MODELING OF THERMOELECTRIC MODULE OPERATION IN INHOMOGENEOUS TRANSIENT TEMPERATURE FIELD USING FINITE ELEMENT METHOD

by

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This paper is the result of research and operation modeling of the new systems for cooling of cutting tools based on thermoelectric module. A copper inlay with thermoelectric module on the back side was added to a standard turning tool for metal processing. For modeling and simulating the operation of thermoelectric module, finite element method was used as a method for successful solving the problems of inhomogeneous transient temperature field on the cutting tip of lathe knives. Developed mathematical model is implemented in the software package PAK-T through which numerical results are obtained. Experimental research was done in different conditions of thermoelectric module operation. Cooling of the hot module side was done by a heat exchanger based on fluid using automatic temperature regulator. After the calculation is done, numerical results are in good agreement with experimental. It can be concluded that developed mathematical model can be used successfully for modeling of cooling of cutting tools.

Key words: *TE module, FEM, heat transfer, cutting tools*

Introduction

While metal processing by cutting in the cutting zone, there is a heat generating which negatively affects durability of cutting tools. Cutting tools is usually cooled by standard cooling and lubrication means (CLM). However, in cases when high quality of a processed surface is needed, processing is done without the use of CLM.

TE modules are used for device cooling in various fields of science, technics and everyday life. Due to that, TE modules are used for cooling of cutting tools when the use of standard means for cooling and lubrication is not possible. TE module is consisted of thermo elements joined in one whole and from the outer side covered with ceramic tiles. During TE module work, the cold side is considered to be the one facing the object the heat is drown from, whereas the other module side is called the hot side. In practice, thermo-physical characteristics of semiconductor material that TE module elements are made of are not known. Characteristics given by module manufacturer in a diagram form are used as initial

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parameters for simulation of TE module operation. Quantity of heat which TE module draws from a cooling object is the linear function of temperature difference on the hot and cold module side.

Cutting tool used for the specific processing case is a turning tool attached to a tool holder. There is a temperature field which appears in the turning tool during metal processing and whose calculation is described in the papers [1-3]. In the case of processing by cutting tool described in the paper, mathematical models given in [4,5] are used. Cutting tip is exposed to the heat sources which are generated in the contact with workpiece and chips. Heat transfer with the environment is done by natural convection and conduction through the tool holder.

Boundary conditions given within heat fluxes on the surfaces between the contact of tool, workpiece and chips are used for heat source modeling. Calculation of the coefficients used for boundary conditions with given convection is presented in [5, 6]. This turning tool model is suitable because it observes the tool separately from chips and workpiece, which in the specific case gives good results with significant simplifying.

Developed cooling methodology of the cutting tool by TE module is successfully implemented in the program PAK-T [7] based on the finite element method. Initial and boundary conditions used for modeling correspond to the conditions given in [8-11] except that TE module operation is introduced in this problem representing an additional boundary condition. Experimental verification of mathematical model is done for a number of processing regimes in seven points on the length of the turning tool. At the end of the paper, there is a comparative analysis of experimental and numerical results.

Basic principle of thermoelectric module operation

During TE module operation in thermo element there are thermoelectric processes (Seebeck's, Peltier's and Thomson's) followed by Joule's effect. Seebeck's process [12] appears between two semiconductors made of different materials. If the temperature at their ends is different, there is a transformation of temperature difference into electric current I. Peltier's process [13, 14] is opposite to Seebeck's process. In this process the heat drawn to an end of a semiconductor is absorbed and discharged at the end of another semiconductor if the electric current I is conducted through a closed circuit. Scientist William Thomson explained the connection between Seebeck's and Peltier's processes. When the electric current is transferred through a semiconductor and if there is a temperature gradient along the semiconductor, heat will be absorbed or discharged from the conductor. Depending on the temperature gradient direction and the direction of electric energy, there will be absorption or discharge of temperature from the conductor. This phenomenon is called Thomson's process.

