FRACTAL HARMONIC LAW AND WATERPROOF/DUSTPROOF

by

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The fractal harmonic law admits that the friction between the pure water and the moving surface is the minimum when fractal dimensions of water in Angstrom scale are equal to fractal dimensions of the moving surface in micro scale. In the paper, the fractal harmonic law is applied to demonstrate the mechanism of waterproof/dustproof. The waterproof phenomenon of goose feathers and lotus leaves is illustrated to verify our results and experimental results agree well with our theoretical analysis.

Key words: *waterproof, fractal, fractal harmonic laws*

Introduction

Waterproof, dustproof, oil proof, *e. g.*, are of significant applications in different areas that have attracted attentions of a lot of researchers [1-3]. Materials with waterproof property, such as lotus leaves, are general in natural so that a lot of scholars focus on bionics design according to their micro-structure [4]. The wild silk can live in the silkworm cocoon for about 3-5 days until it gets out from the shell as a moth. During the process, the chrysalis can be alive and dry in the cocoon whether rainy or snow outside [5].

However, the mechanism of these phenomena is rarely to know in recent research. In our group, we establish a model to demonstrate the mechanism of waterproof and dustproof [6]. In this paper, we use fractal harmonic law to illustrate the mechanism of waterproof/dustproof.

Fractal harmonic laws

A fractal is a mathematical set that has a fractal dimension that usually exceeds its topological dimension and may fall between the integers. Fractals are typically self-similar patterns, where self-similar means they are *the same from near as from far* [7].

Fractal dimensions can be calculated by the mathematic expression:

$$D = \frac{\ln M}{\ln N} \tag{1}$$

where M is the number of new units within the original unit with a new dimension and N – the ratio of the original dimension to the new dimension.

A fractal harmonic law was proposed in 2010 to design a swimming vest and a moving surface with minimum friction [8]. Fractal harmonic laws admit that the friction between

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the pure water and the moving surface is the minimum, which means fractal dimensions of water in Angstrom scale should be equal to fractal dimensions of the moving surface in micro scale, which can be expressed:

$$D_{\text{water}} = D_{\text{surface}} \tag{2}$$

According to [8], the fractal of water is $D_{\text{water}} = 1.92$.

Results and discussion

Figures 1 and 2 are a wild silkworm cocoon photo and the scanning electron microscopy (SEM) graph of the out layer surface of wild silk, respectively. We can see that the silk surface is unsmooth with a lot of nanoparticle-like crystals attached on it. According to the natural property of the wild silk, we know that the surface is water-repellent [9]. This property owns to crystals attached on the surface.

Figure 1. A wild silkworm cocoon

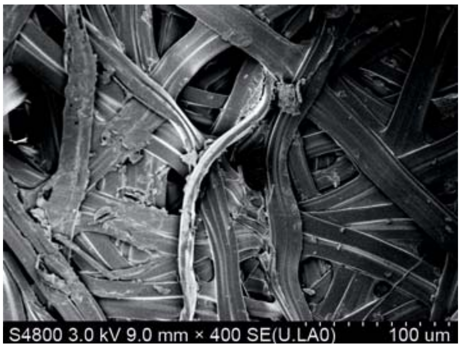


Figure 2. Surface of wild silk

Considering a molecule of water or a particle who is falling down to the surface of the wild silk, as shown in fig. 3(a), according to the model of Lennard-Jones potential of the pair of artificial molecules (as shown in fig. 4) [6]:

$$u(x) = 4\varepsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right] \tag{3}$$

