A MATHEMATICAL MODEL AND SIMULATION OF THE DRYING PROCESS OF THIN LAYERS OF POTATOES IN A CONVEYOR-BELT DRYER

by

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This paper presents a mathematical model and numerical analysis of the convective drying process of small particles of potatoes slowly moving through the flow of a drying agent – hot moist air. The drying process was analyzed in the form of a one-dimensional thin layer. The mathematical model of the drying process is a system of two ordinary non-linear differential equations with constant coefficients and an equation with a transcendent character. The appropriate boundary conditions of the mathematical model were given. The presented model is suitable in the automated control. The presented system of differential equations was solved numerically. The analysis presented here and the obtained results could be useful in predicting the drying kinetics of potatoes and similar natural products in a conveyor-belt dryer.

Key words: *potatoes, drying, thin layer, conveyor-belt dryer, modeling, automatic control*

Introduction

The drying process is a complex process of heat and mass transfer resulting in a direct transfer of humidity from some substance into hot moist air. The heat transfer, necessary for that process, can be direct, convective from the drying agent which flows around the drying material, or indirect, by different procedures. This paper considers the procedure of convective drying of potatoes and also other vegetables, fruits, medical herbs and similar agricultural products in a conveyor-belt dryer using moist air as the drying agent.

The need for drying of different types of natural agricultural products is extremely wide. For centuries, drying has been a widely used method for the preservation of food for long periods. Dried food is resistant to the influence of mould and bacteria and at the same time it retains its biological and nutritional values.

Analysis of phenomena occurring in the "elemental-thin" layer during the drying process is the base for the complex analyses of the mass and heat transfer inside the moist material. Based on these results it is possible to define the drying kinetics for a specific material.

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A lot of work has been dedicated to this problem. Many analytical models based on numerical methods have been presented. In [1, 2], a non-linear partial differential equation was solved numerically and a linear correlation was considered between the thickness of the material and its moisture content. In [3, 4], the importance of internal and external heat transfers in a contact dryer was highlighted. A numerical model for conveyor-dryer solved by a finite-volume method was presented in [5-7]. In [8], a 1-D mathematical model describing heat and mass transfer during drying of a packed bed of porous particles with superheated steam and humid air was presented. In [9] the governing partial differential equations were transformed into a non-similar form by using a special transformation, and then the resulting partial differential equations were solved numerically by using an implicit finite difference method.

Previous experimental and theoretical researches have focused on the convective drying process, especially for drying particles of natural materials. They belong to the class of capillary-porous materials and are the most complex for research. Some of the obtained results have been presented in [10-15].

The aim of this work is to present and solve the mathematical model in an original way, in order to define the essential drying parameters of a movable thin layer of potatoes which could be used for the drying kinetics of potatoes and similar natural products on a conveyor-belt dryer. In that way, the desirable final moisture content can be achieved. Application of the presented model is suitable from the viewpoint of automated control.

Problem formulation

The whole process occurs in the conveyor-belt dryer, which is separated into several segments. In order to provide the necessary flow and temperature of the drying agent, each segment has its own belt conveyor, fan and heater for the drying agent. The velocity of belt conveyors in each segment is different. The schematic view of the first segment of the analyzed conveyor-belt dryer is presented on fig. 1. It consists of a belt transporter for small particles of moist material slowly moving through the flow of the drying agent. Preheated moist air with exactly defined characteristics is the drying agent. The drying agent flows through the belt conveyor, perpendicularly to the moving direction of the moist material.

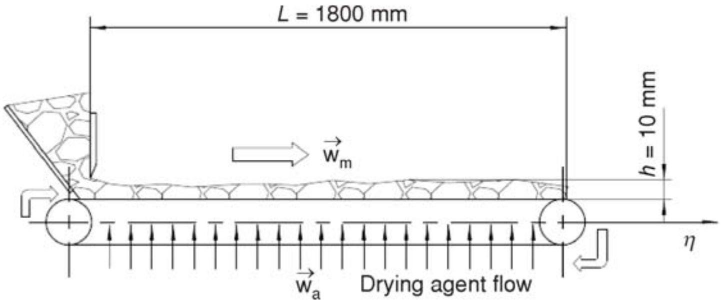

Figure 1. Simplified scheme of functioning of the convective drying process in a conveyor-belt dryer

The mathematical model of the drying process of the "thin layer" of moist potato was developed in the form of four non-linear partial differential equations with constant coefficients. Additionally, one equation of a transcendent character is included. The following assumptions were accepted.

