NUMERICAL COMPARISON OF VISCOSITY MODELS ON MIXED CONVECTION IN DOUBLE LID-DRIVEN CAVITY UTILIZED CuO-WATER NANOFLUID

by

Nader POURMAHMOUD, Ashkan GHAFOURI*, and Iraj MIRZAEE
Department of Mechanical Engineering, Urmia University, Urmia, Iran

Original scientific paper
DOI: 10.2298/TSCI130309048P

Laminar incompressible mixed-convective heat transfer in 2-D lid-driven cavity, filled with nanofluid CuO-water, is studied numerically. Eight different viscosity models are compared to investigate the enhancement in the heat transfer and the increase in the average Nusselt number. The point of view of each model essentially differs in terms of whether it takes various parameters such as temperature effects, Brownian motion of the nanoparticles, the radii of aggregated particles, and the volume-fraction of nanoparticles into account or not. The governing stream-vorticity equations are solved using a second order central finite difference scheme, coupled to the conservation of mass and energy. The main sensitive parameters of interest to investigate the viscosity models are chosen as volume fraction of nanoparticles, and Richardson number. The performance study of the viscosity models and the interpretation of the corresponding results of velocity components are done in a different range of volume fraction of the nanoparticles and Richardson number.

Key words: *nanofluid, viscosity model, mixed convection, lid-driven cavity, heat transfer enhancement*

Introduction

Nanofluids are liquid-solid mixtures which consist of metal or metal oxide nanometric size particles and base liquid. A major limitation of traditional fluids such as, water, mineral oil, and ethylene glycol is the inherently low thermal conductivity. Suspending different type of small solid particles is an innovative way to enhancement the heat transfer. For instance just 0.3% volume fraction of copper nanoparticles cause to an increment up to 40% in the thermal conductivity of ethylene glycol [1]. The nanofluid can be used to various engineering applications such as nuclear system cooling, solar water heating, and improving diesel generator efficiency. These applications and solution methods have been studied extensively in recent years by Putra *et al.* [2], Daungthongsuk and Wongwises [3], Trisaksri and Wongwises [4], Kakac and Pramuanjaroenkij [5], and Saidur *et al.* [6]. Since the flow and thermal behavior of nanofluids are very sophisticated, various theoretical and experimental models have been developed to estimate the thermo-physical properties of nanofluids. These models are based on the temperature, Brownian motion, particle size and shape, and volume fraction of nanoparticles as well as the in-

* Corresponding author; e-mail: a.ghafouri@urmia.ac.ir

teraction between the pure fluid and nanoparticles, which reviewed extensively in the literature [7, 8].

Several studies of convective heat transfer in lid-driven cavities filled with nanofluids have been reported in recent years [9-13]. What is prevalent in most of the mentioned numerical study on nanofluid combined convection problems is the use of the Brinkman [14] viscosity model for a nanofluid and hence, predict enhancement of heat transfer because of the presence of nanoparticles. However, there emerge some incompatible findings concerning the convection characteristics of nanofluids. Nguyen *et al.* [15] in their experimental work, investigated the influence of both the temperature and particle size on the dynamic viscosity of two particular water-based nanofluids, water-Al_2O_3 and water-CuO mixtures. They observed that for water-alumina nanofluid, particle size effects are more important for high particle concentrations. Similar behaviors were found for a CuO-water mixture regarding the effects due to temperature and particle concentration. Finally, they found that Brinkman [14] formula and several other originating from the linear fluid theory seem not to be applicable for nanofluids tested, especially for intermediate to high particle volume fractions. Very recently, Nasrin *et al*. [16] numerically studied steady combined convection in double-lid driven and triangular wavy cavity filled with CuO nanoparticle using two different nanofluid viscosity models, namely the Brinkman [14] model and the Pak and Cho [17] correlation. They found that the increase in the average Nusselt number using the Pak and Cho [17] correlation is higher than the Brinkman [14] model due to the variation of nanoparticles volume fraction. In addition, Sheikhzadeh *et al.* [18] focused on the study of mixed convection heat transfer characteristics in a lid-driven enclosure filled with Al_2O_3-water nanofluid using Nguyen *et al*. [15] correlation and Brinkman [14] viscosity model. Its results show that using various models with corresponding variables have different values of average Nusselt number for a constant solid volume fraction. Also at forced convection, the difference between the two models is higher than that natural convection. It was recognized that at low Richardson numbers, average Nusselt number was more sensitive to the viscosity and the thermal conductivity models. Chamkha and Abu-Nada [19] conducted a study of steady laminar mixed convection flow in single and double lid square cavities filled with water-alumina nanofluid using two viscosity models. They demonstrated with numerical approach that a different trend (reduction of heat transfer) can be predicted using the Pak and Cho [17] correlation which is based on experimental data in comparison with the Brinkman [14] model.

The main motivation of this investigation is to compare numerically the assessor models of CuO-water nanofluid viscosity and their effects on natural, mixed, and forced convections heat transfer. In order to obtain this aim, the eight different theoretical and experimental viscosity models with respective aspects are used to study effects of various parameters such as temperature, Brownian motion, particle size, *etc*., on streamlines, isotherms, velocity, and Nusselt number. The reduction/enhancement in heat transfer rate will be evaluated under wide range of Richardson number and volume fraction of the solid nanoparticles (φ).