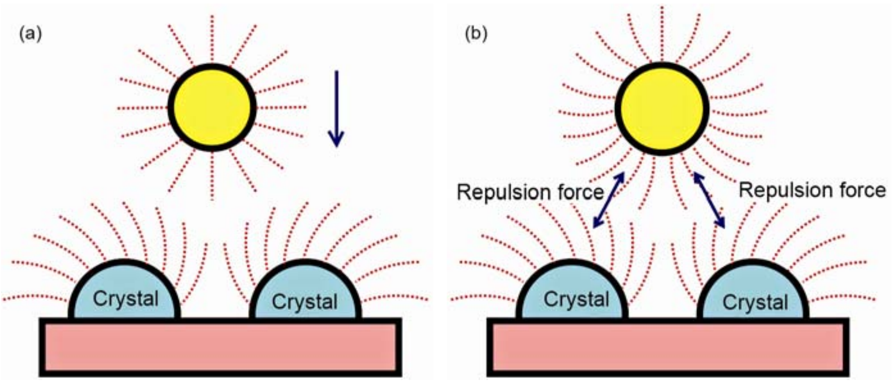


Figure 3. Motor process of water molecular

where ε is the depth of the potential well, σ – the finite distance at which the inter-particle potential is zero, and x – the distance between the particles.

As the distance between the molecule and the crystal is far, the force between the molecule and the crystal is attraction so that the molecule can come down. As the distance between the molecule and the crystal decreases to the limit, the force between them is repulsion, as shown in fig. 3(b).

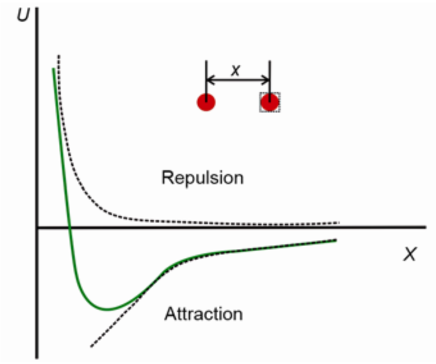


Figure 4. Lennard-Jones potential of the pair of artificial molecules

We accept the criterion that if an object is constituted of several water molecules (as shown in fig. 5), and fractal dimensions of object A are equal to fractal dimensions of water, then we can obtain that object A will stand no pressure in water, whatever is the shape of object A, and the object B will undergo no pressure in object A [10].

Similarly, considering that a liquid water contains some water molecules that comes down to a position whose distance between the molecular and the crystal is equal to the water (fig. 6), and fractal dimensions of the system A and system B are the same so we can obtain that the molecule of water can move with the minimum friction.

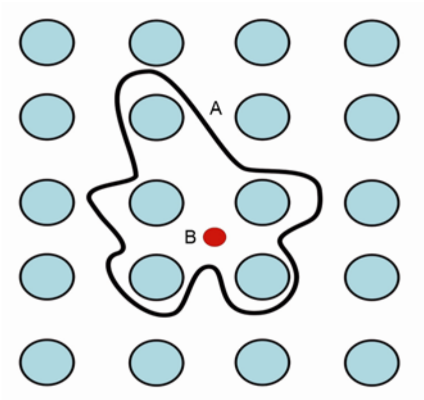


Figure 5. An object A among the water molecules undergoes no pressure

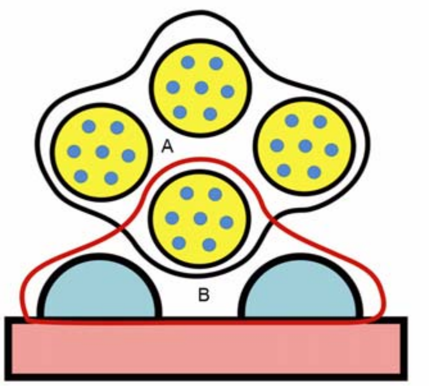


Figure 6. Water molecules and crystals

Verifications

Geese are a kind of poultry that common to see in the countryside. It is common to see that water liquid can easily fall down from a goose feathers with a slight swing. Figure 7 is the SEM photo of a goose feather and we can see that there are many particles attached on the surface. According to the analysis presented in this paper, these particles are the reason of waterproof phenomenon.

