DISCHARGE COEFFICIENT OF SMALL SONIC NOZZLES

by

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Original scientific paper
DOI: 10.2298/TSCI1405505Y

The purpose of this investigation is to understand flow characteristics in mini/micro sonic nozzles, in order to precisely measure and control miniscule flow-rates. Experimental and numerical simulation methods have been used to study critical flow Venturi nozzles. The results show that the nozzle's size and shape influence gas flow characteristics which leading the boundary layer thickness to change, and then impact on the discharge coefficient. With the diameter of sonic nozzle throat decreasing, the discharge coefficient reduces. The maximum discharge coefficient exits in the condition of the inlet surface radius being double the throat diameter. The longer the diffuser section, the smaller the discharge coefficient becomes. Diffuser angle affects the discharge coefficient slightly.

Key words: *small sonic nozzle, discharge coefficient*

Introduction

It is essential to precisely measure and control the miniscule flow-rates gas in the field of medical, chemical, semiconductors, micro-electro-mechanical systems technology and so on. Because of its stability, simple structure, good repeatability and other unique advantages, sonic nozzles are favored in the precise measurement and control of gas flow [1, 2]. Over 70% nozzles are applied as standard gas meters to calibrate other meters in China [3]. The basic principle of sonic nozzle is: when the ratio of stagnation pressure of the upstream of the nozzle and the back pressure of the outlet of the nozzle reaches a critical value, the flow of gas through the nozzle will reach its maximum value, and further increasing the ratio will not affect the nozzle flow. Considering the gas viscosity, friction loss, and the multidimensional of the flow, the actual flow through the nozzle flow q_m is:

$$q_m = C_d q_{mi} = C_d A_t C^* p_0 (\mathrm{R}T_0)^{-1/2} \tag{1}$$

where C_d is the discharge coefficient of nozzle, A_t – the area of the nozzle throat, P_0 and T_0 are stagnation pressure and temperature, respectively, R is the gas constant, and C^* – the critical stream function [4]. ISO 9300 provides reference data on the discharge coefficients for sonic nozzles with Reynolds' numbers larger than $2.1 \cdot 10^4$ and on shapes such as toroidal and cylindrical [5]. The throat sizes of sonic nozzles are larger than 200 μm in ISO 9300.

With the development of micro-processing technology, the nozzle of small scale throat diameter will be used more widely. Geometric parameters are the key components for the nozzle to achieve and maintain critical flow, and they have a significant effect on the flow characteristic of the nozzle. Lavantee *et al.* [6] studied nozzles with throat diameter rang from

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150 μm to 200 μm, and the experimental results showed that the critical pressure ratio was only 0.46, less than the theoretical value of 0.53. Park *et al*. [7] conducted throat diameter of nozzle within 280 μm to 448 μm, and acquired the function of Reynolds number and the discharge coefficient. Li *et al*. [8] found the flow characteristics of nozzle with throat diameter from 150 μm to 500 μm using numerical simulation method, and compared the differences of cross-sectional flow between the square and circular nozzle throat, then discussed the impact of nozzle diffusion angle on the discharge coefficient. Hu *et al.* [9] experimental studied the critical back pressure, and found that the smaller the throat Reynolds number was, the lower the critical back pressure. Meanwhile, the smaller the internal flow scale of nozzle, the effects of flow boundary layer and wall conditions will be more obvious, and the difference between mini scale nozzles with conventional scale will be greater. Now researches focus on the Venturi nozzle about extending the application range of Reynolds number, especially in low Reynolds number region. For small sonic nozzles whose shapes do not conform to the ISO 9300 configuration, their discharge characteristics need to be explored. This paper focuses on the experimental study and numerical simulation of nozzle whose throat diameter is from 180 μm to 960 μm, to clarify the factors influencing on nozzle discharge coefficients in this scope of the diameter.

