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HEAT TRANSFER MODELLING OF TWO-PHASE BUBBLES SWARM CONDENSING IN THREE-PHASE DIRECT-CONTACT CONDENSER

by

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An analytical model for the convective heat transfer coefficient and the two-phase bubble size of a three-phase direct contact heat exchanger was developed. Until the present, there has only been a theoretical model available that deals with a single two-phase bubble and a bubble train condensation in an immiscible liquid. However, to understand the actual heat transfer process within the three-phase direct contact condenser, characteristic models are required. A quasi-steady energy equation in a spherical co-ordinate system with a potential flow assumption and a cell model configuration has been simplified and solved analytically. The convective heat transfer in terms of Nusselt number has been derived, and it was found to be a function to Pecklet number and a system void fraction. In addition, the two-phase bubble size relates to the system void fraction and has been developed by solving a simple energy balance equation and using the derived convective heat transfer coefficient expression. Furthermore, the model correlates well with previous experimental data and theoretical results.

Key words: *direct contact heat transfer, direct contact condenser, cell model, two-phase bubble swarm, heat transfer coefficient, two-phase bubbles size*

Introduction

Direct contact heat exchange from drop evaporation and bubble condensation in immiscible liquids in a stagnant or a flowing system is an efficient thermal exchange method. Generally, the direct contact condensation of the vapour bubbles in immiscible liquids could be a more convincing process than evaporation of drops because the heat transfer resistance, which initiates during the condensation of two-phase bubbles, is less than the build-up during the evaporation of drops. Normally, the condensate formed as a result of the condensation is swept and accumulated at the rear of the two-phase bubble similar to the case of evaporation drops where the vapour produced is concentrated in the upper part of a drop due to the effect of gravity. There are a large number of experimental and theoretical investigations in the literature related to the direct contact condensation of a two-phase bubble in an immiscible liquid. Most of the literature has concentrated on the dynamics and heat transfer of a single two-phase bubble condensed in another liquid. Sideman and Hirsch [1] studied experimentally

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the condensation of a single isopentane bubble in stagnant water column. The instantaneous two-phase bubble condensation, the volumetric heat transfer coefficient and the two-phase bubble rising velocity were measured. Isenberg and Sideman [2] and Moalem *et al.* [3] experimentally and theoretically investigated the heat transfer of a single two-phase bubble condensation rising freely in water and aqueous-glycerol through a stagnant column. Their model was based on the potential flow assumption, and the two-phase bubble was represented as a solid sphere. Similarly, Moalem and Sideman [4] researched theoretically the effect of the rising velocity on the heat transfer of a single two-phase bubble condensation in an immiscible liquid using a potential flow model. Concerning both hydrodynamics and heat transfer, Higeta *et al.* [5] experimentally measured the drag coefficient, half opening angle, and volumetric heat transfer coefficient of a single n-pentane bubble condensed in glycerol and a single water vapour bubble condensed in silicon oil systems. Theoretically, Jacobs and Major [6] have a numerically studied the effect of non-condensable gas distribution on the rate of a single bubble's condensation in a three-phase direct contact heat transfer system. Also, Wanchoo [7] developed expressions for the convective heat transfer coefficient and the collapsing rate of a large two-phase spherical bubble in immiscible liquid. Furthermore, Lerner *et al.* [8] carried out numerical modelling and experimental observations of a single R113 bubbles condensation in a stagnant water column. Their modelling was based on acceleration – deceleration model. Continuing, Lerner and Letan [9] observed experimentally the dynamics of condensation of a single bubble in an immiscible liquid at an intermediate frequency of bubble injection. They noted that a single bubble model could be used to predict the condensation rate history within an intermediate injection frequency. Wanchoo *et al.* [10] measured the drag coefficient and the velocity of rise of a single vapour bubbles collapsing in an immiscible liquid media. Six pairs of the dispersed-continuous systems were tested with a range of Reynolds number from 0.003 to 3000. Kalman and Mori [11] carried out an experimental investigation of the dynamics and heat transfer of a single vapour bubble condensing in both an immiscible and a miscible liquid. They obtained two empirical correlations for drag coefficient based on the Reynolds number and one correlation for instantaneous heat transfer coefficient. Kalman [12, 13] studied both experimentally and theoretically the condensation of a single two-phase bubble in an immiscible liquid using an acceleration-deceleration model as well as the condensation of R113 and hexane bubble trains in a stagnant water column using an enveloped model.
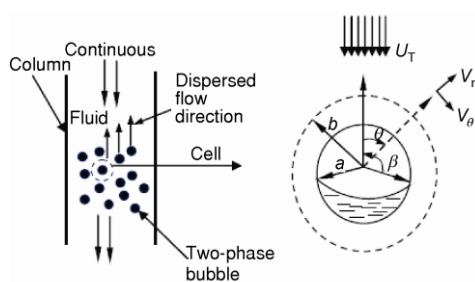

