# SPATIAL AND TEMPORAL DISTRIBUTIONS OF MIGRATION IN BIO-RETENTION SYSTEMS

by

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*Urban bio-retention system is meaningful in reducing rainfall runoff and enhancing infiltration capacity. But the moisture migration in bio-retention systems are not clear under climate change. The spatial and temporal distribution of moisture under different rainfall events in bio-retention systems are studied in this paper based on experimental data in Beijing. Richards model is introduced to simulate the spatial and temporal distribution of moisture including pressure head, hydraulic head and water content under different initial and boundary conditions. As a result, we found that from the depth of the node to the lower boundary, the values of pressure head and hydraulic head increase with depth and decrease with time, while the values of water content represent opposite trends relative to the distribution of pressure head and hydraulic head in bio-retention systems.*

Key words: *spatial and temporal distribution, bio-retention, Richards model, moisture migration*

## Introduction

Bio-retention system which generally consists of soil, gravel, and mulch with a variety of plant species, is one of stormwater best management practices under climate change [1, 2]. Bio-retention system is developed to reduce runoff quantity and improve water quality in a natural, aesthetically pleasing manner [3]. As an effective measurement, bio-retention system with different soil texture provides a shallow depression and plant root in order to reduce runoff peak rate and maintain soil infiltration [4, 5]. When rainfall occurs, a bio-retention system can reduce surface runoff and increase infiltration capacity as surface runoff can be stored in its depression. Bio-retention has a positive influence on the urban sustainable development. Some scholars did some research to stimulate the distribution of pressure head in bio-retention systems [6], but the moisture migration with time in bio-retention systems is seldom researched.

In this paper, Richards model is introduced to simulate the spatial and temporal distribution of moisture migration including pressure head, hydraulic head and water content under different initial and boundary conditions. The model is applied in bio-retention systems in Beijing.

## Richards model for the moisture in a bio-retention system

The Richards model is based on the one dimensional Richards equation, including three variables of pressure head $\varphi$, hydraulic head $h$, water content $\theta$, and basic soil hydraulic

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parameters $S(\theta)$, $K(\theta)$, and $M(\theta)$. The model can be solved by a numerical model based on a spatial discretization method [7, 8]. Using the Richards model, the distribution of pressure head, hydraulic head and water content can be stimulated in bio-retention systems.

### *Richards equation*

The Richards equation is based on the formulation of Darcy's law and the principle of continuity [9]:

$$\frac{\partial\theta}{\partial t}=\frac{\partial\theta}{\partial\varphi}\frac{\partial\varphi}{\partial t}=\frac{\partial}{\partial z}K(\varphi)\frac{\partial}{\partial z}(\varphi-z)-S_r \tag{1}$$

where $\theta$ is the volumetric water content [L$^3$/L$^3$], $z$ – the vertical position [L], $t$ – the time [T], $K(\theta)$ – the hydraulic conductivity of water [L/T], $\varphi$ – the pressure head [L], and $S_r$ – the plant transpiration rate [1/T]. $\partial\theta/\partial\varphi$ is the soil moisture capacity function, and is referred to $K(\theta)$ below.

### *Soil hydraulic parameters*

The soil hydraulic parameters $S(\theta)$, $K(\theta)$, and $M(\theta)$ are needed to be solved in the Richards equation. Here $\theta$ as the volumetric water content is a description of soil porosity. Its maximum value is equal to the porosity of the soil medium.

As an alternative parameter, the saturation $S(\theta)$ can be used between 0 and 1:

$$S(\theta)=\frac{\theta-\theta_r}{\theta_s-\theta_r} \tag{2}$$

where $\theta_s$ is the water content in the saturated condition [L$^3$/L$^3$], and $\theta_r$ – the residual water content [L$^3$/L$^3$].

Then Van Genuchten uses the empirical mathematical form, which frequently appeared in the publications [9]:

$$S(\theta)=\frac{1}{(1+|\alpha\varphi|^n)^m} \tag{3}$$

where $m$ and $\alpha$ are soil specific parameters, and $n = 1 - 1/m$. $m$, $n$, and $\alpha$ are Van Genuchten parameters.