Experimental study

Four Venturi sonic nozzles with throat diameter being 0.18 mm, 0.22 mm, 0.28 mm, and 0.96 mm were produced. Test experiments have been performed in Zhejiang Institute of metrology, China. A piston calibrator was designed to test of the small nozzle [10], whose maximum volumetric flow was $5\ \mathrm{m^3/h}$, and the relative standard uncertainty was 0.05%. The throat sizes of the sonic nozzle were measured using a scanning electron microscope. The discharge coefficient of sonic nozzle represents the ratio of real mass flow-rate to idea mass flow-rate in a sonic condition [11]. The critical back pressure ratio was near 0.45, smaller than the theoretical value.

Numerical simulation

Theoretical model

In order to understand the air flow inside the nozzle, and to analyze the law of the nozzle flow, the numerical simulations have been studied with the nozzle throat diameters of 0.18 mm, 0.56 mm, and 0.96 mm, respectively. Stagnation parameters of temperature and pressure are T_0 = 293.15 K and P_0 = 101325 Pa. The critical Reynolds number range of nozzle throat is within the range of 2096 and $1.2 \cdot 10^4$. According to the findings of Johnson [12], for the flow in a tiny nozzle, when the throat Reynolds number is smaller than $6.6 \cdot 10^4$, flow can be considered as a laminar flow. Therefore, the theoretical model of the numerical simulation is the laminar model, and the gas follows the flowing continuity and momentum equations as [13]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_j) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho u_j) + \frac{\partial}{\partial x_i}(\rho u_i u_j) + \frac{\partial}{\partial x_j}(\rho u_j^2 + p) = \frac{\partial}{\partial x_i}\left[\frac{\mu}{\mathrm{Re}}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + \frac{\partial}{\partial x_j}\left\{\frac{\mu}{\mathrm{Re}}\left[2\frac{\partial u_i}{\partial x_i} - \frac{2}{3}\left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_j}\right)\right]\right\} \tag{3}$$

where ρ is the gas density, u_i and u_j are the speeds in both directions; p is the pressure, μ – the dynamic viscosity, Re – the Reynolds number, and i, j correspond to the x, y coordinate.

Geometry and boundary condition

The shape of the nozzle according to ISO 9300 is shown in fig. 1. Entrance is a circular arc. d is the diameter at the minimum position. Diffusion segment (l) is a straight line and l should be longer than d. θ is diffusion angle, whose range is between 2.5° and 6.0°.

In this paper, l, r, θ will be changed to study the nozzle's internal flow field and the effect on discharge coefficient. Since the geometry of nozzle is an axisymmetric structure, half of nozzle shape is used. The wall grids are encrypted. The conditions of inlet and outlet are both pressure boundary conditions. Wall conditions is adiabatic, no-slip boundary condition and without considering the impact of wall roughness. During the numerical simulation process, we pay attention to the outlet pressure. When the flow rate declines sharply after a value of back pressure ratio, which is the critical back pressure ratio, flow field inside the nozzle reaches a critical state.

