UNSTEADY MIXED CONVECTION IN A POROUS MEDIA FILLED LID-DRIVEN CAVITY HEATED BY A SEMI-CIRCULAR HEATERS

by

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A computational study has been performed on natural convection heat transfer and fluid flow in a porous media filled enclosure with semi-circular heaters by using finite element method. The ceiling of the cavity moves with a constant velocity and it is insulated. Vertical walls temperature is lower than that of heaters. Results are presented via streamlines, isotherms, average Nusselt numbers, and cross-sectional velocity for different governing parameters such as Richardson and Darcy numbers, and dimensionless time. It is observed that both circulation of the flow and heat transfer is strongly affected with time increment and Darcy number inside the cavity.

Key words: *porous media, lid-driven cavity, mixed convection, curvilinear heater*

Introduction

Natural convection is extremely important in thermofluid science due to its wide application areas as cooling of electronic equipments, heat exchangers, building ventilation, cooling processes, and heat exchangers. Heat transfer and fluid flow in porous media filled enclosures or channels have important engineering applications such as filtration, separation processes in chemical industries, solar collectors, heat exchangers, *etc.* Studies on thermal convection in porous media and state-of-the-art reviews are given by Ingham and Pop [1], Bejan [2], and Vafai [3].

Mixed convection heat transfer is an important heat transfer regime that it occurs when forced and natural convection become effective simultaneously in building heating and ventilation, cooling of electronic devices, heat exchangers and air heater solar collectors. The cavity can be filled with a porous media. The lid-driven cavities are also common mathematical model for mixed convection flow and heat transfer [4, 5].

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Jue [6] worked on the convection flow caused by a torsionally-oscillatory lid with thermal stable stratification in an enclosure filled with porous medium for different Grashof, Reynolds, Darcy numbers and porosity. He indicated that the influence of oscillatory frequency is so serious in heat flux variation at some particular frequency corresponding to the resonant frequency. Khanafer and Chamkha [7] performed a work to investigate the laminar, mixed convection in an enclosure with a Darcian fluid-saturated uniform porous medium in a lid-driven cavity in the presence of internal heat generation by using the finite volume approach along with the advanced diagnostic imaging procedure. A computational work has been performed by Oztop [8] to simulate the mixed convection heat transfer in a lid-driven porous media filled and partially heated cavity. He tested the effects of location of partial heater on heat transfer and fluid flow by using finite volume method. It is found that the location of heater can be used as control parameter for heat and fluid flow. Mahmud and Pop [9] studied the mixed convection in vented enclosure filled with a porous medium. In their case, inlet and outlet ports are located in the bottom and top and flow motion resembles to lid-driven cavity flow. Double-diffusive mixed convection in a lid-driven enclosure in porous media filled cavities are studied by Khanafer and Vafai [10] by using finite volume approach. They indicated that the buoyancy ratio, Darcy, Lewis, and Richardson numbers have profound effects on the double-diffusive phenomenon. Unsteady mixed or forced convection heat transfer on lid-driven cavities is very limited. In this context, Shi and Khodadadi [11, 12] made a computational work on mixed convection heat transfer in a lid-driven cavity due to an oscillating thin fin. They investigated the movement of thin fin on flow filed and heat transfer when lid-moves from left to right. They observed that phase of the thin fin is extremely important parameter to control flow and heat transfer and temperature distribution inside the cavity. Different shaped bodies or fins are inserted into cavity or attached to the cavity wall. They may have insulated or constant heat flux or temperature. In this context, Rahman *et al.* [13] made a computational work on MHD mixed convection in a lid-driven cavity with a heated semi-circular source.

The main aim of this work is to examine the unsteady mixed convection in an enclosure which is heated by two semi-circular heaters from bottom and filled with porous media. Results will be presented by streamlines, isotherms, Nusselt number, and velocity profiles.