– The velocity of the drying agent in the direction of co-ordinate axes η is negligibly small.

– The velocity of the drying agent in the direction perpendicular to coordinate η has a constant value.
– The total pressure of humid air has a constant value.
– The moist material velocity in the moving direction (axes η) depends only on time, which means that the material moves along the conveyor without slipping.
– The velocity of the moist material in the moving direction (axes η) is negligibly small.
– The heat transfer (α) and moisture transfer (β) coefficients have constant values.

Convection laws are used for heat and moisture transfer between the surface layer of the material and the drying agent. Dalton's law is used for moisture transfer and Newton's law is used for heat transfer [16]. The surface moisture flux according to Dalton's law is:

$$j_{\mathrm{V}} = \beta(p_{\mathrm{wvs}} - p_{\mathrm{wv}}) \tag{1}$$

The surface heat flux according to Newton's law is:

$$q_{\mathrm{V}} = \alpha(\theta_{\mathrm{a}} - \theta_{\mathrm{m}}) \tag{2}$$

Having in mind that the drying agent and vapor are in the gas phase, it is more common to use the difference between the partial pressure of vapor on the material surface and the partial pressure of vapor of the drying agent, eq. (1), instead of the difference between mass participations of vapor on the material surface and the drying agent. Based on the results presented in [17], the following expressions are used for mass (moisture) transfer coefficient β and heat transfer coefficient α:

$$\beta = \frac{\mathrm{Sh}\rho_{\mathrm{V}} D_{\mathrm{V}} M_{\mathrm{VP}}}{d_{\mathrm{e}} p_{\mathrm{V_0}} M_{\mathrm{SV}}} \tag{3}$$

$$\alpha = \frac{\mathrm{Nu}\lambda_{\mathrm{V}}}{d_{\mathrm{e}}} \tag{4}$$

The amount of delivered moisture from the material surface is given as:

$$\dot{\chi}_{\mathrm{m}} = \beta S_{\mathrm{m}}(p_{\mathrm{wvs}} - p_{\mathrm{wv}}) \tag{5}$$

The amount of delivered heat of the drying agent on the material surface is given as:

$$\dot{\chi} = \alpha S_{\mathrm{m}}(\theta_{\mathrm{a}} - \theta_{\mathrm{s}}) \tag{6}$$

These expressions are also the base for analysis of the drying process of thick layer materials.

The mathematical model of the "thin layer" drying process is a system of two non-linear ordinary differential equations with constant coefficients and additionally one equation of a transcendent character. The model based on [18] was improved [19] in order to obtain appropriate drying kinetics of potatoes. This mathematical model has the following form:

$$\frac{\mathrm{d}x_{\mathrm{m}}}{\mathrm{d}\eta} = -\frac{\beta S_{\mathrm{m}}}{w_{\mathrm{m}}\rho_{\mathrm{dm}}}[p_{\mathrm{wvs}} - R_{\mathrm{wv}}\rho_{\mathrm{da}}x_{\mathrm{a}}(\theta_{\mathrm{a}} + \theta_0)] \tag{7}$$

$$\frac{\mathrm{d}\theta_{\mathrm{m}}}{\mathrm{d}\eta} = \frac{\alpha S_{\mathrm{m}}}{w_{\mathrm{m}}\rho_{\mathrm{dm}}(c_{\mathrm{dm}} + x_{\mathrm{m}}c_{\mathrm{m}})}(\theta_{\mathrm{a}} - \theta_{\mathrm{m}}) - \\ - \frac{\beta S_{\mathrm{m}}}{w_{\mathrm{m}}\rho_{\mathrm{dm}}(c_{\mathrm{dm}} + x_{\mathrm{m}}c_{\mathrm{w}})}[p_{\mathrm{wvs}} - R_{\mathrm{wv}}\rho_{\mathrm{da}}x_{\mathrm{a}}(\theta_{\mathrm{a}} - \theta_0)](r_0 - K_{\mathrm{r}}\theta_{\mathrm{m}}) \tag{8}$$