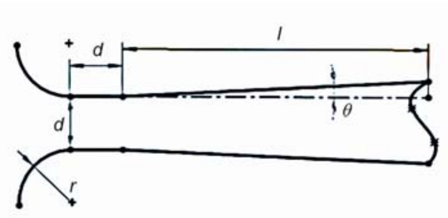

Figure 1. Nozzle figuration

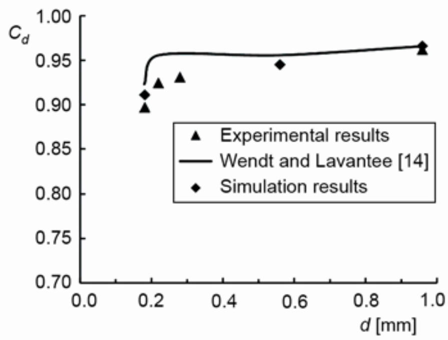

Figure 2. C_d *vs.* d

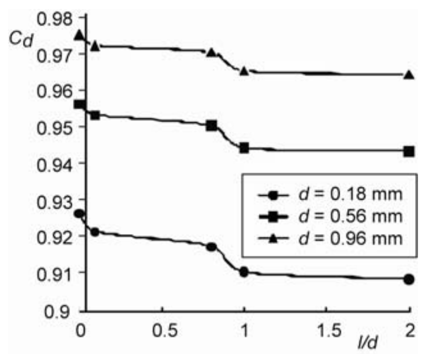

Figure 3. C_d *vs.* l/d

Results and discussion

Comparison of results

The simulated discharge coefficients are compared with the predicted value of Wendt and Lavantee's [14] empirical equation and the experimental results in fig. 2. The simulation discharge coefficients of nozzle with different throat size is in the condition of $l = 2d$, $r = d$, and $\theta = 3°$. The simulate results are very well consistent with the Wendt and Lavantee's expression and experimental results. The discharge coefficient increases in the condition of increasing throat size.

Effect of diffusion length on discharge coefficient

The gas pressure recoveries in the diffuser segment, which will reduce pressure loss. The shape of the diffuser impacts air flow inside nozzle, and makes velocity and pressure changed, at last affects flow rate of the nozzle in the critical state. The simulation results in fig. 3 show that the longer the diffuser, the smaller the discharge coefficient is. When $l < 1d$, the discharge coefficient decreases more obviously, on the country while $l > 1d$, the discharge coefficient has little change. The results shows the decreasing of discharge coefficients of nozzles are 1.73%, 1.25%, and 1.02% with throat diameters being 0.18 mm, 0.56 mm, and 0.96 mm, respectively, when l changes from 0 to $1d$. The results show that the smaller the throat diameter of nozzle, the more obviously the length of diffuser impacts on discharge coefficients.

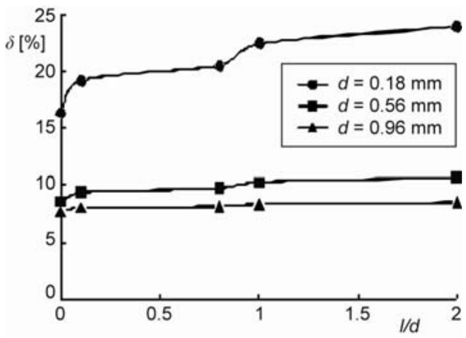

Figure 4. ***δ vs. l/d***

Because of fluid viscous, there is a boundary layer near the nozzle wall, whose velocity value is small and gradient is high. The flow-rate in the boundary layer is far smaller than that in the center axis, and ultimately leading to $C_\mathrm{d} < 1$. The boundary layer thickness in nozzle (δ_a) of different throat diameter varies with the diffusion length. We use $\delta = 2\delta_a/d \cdot 100\%$ to express dimensionless boundary layer thickness. For the nozzles with the same throat diameter, the results indicate that the increasing thicker of the boundary layer lead to the flow-rate decrease and at last the discharge coefficient decline. Figure 4 also shows the longer the diffuser length, the thicker the boundary layer and the lower discharge coefficient become. Meanwhile, with the size of nozzle throat increasing, the relative thickness of the boundary layer decreases, and the influence on discharge coefficient reduces.