Problem description

Figure 1 shows a schematic view of the 2-D lid-driven cavity filled with water based nanofluid containing CuO nanoparticles. The top wall is moving right while the bottom wall is moving leftwards with a uniform velocity, U_m. The height and the width of cavity are given by L. Also the left boundary is heated and maintained at a constant temperature, T_H, higher than the right cold boundary temperature, T_C, whereas top and bottom horizontal boundaries are thermally insulated. The nanofluid in the enclosure is Newtonian, incompressible, and laminar. The thermo-physical properties of the pure water and nanoparticle at temperature of 25 °C are as-

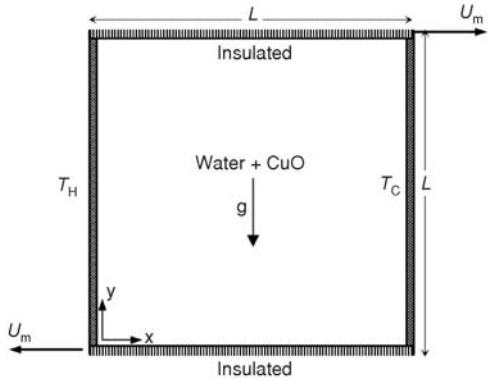

Figure 1. Schematic view of the cavity

Table 1. Thermo-physical properties of fluid and nanoparticles [20]

Physical properties	Fluid phase (water)	CuO
C_p [Jkg^{-1}K^{-1}]	4179	540
ρ [kgm^{-3}]	997.1	6320
k [Wm^{-1}K^{-1}]	0.613	76.5
$\beta \cdot 10^{-5}$ [1/K]	21	1.8

sumed to be constant and are given in tab. 1 [20], whereas the density variation in the buoyancy force term is handled by the Boussinesq approximation.

Mathematical formulation

The non-dimensional governing equations in the Cartesian co-ordinate system for the stream function, Ψ, vorticity function, Ω, and thermal transport, respectively, can be written:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega \tag{1}$$

$$U\frac{\partial \Omega}{\partial X} + V\frac{\partial \Omega}{\partial Y} = \frac{\dfrac{\mu_{nf}}{\mu_f}}{(1-\varphi)+\varphi\dfrac{\rho_s}{\rho_f}}\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \Omega}{\partial X^2}+\frac{\partial^2 \Omega}{\partial Y^2}\right)+\left[(1-\varphi)+\varphi\frac{\beta_s}{\beta_f}\right]\frac{\mathrm{Ra}}{\mathrm{Pr}\,\mathrm{Re}^2}\frac{\partial^2 \theta}{\partial Y^2} \tag{2}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\dfrac{k_{nf}}{k_f}}{(1-\varphi)+\varphi\dfrac{(\rho c_p)_s}{(\rho c_p)_f}}\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{3}$$

The additional dimensionless variables in the previous equations and the effective density, heat capacitance, thermal expansion coefficient, and thermal diffusivity of the nanofluid are defined in our previous work [15]. The effective thermal conductivity of the nanofluid is approximated by the Chon *et al.* [21] model:

$$\frac{k_{nf}}{k_f} = 1 + 64.7\varphi^{0.7640}\left(\frac{d_f}{d_s}\right)^{0.3690}\left(\frac{k_s}{k_f}\right)^{0.7476}\mathrm{Pr}_T^{0.9955}\,\mathrm{Re}_T^{1.2321} \tag{4}$$

where Pr_T and Re_T are defined by:

$$\mathrm{Pr}_T = \frac{\mu_f}{\rho_f \alpha_f}, \quad \text{and} \quad \mathrm{Re}_T = \frac{\rho_f k_B T}{3\pi\mu_f^2 l_f} \tag{5}$$

The symbol k_B is the Boltzmann constant equally to $1.3807\cdot10^{-23}$ J/K, and $l_f = 0.17$ nm is the mean path of fluid particles [21]. This model is superior than earlier thermal conductivity proposed models (*e. g.* Maxwell), because this model considers the effect of nanoparticle size and temperature on nanofluids thermal conductivity with a wide range of temperature between 21 and 70 °C. Accuracy of the Chon *et al*. [21] model was confirmed by the experiments of Angue Minsta *et al.* [22]. This model was found appropriate for studying heat transfer enhancement using CuO-water nanofluids [23].

One of the main goals of this research is to show that diverse predictions can be obtained with using different effective viscosity models of the CuO-water nanofluids. So in this current study eight models are used, which depends on multitude parameters such as antiparticle volume fraction, particle size, temperature, and Brownian motion nanoparticles. These models are presented:

Brinkman [14] proposed that the viscosity of the nanofluid can be approximated as viscosity of a base fluid (μ_f) containing to dilute suspension of fine spherical particles:

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\varphi)^{2.5}} \tag{6}$$

Krieger and Dougherty [24] suggested a power law model for randomly mono-dispersed shape by taking the maximum close packing of $\varphi_m = 0.64$ and for intrinsic viscosity $\eta = 2.5$:

$$\frac{\mu_{eff}}{\mu_f} = \left(1 - \frac{\varphi_s}{\varphi_m}\right)^{-\eta\varphi_m} \tag{7}$$

Considering the effects of the variable packing fractions within the aggregate structure, this equation has been modified by Chen *et al.* [25] and termed modified Krieger and Dougherty [24] equation:

$$\frac{\mu_{eff}}{\mu_f} = \left[1 - \left(\frac{\varphi_s}{\varphi_m}\right)\left(\frac{a_a}{a}\right)^{1.2}\right]^{-\eta\varphi_m} \tag{8}$$

where a_a and a, are the radii of aggregates and primary particles, respectively.