$$A\left(\frac{x_{\mathrm{m}}}{x_{\mathrm{m0}}}\right)^{a_1}\left(\frac{\theta_{\mathrm{m}}}{\theta_{\mathrm{m0}}}\right)^{a_2}\left\{x_{\mathrm{m}}-\left[\frac{-\ln\left(1-\frac{p_{wvs}}{p_1 10^{\frac{p_2\theta_m}{p_3+\theta_m}}}\right)}{k_1(\theta_{\mathrm{m}}+\theta_0)^{k_2}}\right]^{\frac{1}{k_3(\theta_m+\theta_0)+k_4}}\right\}=$$

$$=\frac{\beta S_{\mathrm{m}}}{\rho_{\mathrm{dm}}}[p_{\mathrm{wvs}}-R_{\mathrm{wv}}\rho_{\mathrm{da}}x_{\mathrm{a}}(\theta_{\mathrm{a}}+\theta_0)] \tag{9}$$

As far as it's known, eqs. (7)-(9) can be only solved numerically.

The system of equations defines changes of humidity (x_{m}) and temperature (θ_{m}) of the moist material, and the values of partial pressure of water vapor (p_{wvs}) on the product surface, in dependence on the co-ordinate (η) which determines the position of the particle in the drier. The boundary conditions for this system of equations are:

$$x_{\mathrm{m}}(\eta)\big|_{\eta=0}=x_{\mathrm{m}}(0)=x_{\mathrm{m0}} \tag{10}$$

$$\theta_{\mathrm{m}}(\eta)\big|_{\eta=0}=\theta_{\mathrm{m}}(0)=\theta_{\mathrm{m0}} \tag{11}$$

$$p_{\mathrm{wvs}}(\eta)\big|_{\eta=0}=p_{\mathrm{wvs}}(0)=p_{\mathrm{wvs0}} \tag{12}$$

The boundary contour for which the system of eqs. (7)-(9) is valid, with boundary conditions eqs. (10)-(12), is a rectangle with the length (L), equal to the length of the conveyor segment, and height (h), equal to the height of the "thin layer" on the conveyor (h = 10 mm), which is presented on fig. 2.

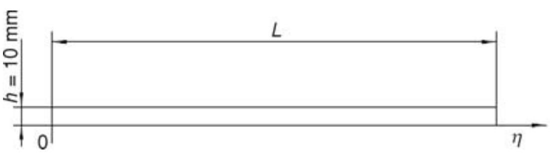

Figure 2. Boundary contour for the system of eqs. (7)-(9), presented as a rectangle with length (*L*) and height (*h*)

From the mathematical viewpoint, the boundary contour in the form of rectangle, presented on fig. 2, could be reduced and further considered as a line with length (L), because the height of a material (h) is small enough and has no influence on the values (x_{m}), (θ_{m}), and (p_{wvs}).

The system of eqs. (7)-(9) with boundary conditions eqs. (10)-(12), is analytically unsolvable. For that reason numerical analysis was chosen as the method for solving the presented system of equations.

Problem solution

At first, the system of eqs. (7)-(9) has to be partially transformed by introducing the following auxiliary values:

$$m_1=\frac{\beta S_{\mathrm{m}}}{w_{\mathrm{m}}\rho_{\mathrm{dm}}} \tag{13}$$

$$m_2 = \frac{\alpha S_{\mathrm{m}}}{w_{\mathrm{m}}\rho_{\mathrm{dm}}} \tag{14}$$

$$m_3 = R_{\mathrm{wv}}\rho_{\mathrm{da}}x_{\mathrm{a}}(\theta_{\mathrm{a}} + \theta_0) \tag{15}$$

$$m_4 = \frac{\beta S_{\mathrm{m}}}{\rho_{\mathrm{dm}}} \tag{16}$$

$$A^* = \frac{A}{x_{m0}^{a_1}\theta_{m0}^{a_2}} \tag{17}$$

After partially transformation the system of eqs. (7)-(9), has the following form:

$$\frac{\mathrm{d}x_{\mathrm{m}}}{\mathrm{d}\eta} + m_1(p_{\mathrm{wvs}} - m_3) = 0 \tag{18}$$

$$\frac{\mathrm{d}\theta_{\mathrm{m}}}{\mathrm{d}\eta} - \frac{m_2}{c_{\mathrm{dm}} + x_{\mathrm{m}}c_w}(\theta_a - \theta_m) + \frac{m_1}{c_{\mathrm{dm}} + x_{\mathrm{m}}x_{\mathrm{w}}}(p_{\mathrm{wvs}} - m_3)(r_0 - K_{\mathrm{r}}\theta_{\mathrm{m}}) = 0 \tag{19}$$

