermal diffusivity, $t$ – the time, and $r$ and $z$ are the co-ordinate variables, respectively.

The boundary conditions for transient point heat source are:

$$t = t_{+0}, \quad z = 0, \quad r = 0: \quad q = q_0 \tag{3}$$

$$t > t_{+0}, \quad z = 0, \quad r = 0: \quad q = 0 \tag{4}$$

$$z = 0, \quad 0 < r < \infty: \quad \alpha_1 T = k \frac{\partial T}{\partial z} \tag{5}$$

$$z = L, \quad 0 \le r < \infty: \quad \alpha_2 T = -k \frac{\partial T}{\partial z} \tag{6}$$

where $q_0$ is the heat liberation rate of the transient point heat source, $k$ – the thermal conductivity of the body, $\alpha_1$ and $\alpha_2$ are the convective heat transfer coefficients on both sides of the slab, respectively, and $t_{+0}$ is the infinitesimal length of time after the initiation of the point heat source.

As for the boundary conditions under the continuous point heat source, change eqs. (3) and (4) into:

$$0 \le t \le t_n \quad z = 0, \quad r = 0: \quad q = q_0 \tag{7}$$

$$t > t_n, \quad z = 0, \quad r = 0: \quad q = 0 \tag{8}$$

where $t_n$ is the length of time after the initiation of a continuous point heat source. The control volume approach [8, 9] is employed for the solution of eqs. (1-8).

## Numerical experiments and analysis

### *Numerical solution for the instantaneous point heat source*

Assume the slab is made of pure copper. Then the following parameters are used: $k$ = 380 W/mK, $\rho$ = 8900 kg/m$^3$, $c$ = 385 J/kgK, $\alpha_1$ = 10 W/Km, $\alpha_2$ = 20 W/Km, $\theta_f$ = 293 K, $L$ = 0.2 m, and $q_0$ = 15000 W. The melting temperature of pure copper is 1356 K. To meet the geometrical characteristics of the infinite flat wall approximately, the cylinder radius is taken as 50 times of the wall's thickness, *i. e.*, the cylinder radius is 10 meters. A grid system, 20 equal-spaced grids along the z-direction (the space step $\Delta z$ = 0.01 m) and 1000 equal-spaced grids along the r-direction (the space step $\Delta r$ = 0.01 m) is used. The time step is $\Delta t$ = 0.1 s. As an approximation, take the heat liberation during the first time step (the value $q_0\Delta t$ = 15000·0.1 = 1500 W) as a transient input heat flux and let $t_{+0} = \Delta t$. The ADI scheme [9] is used to solve the discrete equations, and the convergence criterion is set as $\varepsilon$ = 0.001 in the numerical iterative process.

The numerical results using a FORTRAN program written by the author are illustrated in figs. 2-4. From them, we can conclude:

- When the point source begins to release heat, the surface temperature rise at point $r$ = 0, $z$ = 0 will reach peak value 743 K in 0.1 seconds sharply, then drops quickly.
- At the opposing point $r$ = 0, $z$ = $L$, the surface temperature rise reaches 0.01 K after 32 s, whose peak value is only 0.016 K appearing at 65$^{th}$ second, and then drops rapidly. Thus

the point heat source on one side hardly causes the variation in temperature in the opposing side of the slab.

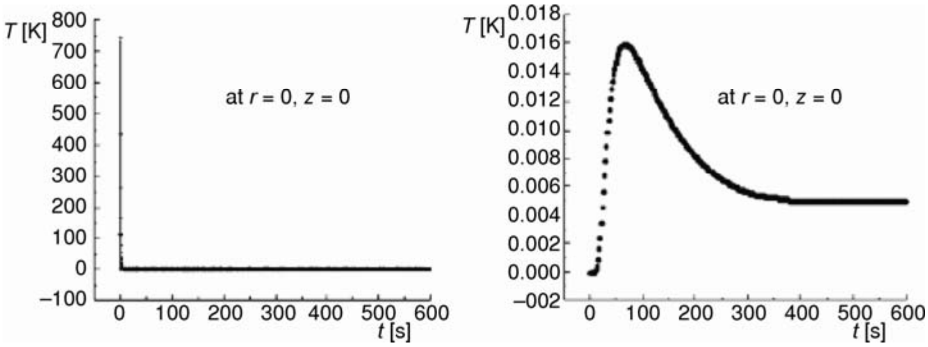


**Figure 2. Temperature rise *vs* time**

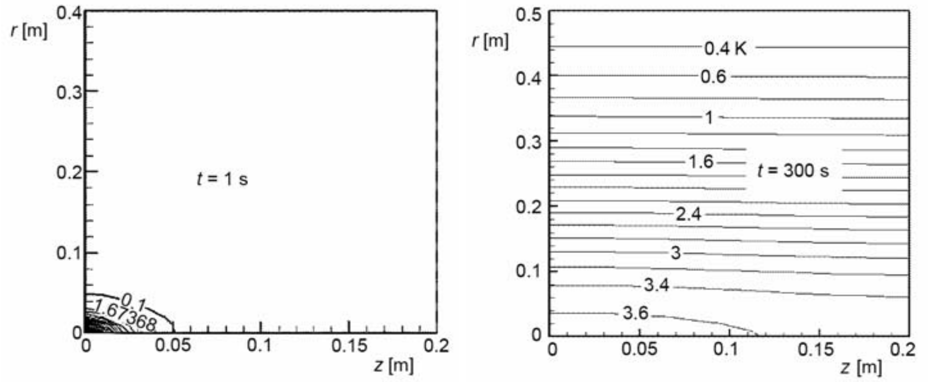


**Figure 3. Partial enlarged drawing of the temperature rise contour**

– The area of temperature influence caused by the point heat source is very finite along the r-direction.