Figure 1. Schematic representation of the cell model

In the present paper, an analytical model for the heat transfer of the direct contact condenser was developed. The impact of the system void fraction has been included and its effect on the heat transfer coefficient and the two-phase bubble size has been tested.

Modelling

Assume that, the two-phase bubbles moved in a potential flow field with a constant reference two-phase bubbles' radius, which means the effect of decreasing a two-phase bubble's size due to condensation is neglected in comparison with its velocity. Using a cell model as shown in fig. 1, the potential velocity of the two-phase bubble in the swarm was given by Milne-Thomson [14] and used by Cai and Wallis [15]:

$$\phi = \frac{\cos\theta}{b^3 - a^3}\left[(a^3 U - b^3 v)r + \frac{a^3 b^3}{2r^2}(U - v)\right] \tag{1}$$

where U and v represent velocities of outer and inner cell boundaries, respectively.

For $v = 0$, eq. (1) reduces to the expression given by [16] and used by [17]:

$$\phi = \frac{U}{b^3 - a^3}\left(a^3 r + \frac{a^3 b^3}{2r^2}\right)\cos\theta \tag{2}$$

and

$$\alpha = \left(\frac{a}{b}\right)^3 \tag{3}$$

Equation (2) becomes:

$$\phi = \frac{U}{1-\alpha}\left[\alpha r + \frac{1}{2}\left(\frac{a^3}{r^2}\right)\right]\cos\theta \tag{4}$$

The velocity components can be found using eq. (4):

$$V_r = -\frac{\partial\phi}{\partial r} = \frac{U}{1-\alpha}\left[\alpha - \left(\frac{a}{r}\right)^3\right]\cos\theta \tag{5}$$

$$V_\theta = -\frac{\partial\phi}{r\partial\theta} = \frac{U}{1-\alpha}\left[\alpha + \frac{1}{2}\left(\frac{a}{r}\right)^3\right]\sin\theta \tag{6}$$

where V_r and V_θ are the radial and tangential velocity components.

Equations (5) and (6) differ from those equations used by [18, 19] when they analyzed the problem of gas bubble swarm.

The energy equation for steady-state heat transfer in a spherical co-ordinate with axial symmetric can be written:

$$V_r\frac{\partial T}{\partial r} + \frac{V_\theta}{r}\frac{\partial T}{\partial\theta} = \varepsilon\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)\right] \tag{7}$$

where T is the temperature, and ε – the thermal diffusivity of continuous phase.