Nielsen [26] derived a generalized equation for the relative viscosity for a concentration of dispersed particles:

$$\frac{\mu_{eff}}{\mu_f} = (1 + 1.5\varphi_s)e^{\frac{\varphi_s}{1-\varphi_m}} \tag{9}$$

where φ_s and φ_m are the volume fraction of particles and the maximum packing fraction, respectively.

Wang *et al.* [27] developed a viscosity model based on the particle volume fraction and shown that this variable is the key factor for improved viscosity:

$$\frac{\mu_{nf}}{\mu_f} = (1 + 7.3\varphi + 123\varphi^2) \tag{10}$$

Azmi *et al.* [28] proposed an empirical correlation for viscosity of nanofluids by cure fitting to 233 experimental data points for Al_2O_3-water nanofluid with particle size of 36 and 47 nm and CuO of 29 nm size:

$$\mu_{nf} = \mu_f\left(1 + \frac{\varphi}{100}\right)^{11.3}\left(1 + \frac{T_{nf}}{70}\right)^{-0.038}\left(1 + \frac{d_s}{170}\right)^{-0.061} \tag{11}$$

Koo and Kleinstreuer [29] established a new theoretical model for nanofluids viscosity prediction consist of a conventional static part as well as a dynamic part which originates from the Brownian motions of nanoparticles.

$$\mu_{eff} = \mu_f(1-\varphi)^{-2.5} + 5\cdot 10^4 \lambda\varphi\rho_f\sqrt{\frac{k_B T}{2\rho_s R_s}}\zeta(T,\varphi) \tag{12}$$

For the water-CuO nanofluid, the two modeling function λ and ζ are experimentally estimated by:

$$\lambda = 0.0137(100\varphi)^{-0.8229} \text{ for } \varphi \le 1\% \text{ and } \lambda = 0.0011(100\varphi)^{-0.7272} \text{ for } \varphi > 1\% \tag{13}$$

$$\zeta(T,\varphi) = (-6.04\varphi + 0.4705)T + (1722.3\varphi - 134.63) \text{ for } 300K \le T \le 325K \tag{14}$$

The correlation for effective dynamic viscosity of CuO-water nanofluid is derived by Haddad *et al*. [23] based on the detailed experimental results reported by Nguyen *et al.* [15]:

$$\mu_{\text{CuO}} = -0.6967 + \frac{15.937}{T} + 1.238\varphi + \frac{1356.14}{T^2} - 0.259\varphi^2 - 30.88\frac{\varphi}{T} - \frac{19652.74}{T^3} + \\ + 0.01593\varphi^3 + 4.38206\frac{\varphi^2}{T} + 147.573\frac{\varphi}{T^2} \tag{15}$$

This viscosity in the previous equation is expressed in centipoise and the temperature in °C. The viscosity of the base fluid (water) considered to vary with temperature:

$$\mu_f = 2.414 \cdot 10^{-5} \cdot 10^{247.8(T-140)} \tag{16}$$

The appropriate dimensionless boundary conditions can be written:

– on the left wall $U = V = \Psi = 0,\ \theta = 1,\ \text{and } \Omega = -\dfrac{\partial^2 \Psi}{\partial X^2}$ (17a)

– on the right wall $U = V = \Psi = 0,\ \theta = 0,\ \text{and } \Omega = -\dfrac{\partial^2 \Psi}{\partial X^2}$ (17b)

– on the top wall $V = \Psi = 0,\ U = 1,\ \dfrac{\partial \theta}{\partial Y} = 0,\ \text{and } \Omega = -\dfrac{\partial^2 \Psi}{\partial Y^2}$ (17c)

– on the bottom wall $U = -1,\ V = \Psi = 0,\ \dfrac{\partial \theta}{\partial Y} = 0,\ \text{and } \Omega = -\dfrac{\partial^2 \Psi}{\partial Y^2}$ (17d)

Numerical method

The set of non-linear coupled governing mass, momentum, and energy equations are developed in terms of stream function-vorticity Ψ-Ω formulation and then solved numerically with the corresponding boundary condition given in eq. (17). The equations are approximated by second-order central difference scheme and successive over relaxation method is used to solve stream function equation. The convergence criterion is defined by the following expression:

$$\varepsilon = \frac{\sum_{j=1}^{j=M}\sum_{i=1}^{i=N}\left|\gamma^{n+1} - \gamma^{n}\right|}{\sum_{j=1}^{j=M}\sum_{i=1}^{i=N}\left|\gamma^{n+1}\right|} \le 10^{-6} \tag{18}$$

where ε represents the tolerance, M and N are the number of grid points in the X- and Y-direction, respectively. The symbol γ denotes any scalar transport quantity namely Ψ, Ω, or θ. An accurate representation of vorticity at the surface is the most critical step in the stream function-vorticity formulation. A second-order accurate formula is used for the vorticity boundary condition. For example, the vorticity at the top wall is expressed:

$$\Omega_{i,M} = \frac{-\Psi_{i,M} + 8\Psi_{i,M-1} - 7\Psi_{i,M-2} - 6\Delta Y}{2(\Delta Y)^2} \tag{19}$$

Similar expression for the lower lid if it is moving (left) and the last term in the numerator vanishes for static lower lid.