Physical model, grid independency, and validation of the code

The physical model consists a square closed cavity with moving top adiabatic ceiling and it has two semi-circular heaters on the bottom wall while vertical walls have lower temperature than that of heaters as presented in fig. 1 with boundary conditions. Remaining parts are adiabatic. The length and width of the 2-D cavity is equal to each other and it is filled with porous medium. The velocity of the lid is taken as constant (u_0). Diameters of the semi-circular heaters are taken as H_1 and H_2, respectively. Gravity acts in y-direction. In the physical model, the half circular cylinders are heated as isothermally and their temperature is lower than that of vertical walls. Grid independency is illustrated in fig. 2 for τ = 0.1, Pr = 0.71, Ri = 1, and Da = 10^{-4}. As shown from the figure, 5498 grid is chosen in the work. Validation of the code was performed against to study of Mahmud and Fraser [14]. The code is adopted according to literature and it is seen that there is a good agreement between literature (on the right column) and present work (on the left column) in fig. 3.

Equations and numerical method

The fluid is considered as Newtonian and incompressible and the flow is laminar and unsteady. The porous bed is also assumed to be uniform, isotropic, saturated with incom-

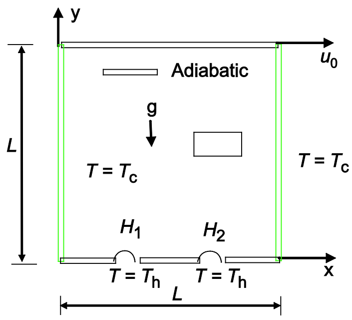

Figure 1. Schematic diagram for the problem with boundary conditions and co-ordinate system

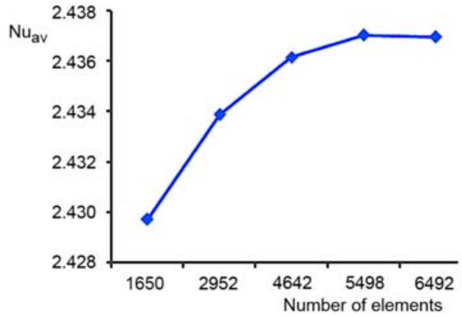

Figure 2. Grid independency study with τ = 0.1, Pr = 0.71, Ri = 1, and Da = 10^{-4}