$$m_4 p_{\mathrm{wvs}} + A^* x_m^{a_1}\theta_m^{a_2}\left[\frac{-\ln\left(1 - \dfrac{p_{\mathrm{wvs}}}{p_1 \cdot 10^{\frac{p_2\theta_m}{p_{3+\theta_m}}}}\right)}{k_1(\theta_{\mathrm{m}} + \theta_0)^{k_2}}\right]^{\frac{1}{k_3(\theta_m + \theta_0) + k_4}} - m_4 m_3 - A^* x_x^{(a_1+1)}\theta_m^{a_2} \tag{20}$$

It should be mentioned that the boundary conditions, eqs. (10)-(12) and boundary contour (fig. 2) have already been determined.

The system of eqs. (18)-(20), with boundary conditions eqs. (10)-(12) can be solved by the method of numerical analysis. The algorithm of the Euler method of simple finite differences was used. The Newton method of a tangent was applied for the additional algebraic eq. (20).

Correct selection of the integration step (Δ) is most important for the process of numerical integration, since the stability of a solution to a great extent depends on it. The mean equivalent diameter of a particle (d_{e}), *i. e.* ($\Delta = d_{\mathrm{e}}$) has been accepted as a natural and logical solution for the integration step (Δ).

The algorithm of numerical integration of the system of equations starts with definition of the network, *i. e.* by dividing the boundary contour into equal segments according to the adopted integration step (Δ). The boundary contour from fig. 2 defined as an integration network is given on fig. 3.

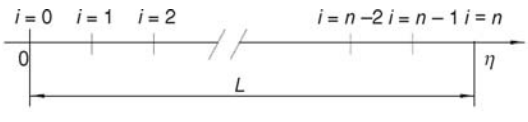

Figure 3. The boundary contour with a network for numerical integration of the system of eqs. (18)-(20), with boundary conditions eqs. (10)-(12)

Derivatives of individual values replaced by finite differences were defined as:

$$\frac{\mathrm{d}x_{\mathrm{m}}}{\mathrm{d}\eta} \approx \frac{x_{m,i} - x_{m,i-1}}{\Delta} \tag{21}$$

$$\frac{\mathrm{d}\theta_m}{\mathrm{d}\eta} \approx \frac{\theta_{m,i} - \theta_{m,i-1}}{\Delta} \tag{22}$$

The Newton's iteration method of tangent was applied for solving equations (18)-(20).

$$x_{k+1} = x_\mathrm{k} - \frac{f(x_\mathrm{k})}{f'(x_\mathrm{k})} \quad (k = 0,\ 1,\ 2, \ldots, n) \tag{23}$$

The iterative formula (23) should be used until two consecutive iterations fall within the set accuracy limits.

In order to apply the previously defined algorithm on eq. (20), it is necessary to perform its partial transformation. Therefore, auxiliary values were introduced, as follows:

$$K_1 = m_4 \tag{24}$$

$$K_2 = A^* x_m^{a_1} \theta_m^{a_2} \tag{25}$$

$$K_3 = k_1 (\theta_\mathrm{m} + \theta_0)^{k_2} \tag{26}$$

$$K_4 = \frac{1}{k_3(\theta_\mathrm{m} + \theta_0) + k_4} \tag{27}$$

$$K_5 = m_4 m_3 \tag{28}$$

$$K_6 = A^* x_m^{(a_1+1)} \theta_m^{a_2} \tag{29}$$

$$p_\mathrm{wvs} = p_1 \cdot 10^{\frac{p_2 \theta_m}{p_3 + \theta_m}} \tag{30}$$

After introduction of auxiliary values, eq. (20) has the following form:

$$K_1 p_\mathrm{wvs} + K_2 \left[\frac{-\ln\left(1 - \dfrac{p_\mathrm{wvs}}{p_\mathrm{wvsz}}\right)}{K_3} \right]^{K_4} - K_5 - K_6 = 0 \tag{31}$$