The local and average heat transfer rates of the cavity can be presented by means of the local and average Nusselt numbers. The local Nusselt number is calculated along the left heated wall, eq. (20), and the average Nusselt number is determined by integrating the local Nusselt number along the heated wall eq. (21):

$$\mathrm{Nu}(X) = \frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} \frac{\partial \theta}{\partial X}\bigg|_{X=0} \tag{20}$$

$$\mathrm{Nu}_{\mathrm{avg}} = \int_0^1 \mathrm{Nu}(X)\mathrm{d}Y \tag{21}$$

where $(k_{\mathrm{nf}}/k_{\mathrm{f}})$ is calculated using eq. (4). To evaluate eq. (21), one third Simpson's rule of integration is implemented. For convenience, a normalized average Nusselt number is defined as the ratio of Nusselt number at any volume fraction of nanoparticles to that of pure water, that is [30]:

$$\mathrm{Nu}^*_{\mathrm{avg}}(\varphi) = \frac{\mathrm{Nu}_{\mathrm{avg}}(\varphi)}{\mathrm{Nu}_{\mathrm{avg}}(\varphi = 0)} \tag{22}$$

Grid testing and code validation

An extensive mesh testing procedure was conducted to guarantee a grid-independent solution. Six distinct uniform grids, 21 × 21, 41 × 41, 61 × 61, 81 × 81, 101 × 101, and 121 × 121 are employed. The present code was tested for grid independence by calculating the average Nusselt number on the left wall for different Richardson number. As in can be observed from fig. 2, an 81 × 81 uniform grid is sufficiently fine to ensure a grid independent solution. Hence, this grid is used to perform all the subsequent calculations.

The present numerical solution is further validated by comparing the present code results for temperature distribution at Ra = 10^5 and Pr = 0.70 against the experiment of Krane and Jessee [31] and numerical simulation of Khanafer *et al*. [9] and also Oztop and Abu-Nada [32]. It is clear that the present code is in good agreement with other works reported in literature as shown in fig. 3. In addition, the governing equations have been solved for the natural convection flow in an enclosed cavity filled by pure fluid, in order to compare the results with those obtained by Khanafer *et al.* [11], Fusegi *et al*. [33], Markatos and Pericleous [34], de Vahl Davis

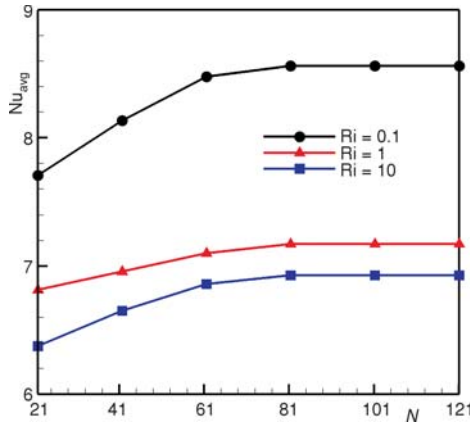

Figure 2. Grid independence study with different Richardson numbers

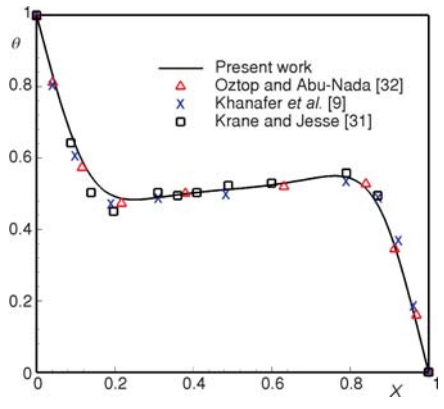

Figure 3. Comparison between present work and other published data for the temperature distribution on the left wall (Ra = 10^5, Pr = 0.7)

[35], and Sheikhzadeh *et al.* [18]. This comparison revealed good agreements between the results, which are shown in tab. 2.

Table 2. Code validation: comparisons of the present results for the average Nusselt number of the hot wall with the results of other investigators

	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$	$Ra = 10^6$
Present work	1.123	2.246	4.521	8.984
Khanafer *et al.* [9]	1.118	2.245	4.522	8.826
Fusegi *et al.* [33]	1.052	2.302	4.646	9.012
Markatos and Pericleous [34]	1.108	2.201	4.430	8.754
De Vahl Davis [35]	1.118	2.243	4.519	8.799
Sheikhzadeh *et al*. [18]	1.120	2.242	4.514	8.790

Results and discussion

A numerical investigation has been performed in this work to study the effect of viscosity models on mixed convection heat transfer in cavity problem utilized CuO-water nanofluid. Calculations were accomplished for Richardson numbers of 0.1, 1, and 10, and nanoparticle volume fraction of 0.01, 0.02, 0.03, 0.04, and 0.05, with fixed Reynolds and Prandtl numbers. Eight different viscosity models were used to approximate nanofluid viscosity that incorporates the effects of nanoparticles concentration, Brownian motion, temperature, nanoparticles size, and radii of aggregated particles. Results are presented by velocity profile, temperature profile, and average and normalized Nusselt numbers.