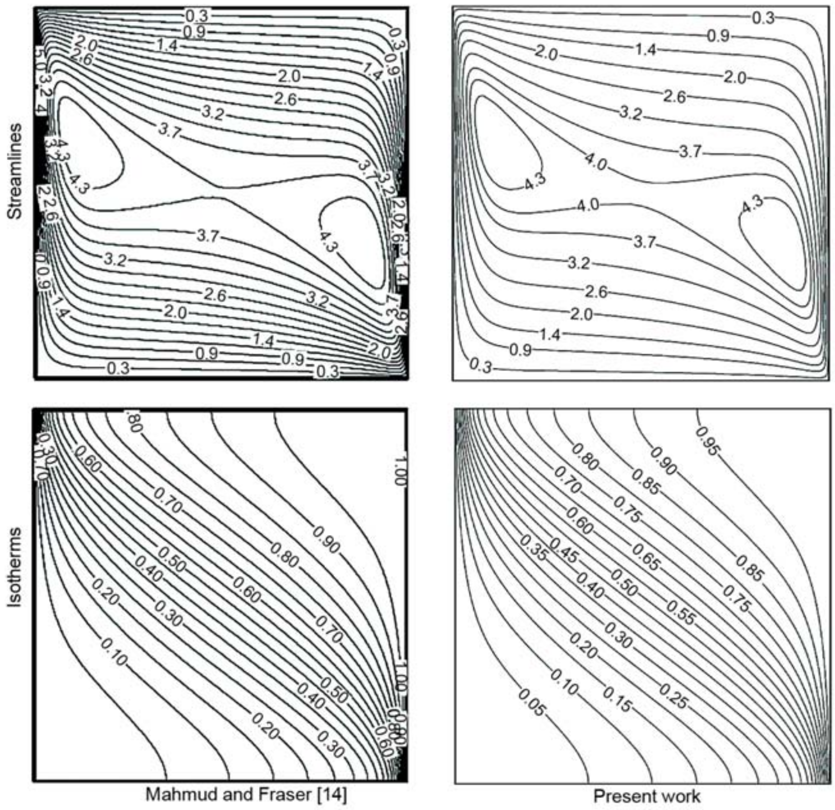

Figure 3. Code validation at Ra_m = 1000 and Ha = 5

pressible fluid. The Boussinesq approximation is applied for the fluid properties based on the variation of density with temperature and to couple the temperature field to the flow field. Taking into account the above declared suppositions and disregarding the Forchheimer inertia term, the governing equations for unsteady 2-D mixed convection flow in a lid-driven porous cavity using conservation of mass, momentum and energy can be written with the following dimensionless forms:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\frac{\partial U}{\partial \tau}+U\,\frac{\partial U}{\partial X}+V\,\frac{\partial U}{\partial Y}=-\,\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\,\nabla^2 U-\frac{1}{\mathrm{Re\,Da}}\,U \tag{2}$$

$$\frac{\partial V}{\partial \tau}+U\,\frac{\partial V}{\partial X}+V\,\frac{\partial V}{\partial Y}=-\,\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\,\nabla^2 V-\frac{1}{\mathrm{Re\,Da}}\,V+\mathrm{Ri}\,\theta \tag{3}$$

$$\frac{\partial \theta}{\partial \tau}+U\,\frac{\partial \theta}{\partial X}+V\,\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re\,Pr}}\,\nabla^2 \theta \tag{4}$$

The transformed initial and boundary conditions are:
- $\tau = 0$, entire domain: $U = V = 0$, $\theta = 0$
- $\tau > 0$, at left and right walls: $U = 0$, $V = 0$, $\theta = 0$
- at top wall and bottom wall other than semi-circle: $U = 0$, $V = 0$, $\partial\theta/\partial Y = 0$
- on semi-circle: $U = V = 0$, $\theta = 1$

The governing equations, initial and boundary conditions are transformed into dimensionless forms using the dimensionless variables:

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{u}{u_0},\quad V=\frac{v}{u_0},\quad P=\frac{p+\rho g y}{\rho\, u_0^{\,2}},\quad \tau=\frac{t u_0}{L},\ \text{and}\ \ \theta=\frac{T-T_\mathrm{c}}{T_\mathrm{h}-T_\mathrm{c}}$$

The dimensionless parameters in the equations are defined:

$$\mathrm{Pr}=\frac{\nu}{\alpha},\qquad \mathrm{Re}=\frac{u_0 L}{\nu},\qquad \mathrm{Ri}=\frac{g\beta(T_\mathrm{h}-T_\mathrm{c})L}{u_0^2},\qquad \mathrm{Da}=\frac{k}{L^2}$$

The average Nusselt number evaluated along the semi-circles can be expressed:

$$\mathrm{Nu_{av}}=-\frac{1}{L_\mathrm{s}}\int_0^{L_\mathrm{s}}\frac{\partial \theta}{\partial Y}\,\mathrm{d}X$$

where L_s is the length of the semi-circle.

Galerkin finite element method [13] is used to complete discretization process of the system of partial differential equations, eqs. (1)-(4), subject to their corresponding initial and boundary conditions. The quadratic triangular element is used to develop the finite element equations. For the velocities and temperature all the six nodes are used and for the pressure only the corner nodes are used. The non-linear algebraic equations arising from the finite element formulation are solved by applying the Newton-Raphson iteration technique. The iteration has been done till the solution becomes convergent. $|\Gamma^{m+1} - \Gamma^{m}| \le 10^{-6}$ where m is the number of iteration and Γ is the general dependent variable.