Neglecting the conduction in   direction with respect to the radial conduction, then expanding the reminder of the right hand side of eq. (7) in addition to assuming a thin boundary layer around the two-phase bubble, and the radial velocity vector becomes zero at a point on the references two-phase surface. Hence, $V_r = 0$, and eq. (7) converts to:

$$\frac{V_\theta}{a}\frac{\partial T}{\partial\theta} = \varepsilon\frac{\partial^2 T}{\partial r^2} \tag{8}$$

Utilizing the binomial theorem and using only first two terms, together with the previous thin boundary layer assumption, $(r - a)/a = y/a \ll 1$, eq. (6) becomes:

$$V_\theta = f(\alpha)U\sin\theta \tag{9}$$

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where

$$f(\alpha) = \frac{\alpha + \frac{1}{2}}{1 - \alpha} \tag{10}$$

Substituting eq. (9) into eq. (8) yields:

$$\sin \theta \, \frac{\partial T}{\partial \theta} = M \, \frac{\partial^2 T}{\partial y^2} \tag{11}$$

where

$$M = \frac{a\varepsilon}{f(\alpha)U} \tag{12}$$

The independent variables in eq. (11) can be converted to other independent variables:

$$Y = y \sin^2 \theta \tag{13}$$

$$X = \int_0^\theta \sin^3 \theta \mathrm{d}\theta = \frac{1}{3} \cos^3 \theta - \cos \theta + \frac{2}{3} \tag{14}$$

Accordingly, eq. (11) changes to:

$$\frac{\partial T}{\partial X} = M \, \frac{\partial^2 T}{\partial Y^2} \tag{15}$$

Equation (15) is a heat equation, which is a parabolic partial differential equation. The new boundary conditions are:

$$T = T_a, \qquad Y = 0, \qquad X \geq 0 \tag{16}$$

$$T = 0, \qquad Y = \infty, \qquad X \geq 0 \tag{17}$$

The solution of eq. (15), [20, 21], is:

$$T = T_a \mathrm{erfc} \left[\frac{Y}{2(MX)^{0.5}} \right] \tag{18}$$

Using eq. (13) and eq. (14) with eq. (18) results in:

$$T = T_a \, \mathrm{erfc} \left[\frac{y \sin^2 \theta}{2M^{0.5} \left(\frac{1}{3} \cos^3 \theta - \cos \theta + \frac{2}{3} \right)^{0.5}} \right] \tag{19}$$

The local heat flux can then be given:

$$q_\theta = -k_\mathrm{c} \left(\frac{\partial T}{\partial y} \right)_{y=0} = k_\mathrm{c} T_a \, \frac{\sin^2 \theta}{(\pi M X)^{0.5}} \tag{20}$$

According to Higeta, *et al.* [22], the average heat transfer can be found using the expression:

$$\bar{q} = \frac{1}{4\pi a^2} \int_0^\beta q_\theta (2\pi a^2) \sin\theta \, d\theta \tag{21}$$

Using eq. (20) for local heat flux, eq. (21) becomes:

$$\bar{q} = \frac{k_c T_a}{2(\pi M)^{0.5}} \int_0^\beta \frac{\sin^3\theta}{\sqrt{X}} \, d\theta \tag{22}$$

Differentiation eq. (14) were regards to the X value results in:

$$dX = (-\cos^2\theta \sin\theta + \sin\theta) d\theta = \sin^3\theta \, d\theta \tag{23}$$

Hence:

$$d\theta = \frac{dX}{\sin^3\theta} \tag{24}$$

The boundary condition can be written:

$$\theta = 0, \qquad X = 0 \tag{25}$$

$$\theta = \beta, \qquad X = \frac{1}{3}\cos^3\beta - \cos\beta + \frac{2}{3} \tag{26}$$

Equation (22) becomes:

$$\bar{q} = \frac{k_c T_a}{2\sqrt{\pi M}} \int_0^{\frac{1}{3}\cos^3(\beta) - \cos(\beta) + \frac{2}{3}} \frac{dX}{\sqrt{X}} \tag{27}$$

Completing the integration, eq. (27) converts to:

$$\bar{q} = \frac{k_c T_a}{\sqrt{\pi M}} \left[\frac{1}{3}\cos^3\beta - \cos\beta + \frac{2}{3} \right]^{0.5} \tag{28}$$