The hydraulic conductivity of water $K(\theta)$ changes with the volumetric water content $\theta$. Van Genuchten has proposed a formula:

$$K(\theta)=K[S(\theta)]^{1/2}\left[1-(1-S(\theta)^{1/m})^m\right]^2=K\left[\frac{(\theta-\theta_r)}{(\theta_s-\theta_r)}\right]^{1/2}\left\{1-\left[1-\left(\frac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{1/m}\right]^m\right\}^2 \tag{4}$$

Here the parameter $K$ represents the conductivity of the saturated soil.

The following formulation is improved Richards equation with water content $\theta$:

$$M(\theta)=\frac{\partial\theta}{\partial\varphi}=\frac{(\theta_s-\theta_r)nm\alpha(-\alpha h)^{n-1}}{(1+|\alpha\varphi|^n)^{m+1}} \tag{5}$$

### *Numerical model*

In this paper, the mixed form of the Richards equation is discretized using a spatial discretization method for polar and non-polar parabolic equation in one space variable created by Skeel and Berzins [7]. This method considers the system of quasilinear partial differential equations:

$$H\left(x,t,C,\frac{\partial C}{\partial x}\right)\frac{\partial C}{\partial t}=x^{-m}\frac{\partial}{\partial x}\left[x^{m}f\left(x,t,C,\frac{\partial C}{\partial x}\right)\right]+S\left(x,t,C,\frac{\partial C}{\partial x}\right) \tag{6}$$

For $a \le x \le b$, where $H$ is a diagonal matrix with non-negative entries and $m$ is non--negative. For parabolic equation like Richards equation, $m$ must be 0, 1, or 2, corresponding to slab, cylindrical, or spherical symmetry, respectively. The boundary condition is:

$$p(x,t,C)+q(x,t,C)f\left(x,t,C,\frac{\partial C}{\partial x}\right)=0 \quad \text{at } x=a,\, b \tag{7}$$

According to eq. (6), the Richards equation is transformed to:

$$\frac{(\theta_S-\theta_r)nm\alpha(-\alpha h)^{n-1}}{(1+|\alpha\varphi|^{n})^{m+1}}\frac{\partial u}{\partial t}=x^{-m}\frac{\partial}{\partial x}\left[\left\{K\left(\frac{\theta-\theta_r}{\theta_S-\theta_r}\right)^{1/2}\left[1-\left\{\left[1-\left(\frac{\theta-\theta_r}{\theta_S-\theta_r}\right)^{1/m}\right]^{m}\right\}\right]^{2}\right\}\right]\cdot$$
$$\cdot\frac{\partial u}{\partial x}-\left[K\left(\frac{\theta-\theta_r}{\theta_S-\theta_r}\right)^{1/2}\left(1-\left\{\left[1-\left(\frac{\theta-\theta_r}{\theta_S-\theta_r}\right)^{1/m}\right]^{m}\right\}\right)^{2}\right] \tag{8}$$

Here boundary condition is $q = 0, 1$; $p = 0, C$; and $C =$ constants.
We can easily use parameters of eq. (6) to express Richards eq. (8).

$$\begin{cases} H\left(x,t,C,\dfrac{\partial C}{\partial x}\right)=\dfrac{(\theta_s-\theta_r)nm\alpha(-\alpha h)^{n-1}}{(1+|\alpha\varphi|^{n})^{m+1}} \\ f\left(x,t,C,\dfrac{\partial C}{\partial x}\right)=\left\{K\left(\dfrac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{1/2}\left\{1-\left[1-\left(\dfrac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{1/m}\right]^{m}\right\}^{2}\right\}\cdot \\ \qquad\qquad \cdot\dfrac{\partial \mu}{\partial x}-\left\{K\left(\dfrac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{1/2}\left\{1-\left[1-\left(\dfrac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{1/m}\right]^{m}\right\}^{2}\right\} \\ S\left(x,t,C,\dfrac{\partial C}{\partial x}\right)=0 \end{cases} \tag{9}$$