According to the algorithm defined for solving equations of a transcendental character, given by eqs. (21) and (22), the iterative algorithm for solving eq. (31) is:

$$p_{wvs}^{K+1} = p_{wvs}^K = \frac{K_1 p_{wvs}^K + K_2 \left[\dfrac{-\ln\left(1 - \dfrac{p_{wvs}^K}{p_{wvsz}^K}\right)}{K_3} \right]^{K_4} - K_5 - K_6}{K_1 + \dfrac{K_2 K_4}{K_3} \left[\dfrac{-\ln\left(1 - \dfrac{p_{wvs}^K}{p_{wvsz}^K}\right)}{K_3} \right]^{(K_4 - 1)} \dfrac{1}{p_{wvsz} - p_{wvs}^K}} \tag{32}$$

The following value was taken as the starting value of the iterative procedure ($K = 0$):

$$p_{wvs}^{K=0} = 0.9999\, p_{\mathrm{wvsz}} \tag{33}$$

The iterative procedure should be applied until the difference between previous and current iterations is less than (10^{-9}).

Now, it is possible to define the complete algorithm for numerical solution of the system of eqs. (18)-(20), with boundary conditions eqs. (10)-(12) and the boundary contour given on fig. 2.

The following algorithm was used.

At the beginning of the co-ordinate system ($i = 0$, see fig. 3) the values of (x_{m}), (θ_{m}), and (p_{wvs}) were defined by boundary conditions eqs. (10)-(12):

$$x_m^{i=0} = x_{m0} \tag{34}$$

$$\theta_m^{i=0} = \theta_{m0} \tag{35}$$

$$p_{wvs}^{i=0} = p_{wvs0} \tag{36}$$

The other values of (x_{m}), (θ_{m}), and (p_{wvs}), for $i = 1$ to n were calculated as follows:

(1) By substituting expressions (21) and (22) in eqs. (18) and (19) the following equations were obtained:

$$x_{m,i} = x_{m,i-1} - \Delta m_1 (p_{\mathrm{wvs},i-1} - m_3) \tag{37}$$

$$\theta_{m,i} = \theta_{\mu,\iota-1} + \frac{\Delta m_2}{c_{\mathrm{dm}} + x_{m,i-1} c_{\mathrm{w}}} (\theta_a - \theta_{m,i-1}) -$$

$$- \frac{\Delta m_2}{c_{\mathrm{dm}} + x_{m,i-1} c_{\mathrm{w}}} (p_{wvs,i-1} - m_3)(r_0 - K_{\mathrm{r}} \theta_{m,i-1}) \tag{38}$$

(2) The obtained values from eqs. (37) and (38) were implemented in eq. (24) to (32) giving the following expressions:

$$K_1 = m_4 = \mathrm{const} \tag{39}$$

$$K_{2,i} = A^* x_{m,i}^{a_1} \theta_{m,i}^{a_2} \tag{40}$$

$$K_{3,i} = k_1 (\theta_{m,i} + \theta_0)^{k_2} \tag{41}$$

$$K_{4,i} = \frac{1}{k_3 (\theta_{m,i} + \theta_0) + k_4} \tag{42}$$

$$K_5 = m_4 m_3 = \mathrm{const} \tag{43}$$

$$K_{6,i} = A^* x_{m,i}^{(a_1+1)} \theta_{m,i}^{a_2} \tag{44}$$

$$p_{wvsz,i} = p_1 \cdot 10^{\frac{p_2 \theta_{m,i}}{p_3 + \theta_{m,i}}} \tag{45}$$

$$p_{wvs,i}^{K+1} = p_{wvs,i}^{K} = -\frac{K_1 p_{wvs,i}^{K} + K_{2,i}\left[\dfrac{-\ln\left(1-\dfrac{p_{wvs,i}^{K}}{p_{wvsz,i}^{K}}\right)}{K_3}\right]^{K_{4,i}} - K_5 - K_{6,i}}{K_1 + \dfrac{K_{2,i}K_{4,i}}{K_{3,i}}\left[\dfrac{-\ln\left(1-\dfrac{p_{wvs,i}^{K}}{p_{wvsz,i}^{K}}\right)}{K_{3,i}}\right]^{(K_{4,i}-1)} \dfrac{1}{p_{wvsz,i} - p_{wvs,i}^{K}}} \tag{46}$$

The following value was taken as the starting value of the iterative procedure ($K = 0$):

$$p_{wvs,i}^{K=0} = 0.9999\, p_{\text{wvsz,i}} \tag{47}$$

The iterative procedure should be applied until the difference between previous and subsequent iterations is less than 10^{-9}.