Results and discussions

A numerical study has been performed to simulate the unsteady mixed convection heat transfer and fluid flow in a lid-driven cavity with curvilinear heaters. With this aim, results will be presented via streamlines, isotherm, and Nusselt numbers in next parts of the work. In this study Reynolds number as taken as constant as 100 and Grashof numbers are varied to supply different Richardson numbers. Figure 4 presents the stream function contours for different values of Darcy numbers at Ri = 0.1 (forced convection dominant regime) and different non-dimensional time. As well-known from the literature, Darcy number is a measure for permeability in flow through porous media. In this work, Darcy number is changed from 10^{-2} to 10^{-4}. As seen from the figure, for Da = 10^{-2}, fig. 4(a), a long disc shaped longitudinal circulation cell is observed for the time beginning (τ = 0.01) and the shape becomes almost same for other values of Darcy number but the strength of the flow decreases with decreasing of Darcy number due to decreasing of resistance. With increasing dimensionless time, still semi-circular heaters are not effective on flow τ = 0.01 due to developing flow. As an expected result, flow strength increases with increasing of the dimensionless time. A small circulation cell is observed at the left corner. This cell becomes bigger for Da = 10^{-3} and Da = 10^{-4}. For τ = 1.0, multiple cell is formed for Da = 10^{-4} due to presence of the semi-circular heaters and low resistance to the flow of the porous medium.

Figure 5 gives stream function contours for Ri = 1 (mixed convection). In this case, effects of natural convection and forced convection can be comparable. Comparison of these results with fig. 4 showed that number of cell and flow strength are affected from the Richardson number for the same dimensionless time and Darcy numbers. Secondary circulating cell is formed in front of the second semi-circular heater due to flowing fluid from left to right by moving lid and up going flow due to natural convection. Again, multiple cells are formed for the lowest value of Darcy number.

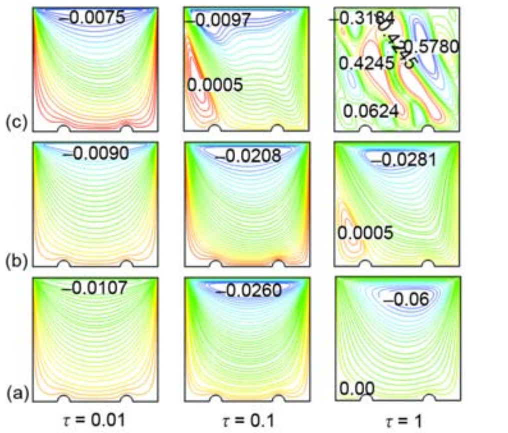

Figure 4. Stream function contours for Ri = 0.1 and (a) Da = 10^{-2}, (b) Da = 10^{-3}, and (c) Da = 10^{-4} *(for color image see journal web-site)*

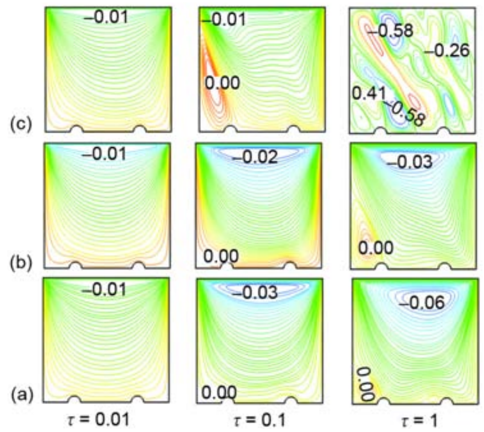

Figure 5. Stream function contours for Ri = 1 and (a) Da = 10^{-2}, (b) Da = 10^{-3}, and (c) Da = 10^{-4} *(for color image see journal web-site)*

Figure 6 illustrates the stream function contours for Ri = 10 (natural convection dominant regime). Natural convection regime becomes dominant in this case and flowing flow due to moving lid becomes very weak due to high Richardson number. Thus, Benard cells are formed above semi-circular heaters. The domain is divided almost two parts for higher values of Darcy numbers as flow due to moving lid (top half) and buoyant flow due to heaters (bottom half). Interaction of flow due to moving lid increases for Da = 10^{-3} and flow distribution is almost same with earlier Richardson number values at Da = 10^{-4}.