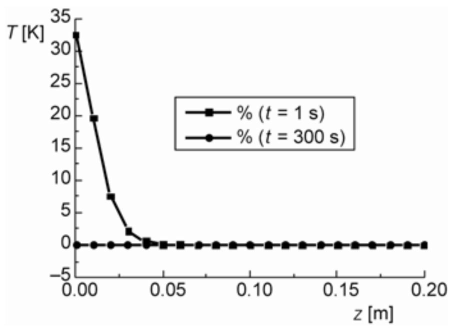


**Figure 4. Temperature rise distribution along z-axis at different time**

### *Numerical solution for the continuous point heat source*

In the case of the continuous point heat source, assume the length of time of the heat release is $t_n = 150$ s. For the purposes of comparison, input heat fluxes are $q_0 = 5000$ W and $q_0 = 7200$ W, respectively. The numerical results are shown in figs. 5-7. From them, we can conclude:

– The surface temperature rise at point $r = 0$, $z = 0$ rises rapidly initially, then changes very slowly as the heating time is prolonged. In the end of the heat release, the surface temperature rise drops quickly, but at the opposing point $r = 0$, $z = L$, the surface temperature rise has shown a drastic change.
– The magnitude of the temperature rise in an infinite slab is dependent mainly on the heat input flux not on the heating time. In the case of $q_0 = 5000$ W, the maximum surface

temperature rise at point $r = 0$, $z = 0$ is 767 K. Consequently, it is not possible for the surface temperature reaching the melting temperature of the material even if heating time is increased. However, in the case of $q_0 = 7200$ W, the surface temperature at the same spot gets to the melting temperature of the material at 21th second.

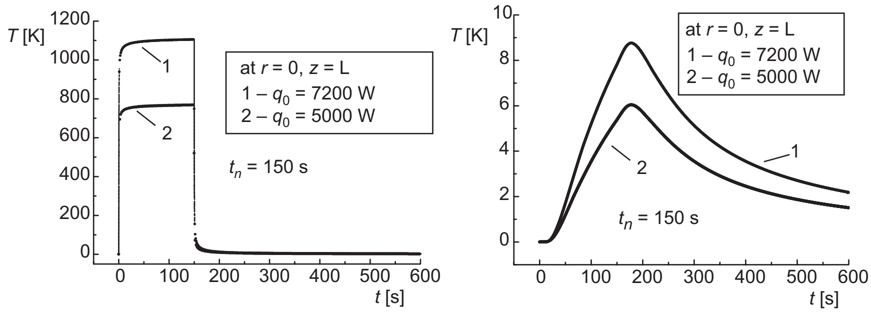


**Figure 5. Temperature rise *vs.* time for different input heat fluxes**

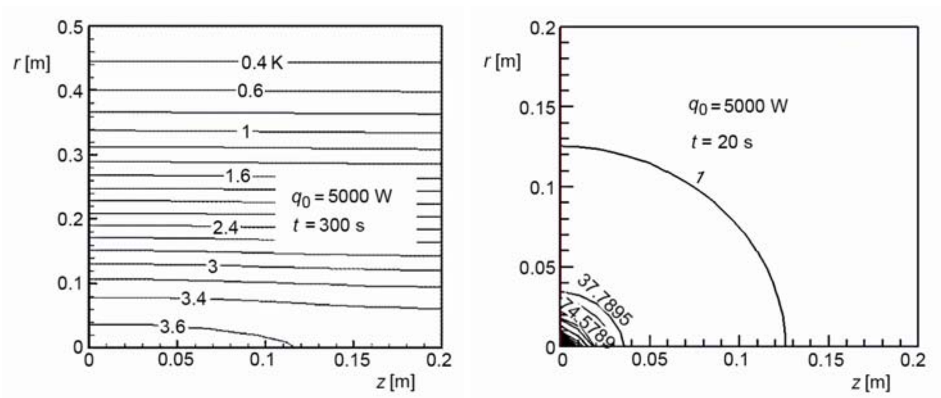


**Figure 6. Partial enlarged drawing of the temperature rise contour**

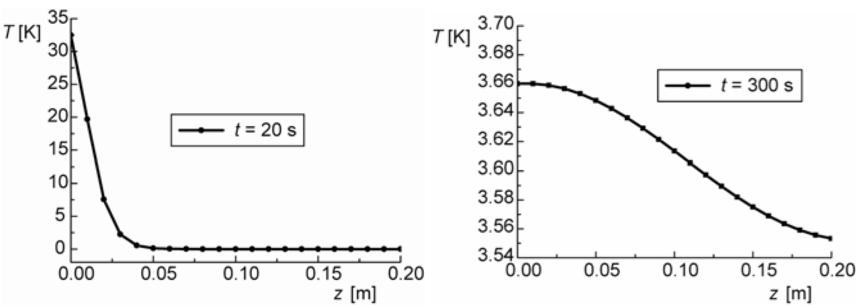


**Figure 7. Temperature rise distribution along *z*-axis at the different time**

– Again from fig. 6, we can see that the area of temperature influence caused by the point heat source is very finite along the r-direction.

## Conclusions

The temperature field in an infinite slab under a point heat source is investigated in this paper. The numerical results show that the characteristics of the temperature response are not identical under different types of point heat sources. The magnitude of the temperature rise in an infinite slab is dependent mainly on the heat input flux rather than the heating time. Meanwhile, the area of temperature influence caused by the point heat source is very finite along the wall of the slab.

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