The average heat transfer coefficient can be calculated:

$$h = \frac{\bar{q}}{T_a} \tag{29}$$

Substituting eq. (28) into eq. (29), the convective heat transfer coefficient can be obtained:

$$h = \frac{k_c}{\sqrt{\pi}} \left[\frac{f(\alpha)U}{a\varepsilon} \right]^{0.5} [f(\alpha)f(\beta)]^{0.5} \tag{30}$$

where

$$f(\beta) = \left[\frac{1}{3}\cos^3(\beta) - \cos(\beta) + \frac{2}{3} \right] \tag{31}$$

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The Nusselt number is then:

$$\mathrm{Nu} = \left(\frac{2}{3}\right)^{0.5} f(\alpha)^{0.5} \left[\frac{\cos^3(\beta) - 3\cos(\beta) + 2}{\pi}\right]^{0.5} \mathrm{Pe}^{0.5} \tag{32}$$

The geometrical term in eq. (32) is relate with the condensation ratio *x*. It was given as [23]:

$$3\cos(\beta) - \cos^3(\beta) + 2 = \frac{4(1-x)}{1 + x(m-1)} \tag{33}$$

Equation (32) now converts to:

$$\mathrm{Nu} = \left(\frac{2}{3\pi}\right)^{0.5} f(\alpha)^{0.5} \left[\frac{4(1-x)}{1 + x(m-1)}\right]^{0.5} \mathrm{Pe}^{0.5} \tag{34}$$

The flowing velocity factor was used by which the solution based on the potential flow assumption was converted to the actual or viscous solution [2-4]:

$$k_v = 0.25\,\mathrm{Pr}^{-\frac{1}{3}} \tag{35}$$

For pure potential flow:

$$k_v = 1 \tag{36}$$

Equation (32), therefore, becomes:

$$\mathrm{Nu} = \left(\frac{2}{3}\right)^{0.5} [k_v f(\alpha)]^{0.5} \left[\frac{\cos^3(\beta) - 3\cos(\beta) + 2}{\pi}\right]^{0.5} \mathrm{Pe}^{0.5} \tag{37}$$

The energy equation of the spherical two-phase bubble condensation in an immiscible liquid, can be written similar to that given by Wanchoo [24]:

$$\frac{da}{at} = \frac{(\rho_{dl} - \rho_{dv})\Delta T}{h_{fg}\rho_{dl}\rho_{dv}}h \tag{38}$$

where *h* represents the heat transfer coefficient found already by eq. (30). The velocity that appears in eq. (30) is a relative velocity. It was given in [25]:

$$U_r = \frac{U_s}{2}\frac{(1-\alpha)^3}{\alpha + 0.5} \tag{39}$$

Substituting eq. (39) into eq. (30), results in:

$$h = \frac{k_c}{\sqrt{2\pi}}\left[\frac{k_v U_s f(\beta)}{a\varepsilon}\right]^{0.5}(1-\alpha) \tag{40}$$

Substituting eq. (40) of the heat transfer coefficient into eq. (38), assuming $\Delta\rho/\rho_{dl} \approx 1$, and completing the integration using the initial condition:

$$a = a_o, \qquad t = 0 \tag{41}$$

eq. (38) then converts to:

$$a = \left\{ a_o^{3/2} - \left\{ \left(\frac{3}{2\sqrt{2\pi}} \right) \left(\frac{\Delta T \, k_c}{h_{fg} \rho_{\mathrm{dv}}} \right) [f(\beta) k_v]^{0.5} \left(\frac{U_s}{\varepsilon} \right)^{0.5} (1-\alpha) t \right\} \right\}^{2/3}$$

(42)