Figures 7-9 display the isotherms for same parameters with figs. 4-6 to answer the question of how parameters are effective on temperature distribution. As indicated earlier part of the paper, the cavity has two semi-circular isothermal heaters. Heat is transferred from heater to vertical walls which have colder temperature. Here, moving lid behaves as mixer for the flow. Distribution of the temperature according to time can be seen from isotherms (left to right column). Isotherms are developed above heaters and they distributed to ceiling. Isotherms become almost constant with time for Ri = 0.1 at all values of Darcy number except Da = 10^{-4}. They show very similar distribution due to small diameter of the semi-circular heater and they behave like partial heater. Thermal boundary layer becomes very thin for lower values of Darcy numbers as seen from the figures.

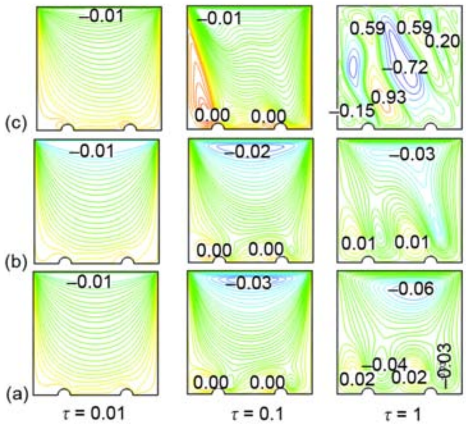

Figure 6. Stream function contours for Ri = 10 and (a) Da = 10^{-2}, (b) Da = 10^{-3}, and (c) Da = 10^{-4}
(for color image see journal web-site)

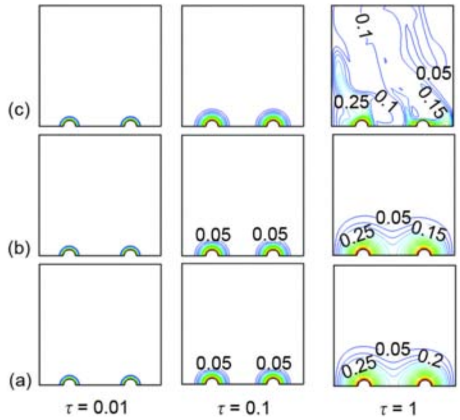

Figure 7. Isotherm contours for Ri = 0.1 and (a) Da = 10^{-2}, (b) Da = 10^{-3}, and (c) Da = 10^{-4}
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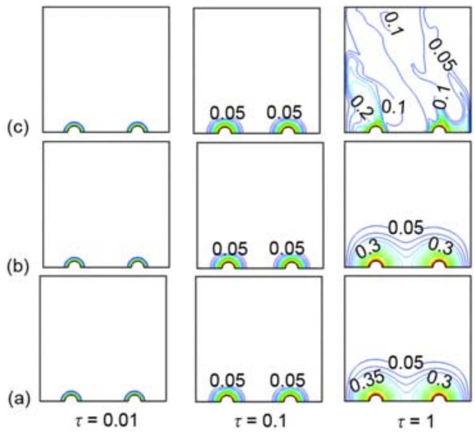

Figure 8. Isotherm contours for Ri = 1 and (a) Da = 10^{-2}, (b) Da = 10^{-3}, and (c) Da = 10^{-4}
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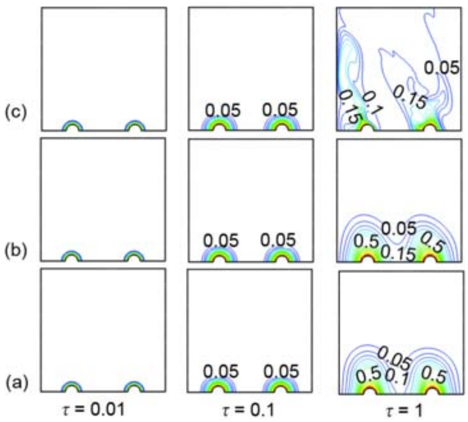