In order to have a better understanding of the flow behavior within the cavity, the variations of horizontal velocity profiles, vertical velocity profiles, and temperature profiles along the mid-section of the cavity are plotted in figs. 4 and 5 for both pure water and the water-CuO nanofluid ($\varphi = 0.05$) at three values of the Richardson number (Ri = 0.1, 1, 10) using the Brinkman [14] model and the Nguyen *et al.* [15] correlation, respectively.

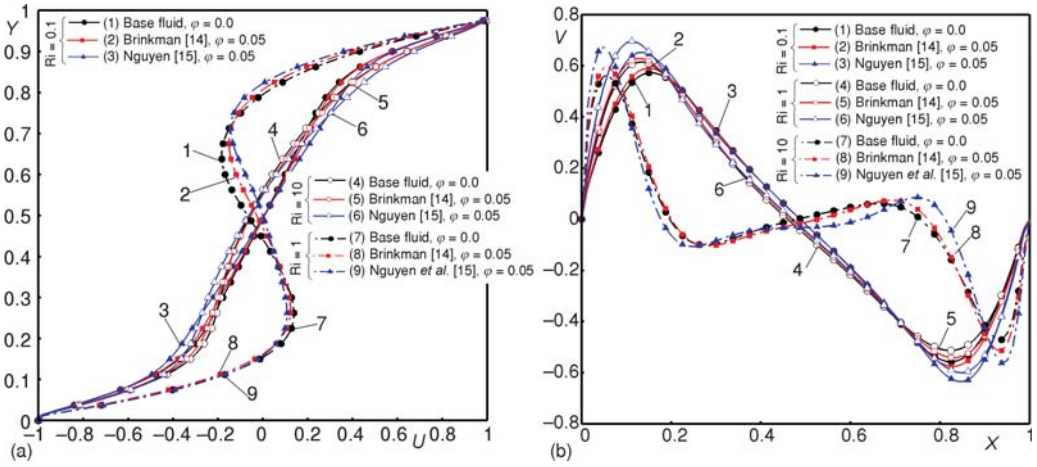

Figure 4. Comparison velocity at the midline section of cavity for base fluid and nanofluid at various Richardson number: (a) U-component, (b) V-component

It is seen from fig. 4(a) that for Ri = 0.1 and 1, the horizontal component of velocity results for the Brinkman [14] model with φ = 5% show negligible variation from those of base fluid whereas the Nguyen *et al.* [15] correlation predicts a considerable effect for which the U velocity increases close to the horizontal moving wall as a consequence of adding 5% nanoparticle volume fractions. Nevertheless, for Ri = 10, the horizontal component of velocity decreases in the bulk of the cavity interior as a result of adding 5% nanoparticle volume fractions and that this decrease is higher for the results obtained using the Nguyen *et al.* [15] correlation than those obtained using the Brinkman [14] model. Figure 4(b) shows that for Ri = 0.1 and 1, higher values of the vertical components of velocity are observed for the nanofluid compared to base fluid. The rising trend of these values is proportional to increasing Richardson number. This increase is higher for the results obtained using the Nguyen *et al.* [15] correlationthan those obtained using the Brinkman [14] model. However, for Ri =10, significant increases are predicted in the vertical component of velocity due to the addition of 5% volume fraction of nanoparticles near the vertical walls of cavity and that these increases are higher using the Nguyen *et al*. [15] correlation than those corresponding to the Brinkman [14] model.

Figure 5 also shows the temperature profile at the mid-section of the cavity for three different values of Richardson number. For Ri = 0.1 and 1, the results obtained using the Brinkman [14] model and the Nguyen *et al*. [15] correlation are essentially different behavior so that the temperature profile increases slightly in Brinkman [14] model while it decreases for Nguyen *et al.* [15] prediction due to the addition of 5% volume fraction of nanoparticles. In addition, at Ri = 10, where the heat transfer is mainly due to natural convection, the Brinkman [14] prediction is associated with a uniform increase of temperature in the middle of the cavity from the right to left wall, while the Nguyen *et al.* [15] prediction shows no uniform rising and falling temperature in the middle of the cavity with greater intensity than Brinkman [14] model.

Figure 6 depicts variation of the average Nusselt number in double-lid driven cavity. Three viscosity models are used for the nanofluid that approximate presence and absence of Brownian motion and temperature effect, Brinkman [14] model, Koo and Kleinstreuer [29], and Nguyen *et al*. [15]. As seen from the figure, average Nusselt number decreases with Richardson number (Ri = 0.1, 1, and 10) and increases with nanoparticle volume fraction (φ = 0, 2%, and 4%). It is indicated that higher average Nusselt number values are observed at forced convection

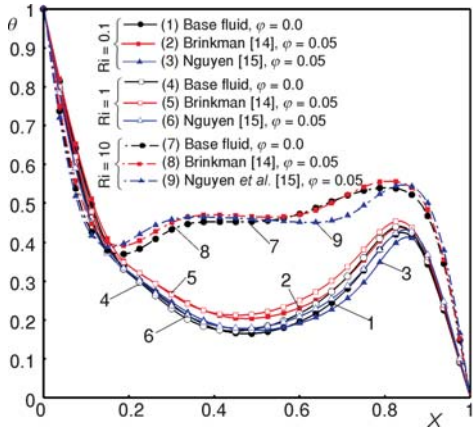

Figure 5. Comparison Temperature at the midline section of cavity for base fluid and nanofluid at various Richardson number

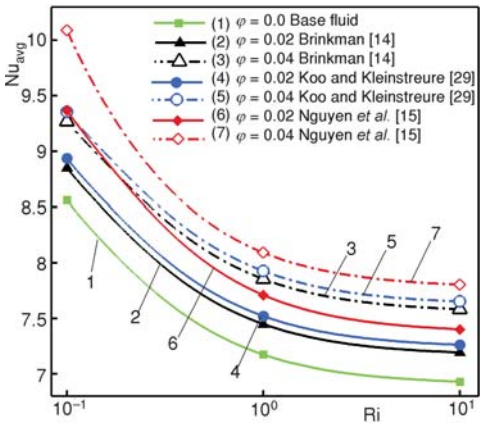

Figure 6. Variation of average Nusselt number with models and nanoparticle volume fractions at different Richardson number

heat transfer with using the Nguyen *et al.* [15] correlation by considering the role of Brownian motion and temperature effect.