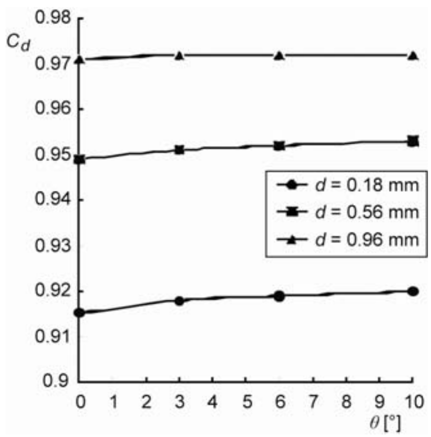

Figure 5. ***C_d vs. θ***

Effect of diffusion angle on discharge coefficient

Another parameter that affects the shape of the nozzle is diffusion angle. Keeping *l* and *r* invariant, the change of discharge coefficient with different diffusion angle have been studied with three kinds of throat diameter. The result in fig. 5 shows that the discharge coefficient of nozzle increases slightly with diffusion angle increasing. The growth rates of three cases are all less than 1%. Figure 6 shows the thickness of boundary layer changing with diffusion angle. The result shows that the effect of the diffusion angle on the boundary layer thickness is smaller than the effect of length of the diffuser on the boundary layer thickness. So the diffusion angle has little effect on the discharge coefficient.

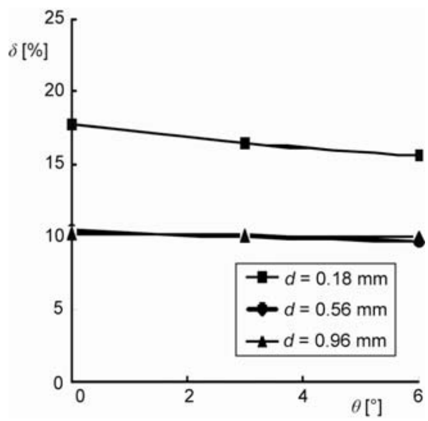

Figure 6. ***δ vs. θ***

Effect of entrance shape on discharge coefficient

Figure 7 shows the variation of discharge coefficient when the radius of curved surface of entrance changes. The entrance is conical when $r = 0$. With the increasing of the radius of curved surface, the discharge coefficient of nozzle increases at beginning and then declines. When $r = 2d$, the discharge coefficient reaches the maximum. For different throat diameters, the effect of the shape of entrance on the discharge coefficients has the same rules. The impact of entrance shape on discharge coefficient is stronger for larger nozzles. Figure 8 shows the change of boundary layer thickness of the nozzle throat with different inlet curved shape. The thickness of boundary layer of nozzle throat changes with *r*. As *r* increasing, the boundary layer thickness decreases. When $r = 2d$, the curve reaches a turning point, after which the boundary layer thickness increases with *r* increasing.

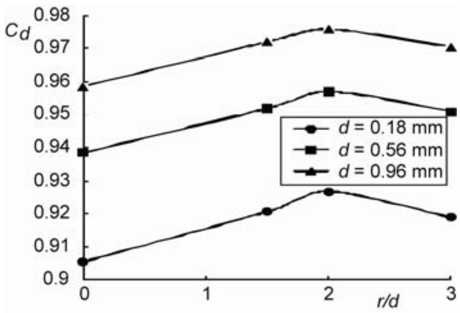

Figure 7. C_d *vs. r*/*d*

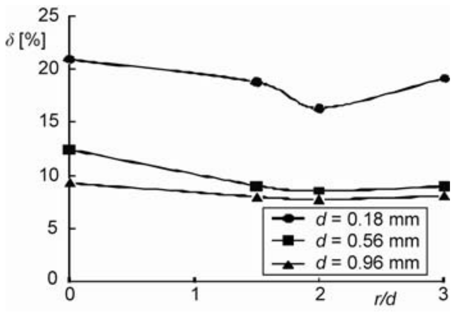

Figure 8. δ *vs. r*

Conclusions

This paper studies the discharge coefficients of small sonic Venturi nozzles with different throat diameters using experimental and numerical simulation methods. The results shows that:

- The smaller the diameter of the nozzle throat, the smaller the discharge coefficient is.
- Changes of the nozzle shape including the entrance shape, diffuser length and angle will affect the discharge coefficient.
- The change of nozzle shape parameters affect flow state inside the nozzle, and the throat boundary layer thickness, resulting in the change of the discharge coefficient.

Acknowledgments

The work was supported by the Major Science Innovation Team of Zhejiang Province (2009R50024).

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Paper submitted: September 30, 2013
Paper revised: April 10, 2014
Paper accepted: July 12, 2014