Due to the particle flow between the hot and cold side of the module, Joule's effect appears. It is of the opposite direction to Peltier's process on the cold module side whereas on the hot module side it has the same direction as Peltier's process.

Thermoelectric cooling is based on the fact that electrifying holders on joint semiconductor-metal exchange energy by absorbing it from the joints or they transfer it to them. For the direction of electric current presented on fig. 1, there is an arrangement of cold and hot joints. Each thermo element has two branches made of two different types of semiconductors: p-type and n-type. Charge carriers in n-type are electrons whereas in p-type these are the holes considered as positive carriers. For presented direction of electric current, electrons absorb the energy in points B and C making these joints cold. Electrons move in the opposite direction of the current direction (towards the positive pole) and in joint A they transfer the energy to the crystal lattice of the metal which is followed by the temperature increase in the joint. Positive carriers move in the direction of the electric current I (towards the negative pole) transferring the energy to the crystal lattice of the metal in the point D whereas the joint D heats up. In order to increase the cooling effect, it is necessary to discharge the heat appearing in hot joints into the environment.

Heat transfer in TE module is done both between the cold and hot side, as well as between environment and each one of them individually (Q_c and Q_h). Cooling effect (as presented on fig. 2) is determined by the quantity of heat Q_c calculated in the cold joint as:

$$Q_c = Q_{pc} - \frac{1}{2}Q_j - Q_\lambda = \alpha_{pn} I T_c - \frac{1}{2} I^2 R - \lambda \Delta T \tag{1}$$

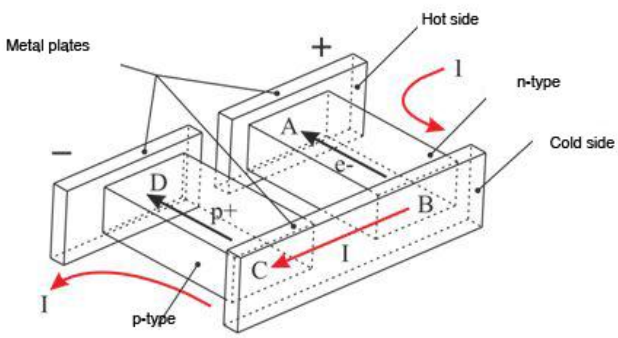

Figure 1. Basic thermo element

Analogically on the hot joint it will be:

$$Q_h = Q_{ph} + \frac{1}{2}Q_j - Q_\lambda = \alpha_{pn} I T_h + \frac{1}{2} I^2 R - \lambda \Delta T \tag{2}$$

Quantity of Peltier's heat on the hot and cold joint is marked as Q_{pc} and Q_{ph} as mentioned in [15,16]. Peltier's heat is proportional to the electric current being conducted through the module and to the temperature in the cold joint T_c or in the hot joint T_h. Seebeck's differential coefficient α_{pn} or thermoelectric coefficient of the power is the characteristic of semi-conductor material depending on the temperature.

Joule's heat Q_j is divided in two parts so it equally affects both the hot and cold module side. Q_λ represents the quantity of heat being generated due to the energy flow between the joints. Direction of Q_λ is opposite to Peltier's heat and depends on the conduction coefficient λ and on the temperature difference ΔT between the hot and cold joint.

Thermo elements in practice are joined into TE module (fig. 3), which represents the smallest unit available on the market. Thermo elements are placed between ceramic tiles. Knowing thermo-physical characteristics of semiconductor material and using the equations (1) and (2), mentioned quantities characterizing TE module operation are determined. In practice, mentioned thermo-physical characteristics are not known so there is a problem to determine mentioned heat quantities in the cold and hot module joints.