One of the main objectives of this paper is evaluation of different viscosity models with corresponding aspects that presented in figs. 7-9. In this section, results of experimental correlations and theoretical models of viscosity concerning temperature, particle size, nanofluid volume fraction, and Brownian effects for CuO-water have been described. As can be seen from fig. 7, for Ri = 10 (natural convection-dominated regime), Chen *et al.* [25], Wang *et al*. [27] and Nielsen [26] viscosity models have lower anticipations about 4%, 3%, and 2%, respectively, than Brinkman [14] model. Meanwhile Koo and Kleinstreuer [29] model and Nguyen *et al*. [15] correlation, indicate that increase in Nu^*_{avg} is more than the Brinkman [14] model about 1.5% and 3%, respectively. The remaining estimates are identical to each other. Figure 8 shows that all assessor models for normalized Nusselt numbers have same pattern at mixed convection heat transfer (Ri = 1), but have difference in amount of evaluation, So that Nguyen *et al*. [15] correlation has highest prediction and lowest estimate of Nu^*_{avg} is related to Chen *et al.* [25] model.

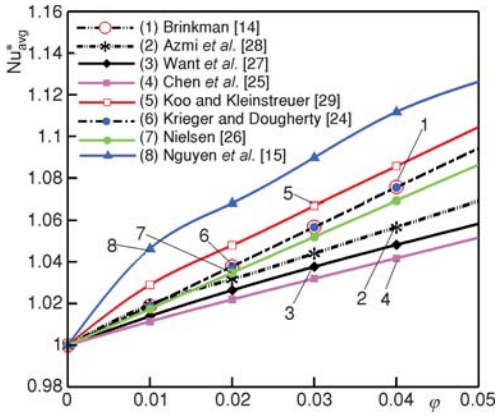

Figure 7. Effect of nanofluid viscosity models on the normalized Nusselt number in natural convection heat transfer at different solid volume fractions

The maximum difference between the estimator models for viscosity of CuO-water nanofluid is observed at forced convection heat transfer that illustrated in fig. 9. As displayed in this figure, it is clearly seen that the values of normalized Nusselt number increase as the value of the φ rises for all assessor models except that for the Nielsen [26] model for which Nu^*_{avg} decreases significantly below that for base fluid reaching minimum at about φ =0.01 and then increases as φ increases further but up to φ = 0.03, they stay below that for base fluid. However,

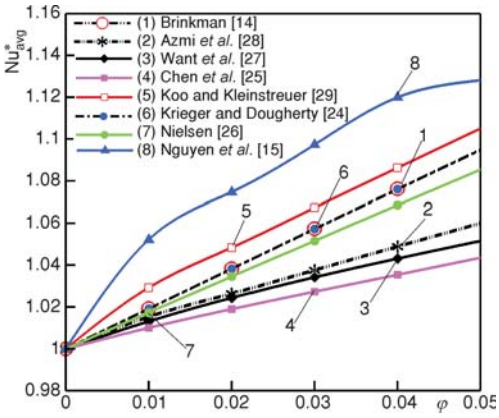

Figure 8. Effect of nanofluid viscosity models on the normalized Nusselt number in mixed convection heat transfer at different solid volume fractions

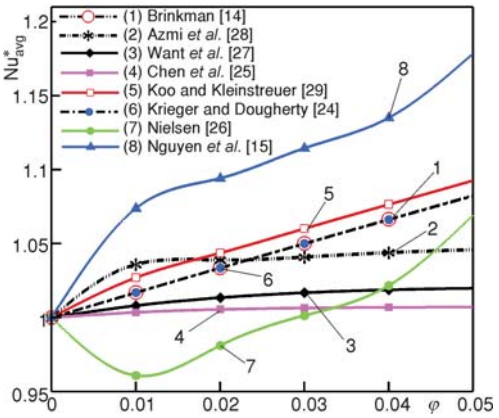

Figure 9. Effect of nanofluid viscosity models on the normalized Nusselt number in forced convection heat transfer at different solid volume fractions

the maximum percent increases in the average Nusselt number at $\varphi = 0.05$ using the Nguyen *et al*. [15] correlation is 18%.

Conclusions

In this study, laminar mixed convection flows of CuO-water nanofluid in double-lid driven cavity was numerically investigated using a second-order accurate finite-difference method. Two horizontal walls of the cavity were kept insulated while the right and left walls were maintained isothermally, but the temperature of the left wall was higher than the right wall. In the double-lid cavity configuration, the top wall moved to the right while the bottom wall moved to the left at the same constant speed. Eight different viscosity models were used to approximate nanofluid viscosity. The study was performed by considering the effects of Brownian motion, temperature, nanoparticles size and radii of aggregated particles and compared to the viscosity models where their effects were neglected. In view of the results, following finding may be summarized.