Results and discussion

The drying process of small potato layers was modeled using the previously presented algorithm. The basic concept was taken from [19]. The dimension of potato pieces was $d_e = 6$ mm, the belt conveyor length was $L = 1.8$ m, while the height of the product layer on the belt conveyor was $h = 10$ mm. The size of one potato piece ($\Delta = d_e$) was chosen for the step of numeric integration, thus the integration net at the boundary contour contains 300 steps. The value of initial humidity and temperature for moist potato pieces were: $x_{m0} = 4.882$ kg_w/kg_{dm}, $\theta_{m0} = 20$ °C.

Table 1. The set of values of the drying agent parameters used for the calculation

x_a [$kg_{wv}kg^{-1}{}_{da}$]	0.05	0.06	0.07	0.08
θ_a [°C]	80	90	100	110

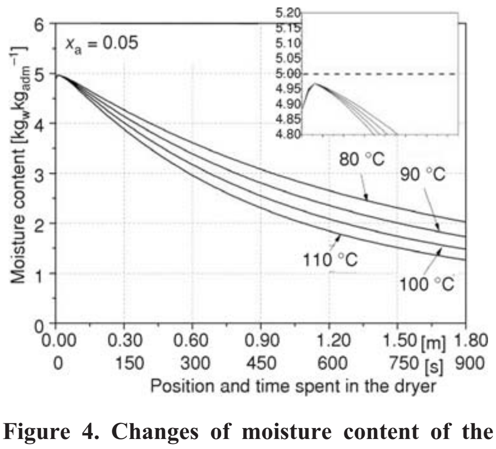

Figure 4. Changes of moisture content of the drying material (x_m) on the first segment

The drying schedule which included most common values for humidity of drying agent (x_a) and temperature of drying agent (θ_a) was adopted in accordance with [12, 19, 20]. The humidity of the drying agent with four different temperatures was used between 0.05 and 0.08, which is presented in tab.1.

The calculated values of changes of moisture (x_m) and temperature (θ_m) for potato pieces as a function of time/position in the drier are presented graphically. The calculated values at the beginning and at the end of the first segment of the conveyor-belt are presented in tab. 2.

On figs. 4-7 changes of humidity of the drying material (x_m) on the first segment of the belt conveyor (η = 0-1.8 m) are presented. For all four different values of humidity of the drying agent the increase of the humidity of the drying material surface occurs in first 15 seconds. The reason for that is condensation, as an effect caused by the difference of the temperatures of

Table 2. Calculated values of moisture and temperature of the drying material

θ_a = 110 °C		x_a [kg$_V$kg$_{dm}$$^{-1}$]							
		0.08		0.07		0.06		0.05	
i	η [m]	x_m [kg$_V$kg$_{dm}$$^{-1}$]	θ_m	x_m [kg$_V$kg$_{dm}$$^{-1}$]	θ_m	x_m [kg$_V$kg$_{dm}$$^{-1}$]	θ_m	x_m [kg$_V$kg$_{dm}$$^{-1}$]	θ_m
0	0	4.882	20.000	4.882	20.000	4.882	20.000	4.882	20.000
1	0.006	5.007	42.902	4.988	40.716	4.968	38.531	4.949	36.345
2	0.012	5.051	54.076	5.024	51.356	4.996	48.504	4.967	45.528
3	0.018	5.037	57.905	5.012	55.613	4.985	53.203	4.958	50.682
4	0.024	5.021	61.168	4.997	59.245	4.972	57.221	4.945	55.099
5	0.030	5.003	63.949	4.980	62.344	4.956	60.652	4.931	58.876
6	0.036	4.984	66.320	4.962	64.986	4.939	63.579	4.915	62.100
…									
296	1.776	1.294	105.330	1.290	105.347	1.287	105.365	1.283	105.383
297	1.782	1.290	105.357	1.286	105.374	1.282	105.391	1.279	105.409
298	1.788	1.285	105.384	1.282	105.400	1.278	105.418	1.274	105.436
299	1.794	1.281	105.410	1.277	105.427	1.274	105.444	1.270	105.462
300	1.800	1.277	105.437	1.273	105.453	1.270	105.470	1.266	105.488