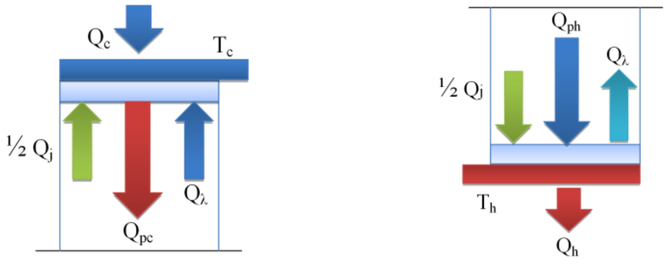

Figure 2. Thermoelectric scheme and heat balance of one thermo element: a) cold and b) hot joint

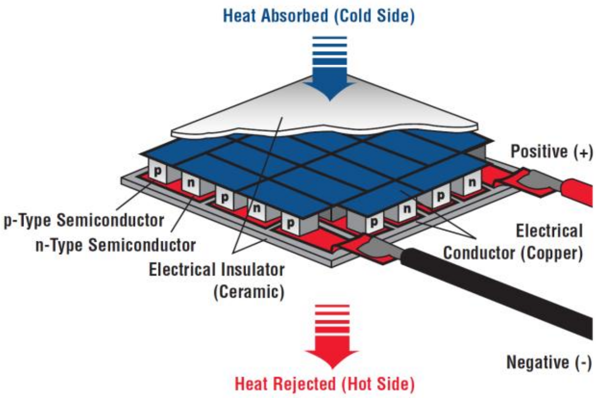

Figure 3. Typical construction of TE module

Analysis of TE module operation

Using characteristics of one TE module (fig. 4), cooling process of an object will be analyzed. It is assumed that there is an inhomogeneous transient temperature field on the surface under the cold module side. Difference in temperature between the hot and cold module side represents a temperature change $\Delta T = T_h - T_c$. Fig. 4 shows that the smaller the difference of mentioned temperatures is, the bigger the heat quantity Q_c drawn by the module from the object is. The biggest quantity is drawn in case when $\Delta T = 0$. The consequence of the previous statement will be the quicker decrease of temperature due to the quicker heat drawing in the points of the object with higher temperature, or the temperatures between the points of the highest and lowest temperature equalize.

Considering the fact that the easiest way to maintain the temperature on the hot side during module operation, higher temperature on the cold side corresponds to a smaller temperature difference between the sides of the module. In these points module will draw more heat whereas in the points on the cold side with lower temperature it will draw less heat. In this manner, differences between minimal and maximal temperature on the cold side decrease resulting in more efficient operation of the very module. Heat must be discharged with greater velocity than velocity of charging into the object to achieve cooling effect. Although a part of the heat is drawn through heat sink, it is necessary for the module to have considerately greater effective power than the drawn heat to fulfill the previous condition.

The quantity of heat drawn by the module from the cooling object in the unit of time (fig. 4) can be presented in a linear function of the form:

$$Q_c = Q_{c0} - K_1 \Delta T \tag{3}$$

where: K_1 - direction coefficient, representing a constant for a specific value of charging voltage and is defined considering fig. 4 in the equation of the form:

$$K_1 = \frac{Q_{c0}}{\Delta T_{\max}} \tag{4}$$

If the temperature on the cold module side is constant along its whole surface, heat flux which the module draws will be even so it can be determined by reduction to the unit of surface:

$$q_c = q_{c0} - k_1 \Delta T \tag{5}$$

where $q_{c0} = Q_{c0} / S$, and $k_1 = q_{c0} / \Delta T_{\max}$

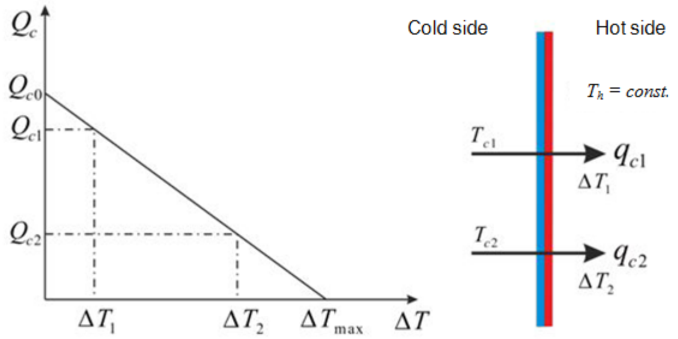

Figure 4. Characteristic of TE module

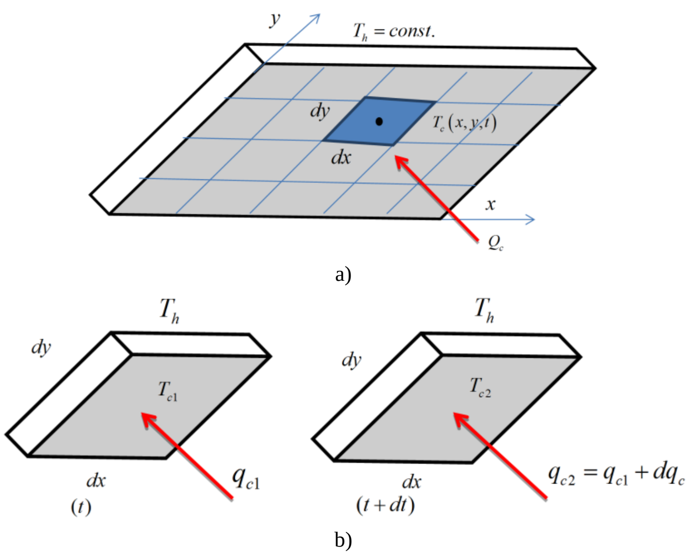

Figure. 5. a) Division of TE module into elements and b) increment q_c of arbitrary finite element

If it is assumed that the module operates on $T_{c(x,y,t)}=const.$, then the temperature differrence along the whole module will be $\Delta T = const$. If the module is divided into finite elements (Fig. 5a), this assumption will be valid for each of them. The consequence is that each of the elements absorbs the same heat quantity q_{ci}, whereas the total heat quantity drawn by the whole module in the unit of time can be determined by summing individual fluxes of all finite elements of the module. If i-th element of the module is observed (fig. 5b) it can be analyzed what will occur if the temperature increases on its cold side. The temperature on the hot side will be maintained as constant.

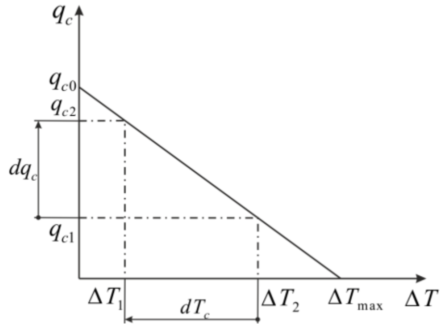

Figure 6. Diagram *qc-ΔT*

At an arbitrary moment of time t, temperatures on observed element are T_h and T_{c1}, and the element draws the heat quantity q_{c1}:

$$q_{c1} = q_{c0} - k_1 \Delta T = q_{c0} - k_1 (T_h - T_{c1}) \tag{6}$$

At another moment of time $t + dt$ on the cold element side, the temperature increases for dT_c resulting in a change of the heat quantity being absorbed by the observed element for the value dq_c. According to the fig. 6, the equations follow:

$$T_{c2} = T_{c1} + dT_c \tag{7}$$

$$q_{c2} = q_{c1} + dq_c \tag{8}$$

$$q_{c2} = q_{c0} - k_1 \left[T_h - (T_{c1} + dT_c) \right] \tag{9}$$

The increment of the absorbed heat quantity for the time dt can be determined according to the fig. 6 as:

$$dq_c = q_{c2} - q_{c1} = k_1 dT_c \tag{10}$$

Integration of the previous equation:

$$\int_{q_{c1}}^{q_c} dq_c = \int_{T_{c1}}^{T_c} k_1 dT_c \tag{11}$$

leads to the final equation in the following form:

$$q_c = q_{c0} - k_1 (T_h - T_c) \tag{12}$$

At an arbitrary moment of time t, temperature on the cold side of the module $T_c = T_{c(x,y)}$ is the function of the point position on the surface. Temperature T_h is very easily adjusted with the help of the automatic temperature regulator located on the heat exchanger.