In dimensionless form, eq. (42) becomes:

$$\frac{a}{a_o} = \left\{ 1 - \left[\left(\frac{3}{4} \right) \left(\frac{k_v}{\pi} \right)^{0.5} f(\beta)^{0.5} \, \mathrm{Ja} \, \mathrm{Pe}^{0.5} \, (1-\alpha)\tau \right] \right\}^{2/3}$$

(43)

where

$$\tau = \frac{\varepsilon t}{a_o^2}$$

(44)

Isenberg and Sideman [2] demonstrated a modified time constant:

$$\hat{\tau} = \mathrm{Ja} \, \mathrm{Pe}^{0.5} \, \tau$$

(45)

Therefore, eq. (43) can be written in another form:

$$\frac{a}{a_o} = \left\{ 1 - \left[\left(\frac{3}{4} \right) \left(\frac{k_v}{\pi} \right)^{0.5} f(\beta)^{0.5} \, (1-\alpha)\hat{\tau} \right] \right\}^{2/3}$$

(46)

For a single two-phase bubble $\alpha \to 0$, and for $\beta \to \pi$ (maximum heat exchange), eq. (46) reduces to:

$$\frac{a}{a_o} = \left\{ 1 - \left[0.866 \left(\frac{k_v}{\pi} \right)^{0.5} \hat{\tau} \right] \right\}^{2/3}$$

(47)

Results and discussions

To verify our analytical model, comparisons with available theoretical model and experimental data were made. Figures 2 and 3 show a comparison between the present model results for the convective heat transfer coefficient in terms of Nusselt number and the experimental results of [22] for two different fluids pairs, n-pentane-glycerol and methanol-silicone. It is clear, that Nusselt number increases with increasing Pecklet number, and the agreement between the two results has shown at a high Pecklet number for both systems. It is, probably, as a result of a potential flow assumption implemented in the present model which, of course, is more valid when bubbles move at a high velocity. However, the n-pentane-glycerol system recorded the better agreement. That could be due to the assumption of the same half opening angle or the same progress of condensation for both systems in the present model, while practically it is different, depending on the physical and thermodynamic properties of the contacting fluids. Figure 3 illustrates a comparison of the present model Nusselt number as a function of the Pecklet number with experimental data given by [26] for n-pentane-water system. An asymptotic agreement has been achieved for all ranges of Pecklet number, which indicates that the present analytical model is more suitable to the case of a low viscosity contacting fluids than high ones (as seen in figs. 2, 3, and 4). Further validation of the present model when ($\alpha = 0$), by comparisons with the available theoretical expressions is shown in fig. 5. Our model has a satisfactory agreement with a solid sphere correlation given by [30] and with [32] for a solid sphere

rear correlation. After model validation, the effect of the void fraction on the convective heat transfer coefficient is demonstrated in fig. 6. An increase of the Nusselt number with an increase of the void fraction is shown clearly in this figure. This can be reasonably justified by a simple argument. The higher the void fraction, the higher the vapour volume in the column. Subsequently higher heat convection through the vapour/liquid interface of higher area.
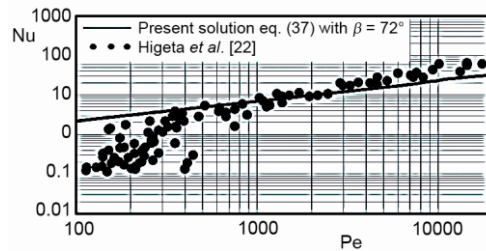

Figure 2. Comparison between present model results and experimental results of [22] for n-pentane-glycerol system
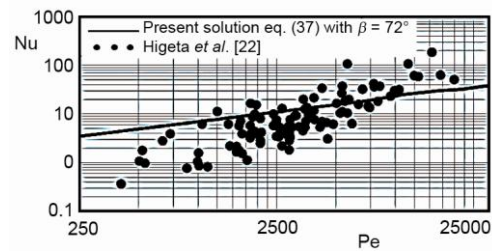