Figure 9. Isotherm contours for Ri = 10 and (a) Da = 10^{-2}, (b) Da = 10^{-3}, and (c) Da = 10^{-4}
(for color image see journal web-site)

Figures 10(a)-(c) are plotted to see the variation of average Nusselt number with Richardson numbers for different values of Darcy number at different dimensionless time. As seen from the figure, average Nusselt number becomes almost constant along the Richardson number for all values of Darcy number. However, Nusselt number becomes higher for lower

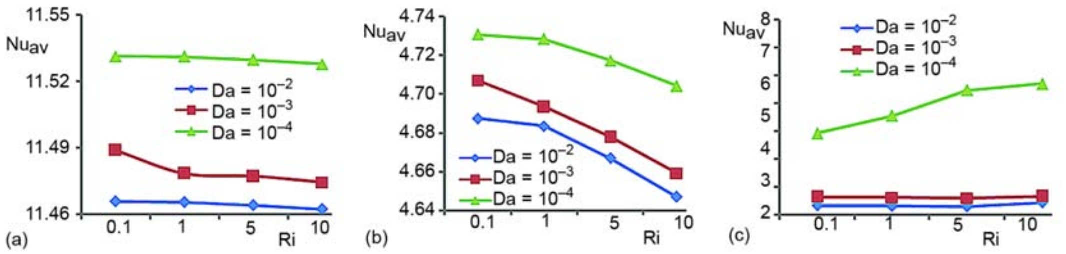

Figure 10. Variation of average Nusselt number with Richardson number for different values of Darcy number at (a) τ = 0.01, (b) τ = 0.1, and (c) τ = 1

values of Darcy number. Values of average Nusselt number is decreased with increasing of the dimensionless time increment and Nu_{av} value is decreased with increasing of Richardson number due to decreasing of flow kinetic energy as given by Oztop and Dagtekin [5]. For τ = 1.0, heat transfer becomes almost same effort the higher values of Darcy number but a huge increment is observed with the lowest value of Darcy number due to non-porosive structure or very low resistance to the flow.

Conclusions

A computational work has been done by using finite element method to investigate the unsteady mixed convection flow and heat transfer in a lid-driven porous cavity with two isothermal curvilinear heaters. The main findings can be drawn from the work as follows.

- Multiple circulation cells are formed for the lowest value of Darcy number due to domination of natural convection heat transfer.
- Heat transfer decreased with decreasing of Darcy number and Richardson number.
- Flow strength increases with increasing of dimensionless time increment.
- Semi-circular heaters make small effects on mixed convection flow and heat transfer due to small diameters. And, they behave as partial heaters.

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Nomenclature

Da – Darcy number ($= k/L^2$)
g – gravitational acceleration, [ms^{-2}]
H_1 – semi-circular heater (left)
H_2 – semi-circular heater (right)
k – fluid conductivity, [Wm^{-1}K^{-1}]
L – length of the cavity, [m]
L_s – length of the semi-circular, [m]
Nu – Nusselt number ($= hLk^{-1}$)
P – non-dimensional pressure
p – pressure, [Nm^{-2}]
Pr – Prandtl number, ($= \nu/\alpha$)
Ri – Richardson number [$g\beta(T_h - T_c)L/u_0^2$]
T – temperature, [K]
t – time, [s]
U, V – dimensionless velocity components
u, v – velocity components, [ms^{-1}]
u_o – lid velocity, [ms^{-1}]
X, Y – dimensionless co-ordinates
x, y – Cartesian co-ordinates

Greek symbols

α – thermal diffusivity, [ms^{-1}]
β – thermal expansion coefficient, [K^{-1}]
θ – non-dimensional temperature
ν – kinematic viscosity, [ms^{-1}]
ρ – density, [kgm^{-3}]
τ – dimensionless time

Subscripts

av – average

h – hot c – cold

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