- Decreasing the value of Richardson number enhances the heat transfer keeping other parameters fixed at different predictor models.
- The presence of nanoparticles causes increase in heat transfer rate absolutely only at natural and mixed convections while at Ri = 0.1 (forced convection dominated regime) the Nielsen [26] model predicts that the average Nusselt number decreases with increasing φ up to 0.03. However, other remained models have same trends with natural and mixed convections.
- It is found that higher heat transfer is formed when Brownian motion and temperature effects are considered. Consequently, the experimental correlation of Nguyen *et al.* [15] and Koo and Kleinstreuer [29] model have significantly highest values of average Nusselt number, respectively.
- Normally, lowest assess is related to Chen *et al.* [25] model. As well as, the Brinkman [14] and the Krieger and Dougherty [24] predictions are identical.
- Based on the present numerical study, the empirical correlation of Nguyen *et al.* [15] is the most accurate predictor model by taking account the effects of nanoparticle diameter and nanofluid temperature for water-CuO nanofluid.

Nomenclature

C_P – specific heat at constant pressure, [Jkg^{-1}K^{-1}]
d – diameter of nanoparticle, [m]
g – gravitational acceleration, [ms^{-2}]
k_B – Boltzmann's constant (=1.38065·10^{-23})
k – thermal conductivity, [Wm^{-1}K^{-1}]
Nu – Nusselt number (= hL/k_f), [–]
Pr – Prandtl number (= ν_f/α_f), [–]
Ra – Rayleigh number (= Gr Pr), [–]
Re – Reynolds number (= U_mL/ν_f), [–]
Ri – Richardson number (= Gr/Re^2), [–]
R_s – radios of nanoparticle, [nm]
T – dimensional temperature, [°C]
U, V – dimensionless velocities, [–]
U_m – lid velocity, [ms^{-1}]
u, v – dimensional x and y components of velocity, [ms^{-1}]
X, Y – dimensionless co-ordinates, [–]

Greek symbols

α – fluid thermal diffusivity, [m^{2}s^{-1}]
β – thermal expansion coefficient, [K^{-1}]
γ – transport quantity
ε – numerical tolerance, [–]
ξ – modeling function, eq. (14)
η – intrinsic viscosity, [–]
θ – dimensionless temperature, [–]
λ – modeling function, eq. (13)
μ – dynamic viscosity, [Nsm^{-2}]
ρ – density, [kgm^{-3}]
φ – volume fraction
Ψ – dimensionless stream function, [–]
Ω – dimensionless vorticity, [–]

Subscripts

avg – average
C – cold
eff – effective
f – fluid
H – hot
m – maximum
nf – nanofluid
s – solid particle

Superscripts

* – normalized

References

[1] Choi, S. U. S., Enhancing Thermal Conductivity of Fluids with Nanoparticles, *ASME Fluids Engineering Division, 231* (1995), Oct., pp. 99-105
[2] Putra, N., *et al*., Natural Convection of Nanofluids, *Heat and Mass Transfer, 39* (2003), 8, pp. 775-784
[3] Daungthongsuk, W., Wongwises, S., A Critical Review of Convective Heat Transfer Nanofluids, *Renewable & Sustainable Energy Reviews, 11* (2007), 5, pp. 797-817
[4] Trisaksri, V., Wongwises, S., Critical Review of Heat Transfer Characteristics of Nanofluids, *Renewable & Sustainable Energy Reviews, 11* (2007), 3, pp. 512-523
[5] Kakac, S., Pramuanjaroenkij, A., Review of Convective Heat Transfer Enhancement with Nanofluids, *International Journal of Heat and Mass Transfer, 52* (2009), 13-14, pp. 3187-3196
[6] Saidur, R., *et al*., A Review on Applications and Challenges of Nanofluids, *Renewable & Sustainable Energy Reviews, 15* (2011), 3, pp. 1646-1668
[7] Mahbubul, I.M., *et al*., Latest Developments on the Viscosity of Nanofluids, *International Journal of Heat and Mass Transfer, 55* (2012), 4, pp. 874-885
[8] Mukesh Kumar, P. C., *et al*., Review on Nanofluid Theoretical Viscosity Models, *International Journal of Engineering Innovation and Research, 1* (2012), 2, pp. 128-134
[9] Khanafer, K., *et al.*, Buoyancy-Driven Heat Transfer Enhancement in a Two-Dimensional Enclosure Utilizing Nanofluids, *International Journal of Heat and Mass Transfer, 46* (2003), 19, pp. 3639-3653
[10] Abu-Nada, E., *et al*., Effect of Nanofluid Variable Properties on Natural Convection in Enclosures, *International Journal of Thermal Science, 49* (2010), 3, pp. 479-491
[11] Talebi, F., *et al*., Numerical Study of Mixed Convection Flows in a Square Lid-Driven Cavity Utilizing Nanofluid, *International Communications in Heat and Mass Transfer, 37* (2010), 1, pp. 79-90
[12] Mahmoodi, M., Numerical Simulation of Free Convection of a Nanofluid in L-Shaped Cavities, *International Journal of Thermal Science, 50* (2011), 9, pp. 1731-1740
[13] Pourmahmoud, N., *et al*., Numerical Study of Mixed Convection Heat Transfer in Lid-Driven Cavity Using Nanofluid; Effect of Type and Model of Nanofluid, *Thermal Science, 19* (2015), 5, pp. 1575-1590
[14] Brinkman, H. C., The Viscosity of Concentrated Suspensions and Solutions, *Journal of Chemical Physics, 20* (1952), 4, pp. 571-581
[15] Nguyen, C. T., *et al.*, Temperature and Particle-Size Dependent Viscosity Data for Water Based Nanofluids-Hysteresis Phenomenon, *International Journal of Heat and Fluid Flow, 28* (2007), 6, pp.1492-1506
[16] Nasrin, R., *et al.*, Combined Convection Flow in Triangular Wavy Chamber Filled with Water-CuO Nanofluid: Effect of Viscosity Models, *International Communications in Heat and Mass Transfer, 39* (2012), 8, pp. 1226-1236
[17] Pak, B. C., Cho, Y., Hydrodynamic and Heat Transfer Study of Dispersed Fluids with Submicron Metallic Oxide Particle, *Experimental Heat Transfer, 11* (1998), 2, pp. 151-170
[18] Sheikhzadeh, G. A., *et al.*, Numerical Study of Mixed Convection Flows in a Lid-Driven Enclosure Filled with Nanofluid Using Variable, *Results in Physics, 2* (2012), 1, pp. 5-13
[19] Chamkha, A. J., Abu-Nada, E., Mixed Convection Flow in Single and Double-Lid Driven Square Cavities Filled with Water-Al_2O_3 Nanofluid: Effect of Viscosity Models, *European Journal of Mechanics – B/Fluids, 36* (2012), Nov.-Dec., pp. 82-96
[20] Abu-Nada, E., Rayleigh-Benard Convection in Nanofluids: Effect of Temperature Dependent Properties, *International Journal of Thermal Science, 50* (2011), 9, pp. 1720-1730
[21] Chon, C. H., *et al.*, Empirical Correlation Finding the Role of Temperature and Particle Size for Nanofluid (Al_2O_3) Thermal Conductivity Enhancement, *Applied Physics Letters, 87* (2005), 15, pp. 153107-1-3
[22] Angue Minsta, H., *et al*., New Temperature and Conductivity Data for Water-Based Nanofluids, *International Journal of Thermal Science, 48* (2008), 2, pp. 363-373