Considering the $a$ spatial mesh $a = x_0 < x_1 < \ldots < x_j = b$, we can seek difference scheme mentioned by Skeel and Berzins [7] that is accurate as possible for the spatial case $f(x, C, \partial C/\partial x) = F(x, C)\, \partial C/\partial x$. For Richards equation we consider it can be expressed as two first-order partial differential equations:

$$\begin{cases} \dfrac{\partial C}{\partial x} = F(x,C)\cdot f\left(x,C,\dfrac{\partial C}{\partial x}\right) \\ \dfrac{\partial (x^m \cdot v)}{\partial x} = x^m \cdot H\left(x,C,\dfrac{\partial C}{\partial x}\right)\cdot\dfrac{\partial C}{\partial t} - f\left(x,C,\dfrac{\partial C}{\partial x}\right) \end{cases} \tag{10}$$

Then we assemble the element equations into difference equations by eliminating the unknown values of $f(x, C, \partial C/\partial x)$ at mesh points. Finally, the solution is gotten by Shampine and Reichelt method [8]. Those are methods used in calculating partial differential equation in MATLAB, then we use finite element method to calculate Richards equation.

## Application

### *Simulation*

Based on experimental data in bio-retention systems in Beijing, the spatial and temporal distribution of moisture on pressure head, hydraulic head and water content were simulated by above Richards model.

In this case, soil properties $\alpha$ = 0.0315, $n$ = 2.5856, $K$ = 682.58 cm/h, $\theta_r$ = 0.0473, $\theta_s$ = 0.332. The initial condition is $h(z, 0)$ = – 1000 cm, and the lower boundary condition is –75 cm. $T$ = 24 hours. Figures 1-3 show the stimulation results.

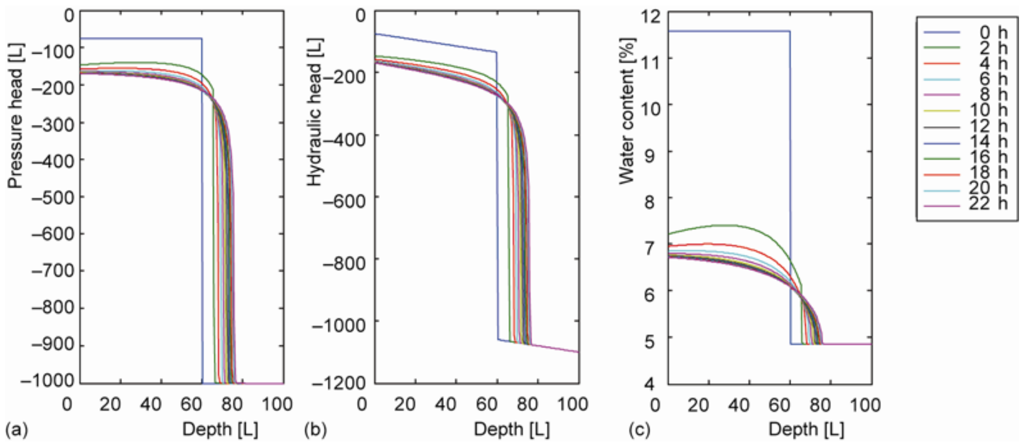


**Figure 1. Spatial and temporal distribution of moisture in bio-retention systems when infiltration rate is 0.05 cm/h; (a) pressure head, (b) hydraulic head, and (c) water content**

For fig. 1(a), when infiltration happens, pressure head is at a stable value of –75 cm. Then it begins to rise rapidly and stops at –1000 cm in about 60 cm depth. From the depth of the node to the lower boundary, pressure head increases with depth and decreases with time. For fig. 1(b), the distribution of hydraulic head is much more like to pressure head. For fig. 1(c). From the depth of the node to the lower boundary, water content decreases with depth and increases with time, which is opposite to the distribution of pressure head and hydraulic head.

Figures 2 and 3 represent the distribution of pressure head, hydraulic head, and water content when infiltration rate adds to 0.1 cm/h, 0.2 cm/h. The values of pressure head and hydraulic head decrease faster with time while the values of water content increase faster when infiltration rate increases. In general, the infiltration processes of three soil hydraulic parameters in figs. 2 and 3 are much more like to fig. 1.