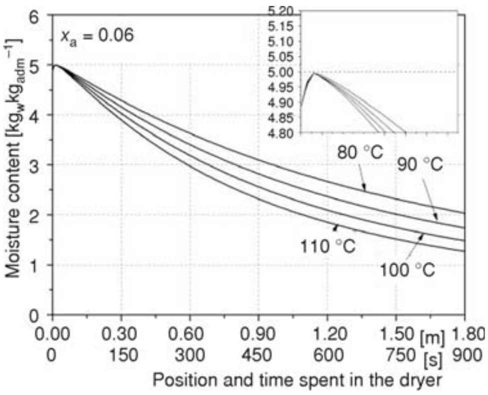

Figure 5. Changes of moisture content of the drying material (x_m) on the first segment

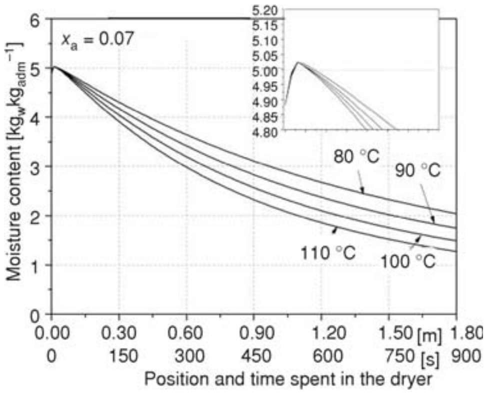

Figure 6. Changes of moisture content of the drying material (x_m) on the first segment

the very hot and wet drying agent and the cold drying material. On the one hand, it is unfavorable because of the increase in moisture content, while on the other hand, it has a positive effect because the water vapor from the drying agent during the condensation releases latent heat of phase change and heats the drying material. This phenomenon is short and occurs within the first few centimeters on the belt conveyor. After that the temperature of the drying material increases

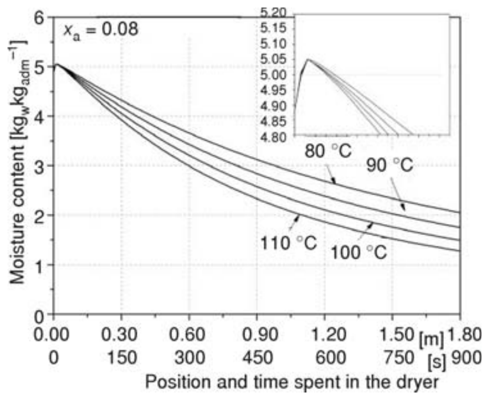

Figure 7. Changes of moisture content of the drying material (x_m) on the first segment

and it starts to lose its humidity. The increase of the humidity of the drying material surface is better explained by the presented model eqs. (37), (38) and (46) compared to other models [10-12] which could be significant from the aspect of regulation of the drying process.

It is obvious that the moisture loss is higher at the beginning of the process. The main reason is effortless transfer of moisture from the surface of the wet material to the drying agent. Later on it is connected with the transfer of moisture from the inside of the drying material which requires much more energy and time.

On figs. 8 and 9 temperature changes of the drying material (θ_m) on the first segment of the belt conveyor (η = 0-1.8 m) are presented. The highest heat transfer is in the first two minutes of the drying process. Later on the temperature rises slowly and in the end of the belt conveyor, slowly comes close to the temperature of the drying agent.