Figure 3. Comparison between present model results and experimental results of [22] for methanol-silicone oil system
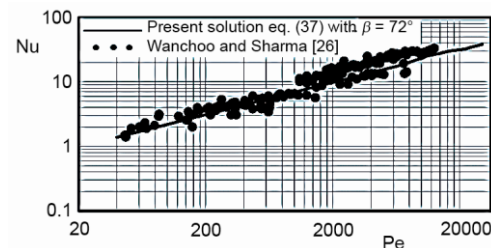

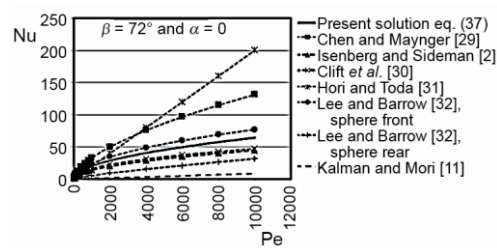**Figure 4. Comparison between the present model results and the experimental results of [26] for pentane-water system**

Figure 5. Comparison between present model for $\beta = 72°$, $\alpha = 0$ and different theories

The two-phase bubble size has been modelled analytically according to the heat transfer coefficient expression given by eq. (42). Although a number of theoretical models have been developed to predict the two-phase bubble size (radius), no general formula can be used successfully for different cases. This is because of a complex heat transfer process associated with the two-phase bubble condensation and a large number of the effective parameters in such a process [11]. However, these expressions have a general form:

$$\frac{a}{a_o} = (1 - c \, \text{Ja} \, \text{Pe}^{0.5} \, \tau)^{\gamma} \tag{48}$$

where c and γ are constants. Most of the investigators [6, 7, 27, 28] have mentioned that $\gamma = 2/3$. On the other hand, different investigators have found different values for the constant c. Accordingly, eq. (43) was developed, is familiar with the general form of eq. (48), and is identical with [6] relations at specific conditions.

Figure 7 shows the comparison of the present analytical model for the two-phase bubble area and the experimental data of [1] for two different initial isopentane bubble radii of 5.5 mm and 3.38 mm. A good agreement has been obtained for two cases during the condensation period of the two-phase bubble. Figure 8 represents the comparison of the time-dependent,

dimensionless two-phase bubble radius with experimental data [1] and with theoretical model results of [24] when a single isopentane bubble with diameter 5.5 mm condenses in a stagnant water column. It can be seen from fig. 8 that the present model agrees well with the numerical model [24] and the experimental data [1], whereas it is divergent from the numerical results [24], which used a Stokes approximation. This could be because the condensation of small bubbles occurs quickly. Therefore, the quasi-steady assumption might fail to predict the bubble size history. In addition the present model is built on the assumption of a potential flow configuration around the condensing bubbles, while a creeping flow is implemented in another [24] and it is more likely to be correct for smaller bubbles which have low Reynolds number.
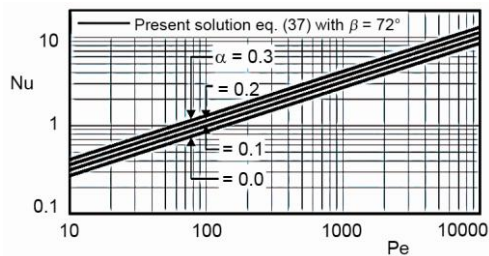

Figure 6. The variation of Nusselt number with Pecklet number for a different void fraction values
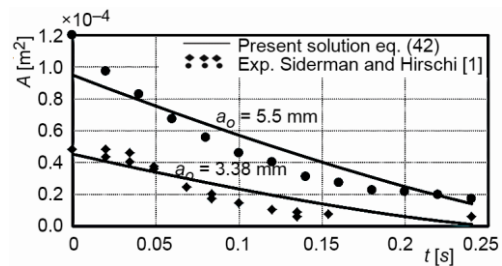

Figure 7. The variation of the two-phase bubble volume with the time for isopentane-water system