Implementation of the finite element method

The finite element method has been applied to solve partial differential equations:

$$-\rho C_p \frac{dT}{dt} + \sum_{j=1}^{3} \frac{\partial}{\partial \mathrm{x}_j} \left(\lambda_j \frac{\partial T}{\partial \mathrm{x}_j} \right) + q = 0 \tag{13}$$

In the practical problem solving it is the solution for the temperature field *T(x,y,z,t)* that is searched for satisfying given initial and boundary conditions and representing a unique solution. Initial conditions are given only for transient problems and they mean that temperature distribution at the initial moment $t = 0$ is known:

$$T(\mathrm{x},\mathrm{y},\mathrm{z},0) = T_0(\mathrm{x},\mathrm{y},\mathrm{z}) \tag{14}$$

Boundary conditions in the case of thermoelectric module operation modeling are, fig. 9:

a) given fluxes on the contact surface with chips and workpiece:

$$q_{1A} = q_{1A}(x,y,z), \quad q_{3A} = q_{3A}(x,y,z) \tag{15}$$

b) given flux from thermoelectric module:

$$q_c = q_{c_0} - k_1 \Delta T = q_{c_0} - k_1 (T_h - T_c) \tag{16}$$

c) given convection on all other surfaces of the turning tool:

$$q_{\alpha_i} = \alpha_i (T_{amb} - T_s) \tag{17}$$

In the equation (17), α_i represents convection coefficient, T_{amb} is the temperature of the environment and T_s is the temperature of the cutting tool. Using Galerkin method, differential equation (18) transforms into the sysmet of algebraic equations as presented in [8-11].

Modeling of the cooling process of cutting tools by TE module

Turning tool cooled by TE module is presented on fig. 8. TE module is attached to the turning tool with cooper inlay. During the installation of the cooling system, it is important to set quality thermal contacts with the module. Isolation of the space where the module is located is also important. Module installation is done according to the procedure defined by the manufacturer. The body of the tool is with its one part of the lower surface (fig. 8) leaning on the tool holder and the attachment is done with bolted connections. Material characteristics of the turning tool, copper inlay and cutting tiles are given in the table 1.

Boundary conditions from the equations (15) and (17) given on the model of the turning tool are presented on fig. 9. Heat fluxes q_{1A} and q_{3A} are given at the top of the cutting tip P20, and they depend on the coordinate *x* and the contact length of the tool and chips l_c as presented on the fig. 10. Power of heat fluxes q_{1A} and q_{3A} is calculated according to the mathematical models from [4, 5] and is given for the specific case of processing in a diagram (fig. 11). On other surfaces of the turning tool, the convection q_6, q_7, q_{7A}, q_8, q_8', q_9 and q_{10} is given in accordance with (17). In the same manner, the convection on all free surfaces of copper inlay is given except for the isolated surface around the module. Boundary condition from the quation (16) is given on the surface covered by TE module as presented on fig. 12. TE module is connected with the copper inlay as presented on the fig. 12. Surface covered by TE module is 40x40 mm in size and is centered set in regard to the back inlay surface. TE module HP-199-1.4-0.8 was used in the experiment by the manufacturer TE Technology, MI, USA. Specifications of TE module are given in [17].