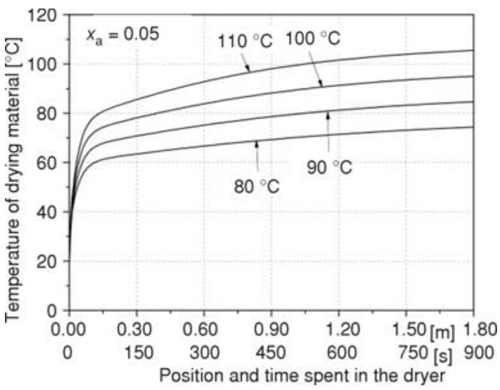

Figure 8. Temperature changes of the drying material (x_m) on the first segment

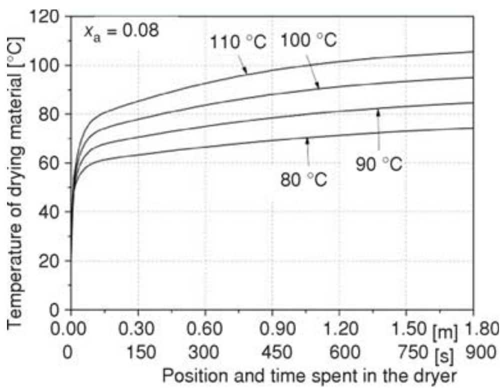

Figure 9. Temperature changes of the drying material (x_m) on the first segment

Having in mind figs. 4 to 7 and results presented in tab. 2., it is obvious that the influence of humidity of the drying agent is not significant in the end of first segment of the belt conveyor (η = 0- 1.8 [m]). The obtained values for the moisture content in the end of the belt conveyor is very similar for the same drying agent temperature and different drying agent humidity. It is the first phase of the drying process when the drying velocity depends mainly on the heat transfer from the drying agent to the drying material [21, 22].

Conclusions

The presented original mathematical model described changes in thin layers of natural materials. This is significant as kinetic changes occurring during drying differ on the surface and inside the material. Here presented model treats heat and mass transfer on the surface and inside the material as one continuum, but with specific differences. It is suitable for application in automated control of belt conveyer dryers. The obtained results using the developed mathematical model give a clear picture of moisture and temperature change in the thin layer of the material and present a good basis for further analysis of changes occurring inside a thick layer of moist materials undergoing the drying process.

Acknowledgments

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Nomenclature

A, a_1, a_2 – constants, [–]
A^* – constant, [–]
c_{dm} – specific heat capacity of dry material, [$Jkg^{-1}K^{-1}$]
c_w – specific heat capacity of water, [$Jkg^{-1}K^{-1}$]
D_V – diffusion coefficient, [m^2s^{-1}]
d_e – equivalent diameter of a particle, [m]
h – height of moist material on belt transporter, [m]
k_1, k_2, k_3, k_4 – constants, [–]
$K_1, K_2, K_3, K_4, K_5, K_6$ – constants, [–]
L – length of belt conveyor, [m]
m_1, m_2, m_3, m_4 – constants, [–]
Nu – Nusselt number ($= \alpha d_e/\lambda_v$), [–]
p_{V_0} – atmospheric pressure, [Pa]
p_{wv} – partial pressure of water vapor of drying agent, [Pa]
p_{wvs} – partial pressure of water vapor on surface of moist material, [Pa]
p_{wvsz} – partial pressure of water vapor on surface of moist material in saturated state, [Pa]
p_1, p_2, p_3 – constants, [–]
r_0 – latent heat of evaporation of water at 0 [°C], [Jkg^{-1}]
K_r – constant, [–]
M_{VP} – molar mass of water vapor, [$kgmol^{-1}$]
M_{SV} – molar mass of dry air, [$kgmol^{-1}$]
R_{wv} – gas constant of water vapor, [$Jkg^{-1}K^{-1}$]
Sh – Sherwood number ($= \beta d_e$/D), [–]
S_m – specific surface area of evaporation (surface/volume), [m^2m^{-3}]
w_m – velocity of moist material [ms^{-1}]
x_a – absolute humidity of moist air [$kg_{wv}kg_{da}^{-1}$]
x_m – absolute humidity of moist material [$kg_w kg_{dm}^{-1}$]

Greek symbols

α – heat transfer coefficient, [$Wm^{-2}K^{-1}$]
β – mass (moisture) transfer coefficient, [$kgm^{-2}Pa^{-1}s^{-1}$]
Δ – step of numerical integration, [–]
η – space co-ordinate, [m]
θ_a – temperature of drying agent, [°C]
θ_m – temperature of moist material, [°C]
θ_s – temperature on the surface of moist material, [°C]
θ_0 – absolute zero, [°C]
λ_v – thermal conductivity, [$Wm^{-1}K^{-1}$]
ρ_{da} – partial density of dry air, [$kg_{da}m^{-3}{}_a{}^{-1}$]
ρ_{dm} – partial density of dry material, [$kg_{dm}m^{-3}m^{-1}$]
ρ_V – density of drying agent, [kgm^{-3}]
$\dot{\chi}_m$ – amount of delivered moisture from the material surface [$kgm^{-3}s^{-1}$]
$\dot{\chi}_t$ – amount of delivered heat of the drying agent on the material surface [Wm^{-3}]

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