[23] Haddad, Z., *et al.*, Natural Convection in Nanofluids: Are the Thermophoresis and Brownian Motion Effects Significant in Nanofluid Heat Transfer Enhancement?, *International Journal of Thermal Science, 57* (2012), July, pp. 152-162
[24] Krieger, I. M., Dougherty, T. J., A Mechanism for Non-Newtonian Flow in Suspensions of Rigid Spheres, *Transactions of the Society of Rheology, 3* (1959), 1, pp. 137-152
[25] Chen, H., *et al*., Rheological Behavior of Nanofluids, *New Journal of Physics, 9* (2007), 10, p. 25
[26] Nielsen, L. E., Generalized Equation for the Elastic Moduli of Composite Materials, *Journal of Applied Physics, 41* (1970), 11,pp. 4626-4627
[27] Wang, X., *et al*., Thermal Conductivity of Nanoparticle-Fluid Mixture, *Journal of Thermophysics and Heat Transfer, 13* (1999), 4, pp. 474-480
[28] Azmi, W. H., *et al*., Correlations for Thermal Conductivity and Viscosity of Water Based Nanofluids, *Materials Science and Engineering, 36* (2012), 012029
[29] Koo, J., Kleinstreuer, C., Laminar Nanofluid Flow in Micro Heat-Sinks, *International Journal of Heat and Mass Transfer, 48* (2005), 13, pp. 2652-2661
[30] Abu-Nada, E., Effects of Variable Viscosity and Thermal Conductivity of CuO-Water Nanofluid on Heat Transfer Enhancement in Natural Convection: Mathematical Model and Simulation, *Journal of Heat Transfer, 132* (2010), 5, 052401
[31] Krane, R. J., Jessee, J., Some Detailed Field Measurements for a Natural Convection Flow in a Vertical Square Enclosure, *Proceedings*, 1st ASME-JSME Thermal Engineering Joint Conference, Honolulu, Hi., USA, 1983, Vol. 1, pp. 323-329
[32] Oztop, H. F., Abu-Nada, E., Numerical Study of Natural Convection in Partially Heated Rectangular Enclosures Filled with Nanofluids, *International Journal of Heat and Fluid Flow, 29* (2008), 5, pp. 1326-1336
[33] Fusegi, T., *et al.*, A Numerical Study of Three Dimensional Natural Convection in a Differentially Heated Cubical Enclosure, *International Journal of Heat and Mass Transfer, 34* (1991), 6, pp.1543-1557
[34] Markatos, N. C., Pericleous, K. A., Laminar and Turbulent Natural Convection in an Enclosed Cavity, *International Journal of Heat and Mass Transfer, 27* (1984), 5, pp. 772-775
[35] Vahl Davis, G. De., Natural Convection of Air in a Square Cavity, a Benchmark Numerical Solution, *International Journal for Numerical Methods in Fluids, 3* (1983), 3, pp. 249-264

Paper submitted: March 9, 2013
Paper revised: March 20, 2014
Paper accepted: April 14, 2014