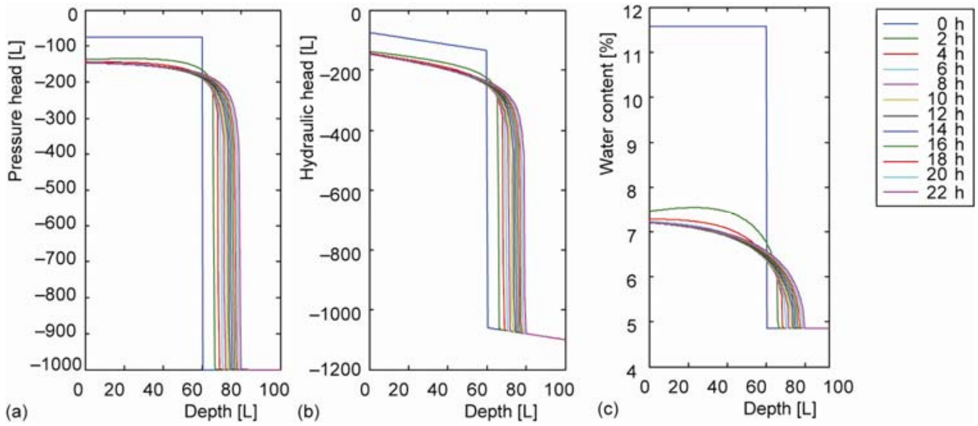


**Figure 2. Spatial and temporal distribution of moisture in bio-retention systems when infiltration rate is 0.1 cm/h; (a) pressure head, (b) hydraulic head, and (c) water content**

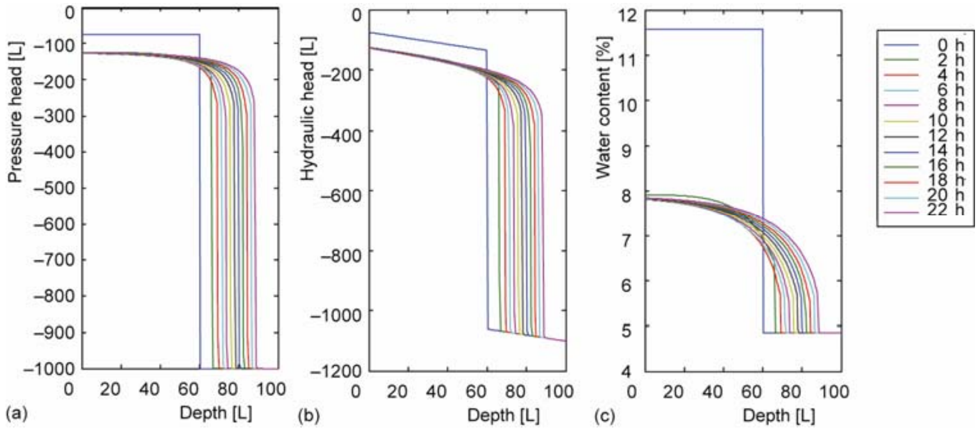


**Figure 3. Spatial and temporal distribution of moisture in bio-retention systems when infiltration rate is 0.2 cm/h; (a) pressure head, (b) hydraulic head, and (c) water content**

### *Discussion*

The Richards model provides a good way to stably stimulate the distribution of the pressure head, hydraulic head and water content in small scale like bio-retention systems. At the same time, by composing the Richards equation and numerical model, the change of pressure head, hydraulic head, and water content in bio-retention systems with time under different rainfall events could be stimulated directly. So the Richards model mentioned in this paper can be applied to stimulate the migration of the water in bio-retention systems.

It is obvious that different soil has different moisture migration process, which causes different surface runoff and different groundwater recharge. Future research should stimulate moisture migration under different bio-retention systems with different soil texture. As bio-retention systems with depression storage can help reduce rainfall runoff and increase infiltration capacity, city designers and planners can create stormwater management landscapes using various kinds of bio-retention systems.

## Conclusions

The Richards model is used to analyze the spatial and temporal distribution of moisture in bio-retention systems based on experimental data in Beijing. The main conclusions can be drawn as follows.

- The Richards model provides a good way to stably stimulate the distribution of the pressure head, hydraulic head, and water content based on experimental data in bio-retention systems in Beijing.
- From the depth of the node to the lower boundary, the values of pressure head, hydraulic head increase with depth and decrease with time based on the Richards model. The values of water content decrease with depth and increase with time, which are opposite to the distribution of pressure head and hydraulic head.
- The study results of the spatial and temporal distribution of moisture in bio-retention systems will provide guidance for urban water resources allocation and management.

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