A nearly identical agreement was obtained between the present model results and the numerical results of [6], while a broad divergence from that of [27] can be seen clearly in fig. 9. The later model [27] was attempted with restricting assumptions, which might have caused this large divergence with other results.
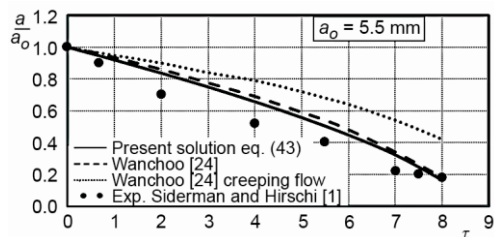

Figure 8. The variation of dimensionless two-phase bubble radius with the time
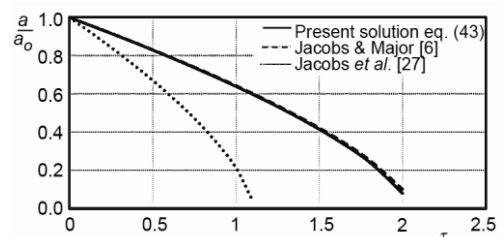
Figure 9. The variation of dimensionless two-phase bubble radius with the time

Conclusions

An analytical model has been developed for the heat transfer of a two-phase bubbles swarm condensing in an immiscible liquid. According to the results, the following conclusions can be made:

- The convective heat transfer coefficient in terms of Nusselt number increases with an increase of system void fraction.
- The half opening angle values have a vital effect on the Nusselt number values.

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- The assumption that the front two-phase bubble heat exchange dominates seems to be suitable for predicting the Nusselt number, which completely agrees with previous observation of Higeta *et al.* [5, 22].
- The constant two-phase bubble rising velocity during the condensation process gives an acceptable prediction of two-phase bubble size.
- The two-phase bubbles size increases with increasing of the system void fraction.
- Generally, good agreements have been obtained between present model results and others available from theoretical results and experimental data.

Nomenclature

A – bubble area, $[m^2]$
a_o – initial bubble radius, $[m]$
b – cell radius, $[m]$
$f(\alpha)$ – function
$f(\beta)$ – function
h – heat transfer coefficient, $[kJm^{-2}s^{-1}K^{-1}]$
h_{fg} – latent heat of condensation, $[kJkg^{-1}]$
Ja – Jacobs number, $[-]$
k_c – thermal conductivity of continuous phase, $[Wm^{-1}K^{-1}]$
k_v – variable (eq. 35)
M – constant (eq. 12)
M – ratio of the continuous phase to a disperse phase density
Nu – Nusselt number, $[-]$
Pe – Pecklet number, $[-]$
Pr – Prantdl number, $[-]$
$\bar{q}$ – average heat transfer flux, $[kJm^{-2}]$
q – local heat transfer flux, $[kJm^{-2}]$
r – radial co-ordinate, $[m]$
T – temperature, $[°C]$
ΔT – temperature differences, $[°C]$

t – time, $[s]$
T_a – initial temperature, $[°C]$
U_r – relative velocity, $[ms^{-1}]$
U_s – terminal velocity of a single bubble, $[ms^{-1}]$
v – inner boundary velocity of a cell, $[ms^{-1}]$
V_r – radial velocity component, $[ms^{-1}]$
V – tangential velocity component, $[ms^{-1}]$
X – independed variable
x – condensation ratio
Y – independent variable, $[mm]$
y – boundary layer thickness, $[mm]$

Greek symbols

α – hold up ratio
β – half opening angle, $[°]$
ε – thermal diffusivity, $[m^2s^{-1}]$
θ – angular angle, $[°]$
ρ_{dl} – dispersed phase liquid density, $[kgm^{-3}]$
ρ_{dv} – dispersed phase vapour density, $[kgm^{-3}]$
τ – dimensionless time constant
$\hat{\tau}$ – modified dimensionless time constant

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