An alternative form of the Darcy equation

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Abstract

This study presents an alternative form of the Darcy equation. This alternative form will be presented with the use of Bejan number (*Be*) in the Left Hand Side (LHS) of the equation. The main advantage in this alternative form of the Darcy equation is presenting both the Left Hand Side (LHS) and the Right Hand Side (RHS) as dimensionless quantities. For instance, this is similar to the relation of Fanning friction factor with Reynolds number for Hagen-Poiseuille flow (fully developed laminar flow in a circular pipe).

Keywords

alternative form; Darcy equation; Bejan number.

Nomenclature

Be	Bejan number
K	intrinsic permeability of the medium (m^2)
L	flow length (m)
q	flux (discharge per unit area) ($m\ s^{-1}$)
Re	Reynolds number
V	intrinsic velocity ($m\ s^{-1}$)
v	filtration velocity ($m\ s^{-1}$)
Greek symbols	
ΔP	pressure drop (Pa)
μ	dynamic viscosity ($kg\ m^{-1}\ s^{-1}$)
ν	momentum diffusivity ($m^2\ s^{-1}$)
Π	pressure drop number
ρ	density ($kg\ m^{-3}$)
ϕ	porosity

In this study, an alternative form of the Darcy equation will be presented with the use of Bejan number (*Be*) in the Left Hand Side (LHS) of the equation. Darcy's law is a phenomenologically derived constitutive equation, which describes the fluid flow through a porous medium. This law was formulated by Henry Darcy in 1856 [1] based on his observations on the public water supply at Dijon and experiments on steady-state unidirectional flow. The refined modern form popularized in the 1937 book by Muskat [2], can be expressed as follows:

$$\frac{\Delta P}{L} = \frac{\mu}{K} q \tag{1}$$

In Eq. (1), the flux (discharge per unit area), q, is also called the filtration velocity (v). Nowadays, the term usually used is the Darcy velocity, which is the velocity averaged over a representative elementary volume (REV) containing both fluid and solid phases. This filtration velocity, v, (velocity averaged over the medium) is related to the intrinsic velocity, V, (velocity averaged over the pore space) by the following relation:

$$v = \phi V \tag{2}$$

It should be noted that Darcy's law means that the drag is linearly proportional to the velocity. This holds for small velocities only. Experimental measurements of Ward [3] have shown that Darcy's law is valid as long as the Reynolds number based on the square root of intrinsic permeability of the medium as the characteristic length (Re_K) is less than unity. However, Darcy's law breaks down for larger velocities. If the Reynolds number based on the square root of intrinsic permeability of the medium as the characteristic length (Re_K) exceeds the order 1, interial influences flatten the friction factor versus the Reynolds number curve in a manner reminiscent of the friction factor versus the Reynolds number curve in turbulent flow over a rough surface (Moody chart [4]). The transition from Darcy flow to Darcy-Forchheimer flow occurs when the Reynolds number based on the square root of intrinsic permeability of the medium as the characteristic length (Re_K) is of order 10^2. This transition is associated with the occurrence of the first eddies in the fluid flow, for instance, the rotating fluid behind an obstacle or a backward facing step. The magnitude order $Re_K \sim 10^2$ is one in a long list of constructal theory results, which show that the laminar-turbulent transition is associated with a universal local Reynolds number of order 10^2 (Bejan [5]).

On the other hand, the Bejan numbers (Be) is named after Duke University Professor Adrian Bejan. It represents the dimensionless pressure drop along a channel of length L. Historically, Bhattacharjee and Grosshandler [6] performed the scale analysis of a wall jet in 1988. The researchers discovered the new dimensionless group

$$Be = \frac{\Delta P L^2}{\mu \nu} \tag{3}$$

They recognized the general importance of this group throughout forced convection (note the ΔP), and they named it "Bejan number" because of the method of scale analysis that they employed based on Bejan's 1984 book [5].

While unaware of Bhattacharjee and Grosshandler's discovery of the Be dimensionless group, Bejan and Sciubba [7] discovered in 1992 the same dimensionless group (more generally, for any Pr) in the scale analysis and intersection of asymptotes of parallel plates channels with optimal spacings and forced convection. They recognized the general role of this group, and named it pressure drop number (Π). The coincidence between Refs. [6] and [7], and the fact that "optimal spacings" became a fast growing field is why the Bejan number term gained wide acceptance. Today, the Bejan number (Be) has spread because it is general, like the scale analysis, which gave birth to it.

Expressing the dynamic viscosity (μ) in the denominator as a product of the fluid density (ρ) and the momentum diffusivity of the fluid (ν), Awad and Lage [8] wrote the original Bejan number (Be) as

$$Be = \frac{\Delta P L^2}{\rho \nu^2} \tag{4}$$

This new form is more akin the physics it represents and has the advantage of having one single viscosity coefficient in it.

By multiplying the Right Hand Side (RHS) of Eq. (4) by *L/L*, we can rewrite Eq. (4) in the following form

$$Be = \left(\frac{\Delta P}{L}\right)\left(\frac{L^3}{\rho \nu^2}\right) \tag{5}$$

Substituting Eq. (1) into Eq. (5), we obtain

$$Be = \frac{qL^3}{K\nu} \tag{6}$$

Eq. (6) represents an alternative form of Darcy’s law. The main advantage in Eq. (6) is presenting both the Left Hand Side (LHS) and the Right Hand Side (RHS) as dimensionless quantities. For example, this is similar to the relation of Fanning friction factor with Reynolds number for Hagen-Poiseuille flow (fully developed laminar flow in a circular pipe). This alternative form of Darcy’s law can be used in many applications such as water flow through an aquifer (groundwater flow), oil, water, and gas flows through petroleum reservoirs because Darcy’s law forms the scientific basis of fluid permeability used in the earth sciences, particularly in hydrogeology.

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