In order to calculate the fractal dimensions of the goose feather, we choose part A in fig. 8 and amplify it. Then we can obtain the fractal dimensions of the goose feather:

$$D_{\text{feather}} = 1.7 \tag{4}$$

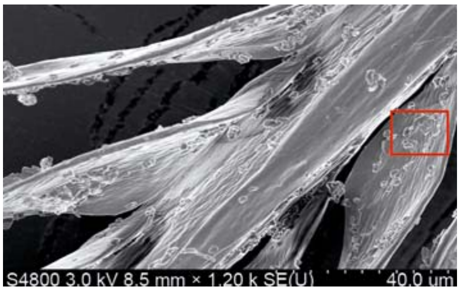


Figure 7. SEM photo of a goose feather surface

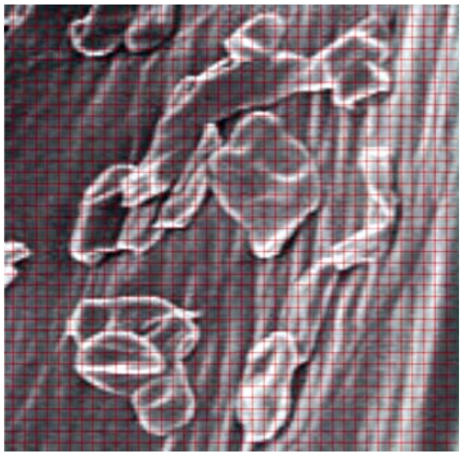
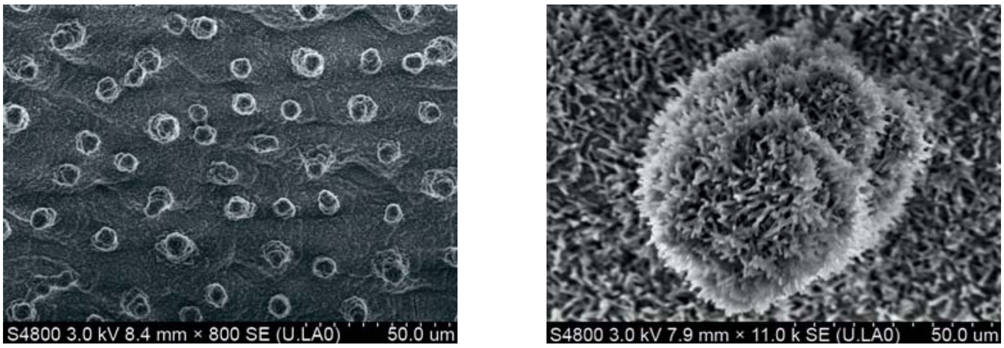


Figure 8. Amplification of part A in fig. 5

The fractal dimensions of pure water are $D_{\text{water}} = 1.92$. We can see that the fractal dimensions of the goose feather are approximate to the fractal dimensions of water. In reality, impurities in the water can change the fractal dimensions of water, and the contacted angle between the surface and water is indeterminate so that the contact area can be changeable. These conditions result in the error of fractal dimensions between the surface and water liquid.

Another plant, lotus leaf, has the property of waterproof. We can see the morphology of the surface of a lotus leaf in fig. 9 and we can obtain the fractal dimensions of the leaf.



(a) (b)

Figure 9. SEM graph of a lotus leaf

The fractal dimension of the lotus leaf in figs. 9(a) and 9(b) are $D_a = 1.776$ and $D_b = 1.778$, respectively. Fractal dimensions of the leaf are approximate to the fractal dimensions of water so that water liquid can slide smoothly on the surface of lotus leaves.

All these experiments can verify the point that if fractal dimensions of the water/particles are equal to the fractal dimensions of the contacted surface, the water liquid/ particle can move with the minimal friction force. The surface has the property of waterproof/dustproof.

Conclusions

A surface with nanoparticles or short nanocylinders, or nanoparticle structure has the properties of waterproof and dustproof. In the paper, fractal harmonic laws are proposed to illustrate the mechanism of waterproof/dustproof. We find that if fractal dimensions of the water/particles are equal to the fractal dimensions of the contacted surface, the water liquid/particle can move with the minimal friction force on the surface, which means the water/particles can move with their own morphology.

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