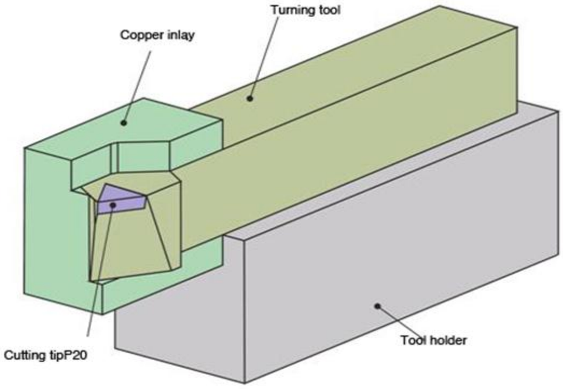

Figure 8. Lathe tool model

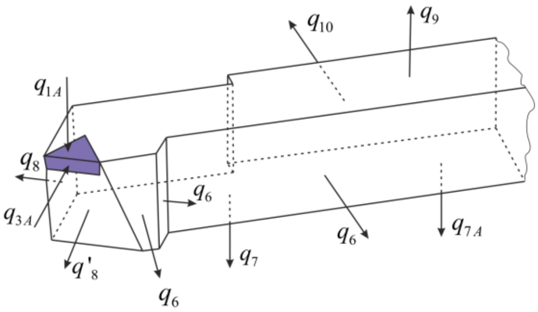

Figure 9. Boundary conditions used in the model

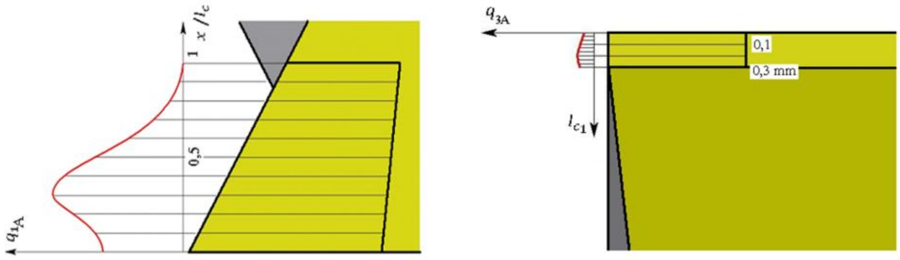

Figure 10. Heat fluxes given on front and back surface of cutting tip

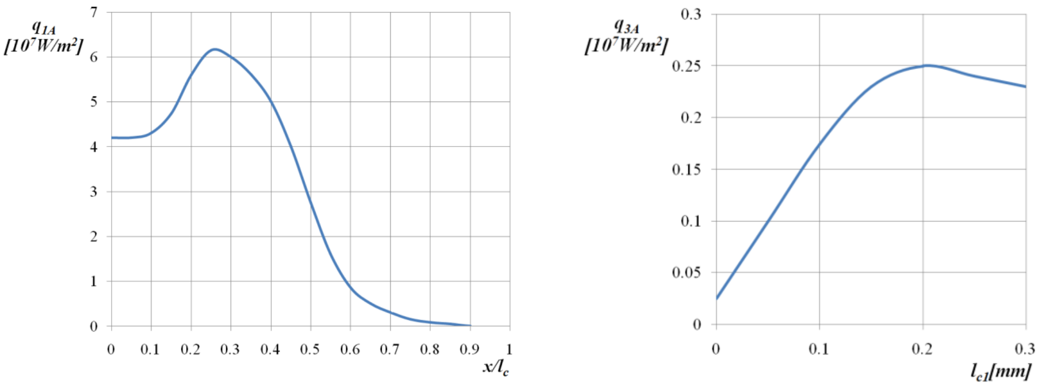

Figure 11. Power of heat sources on front and back clearance

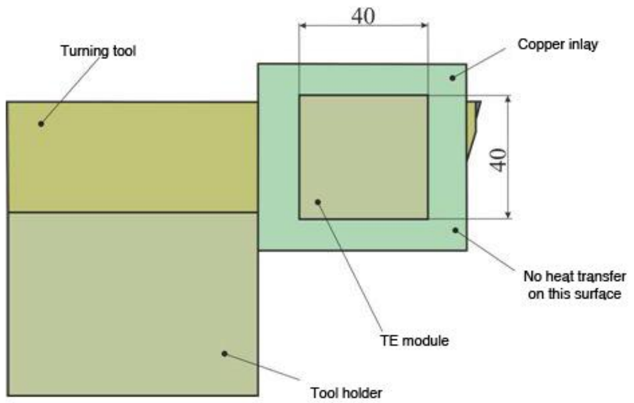

Figure 12. Position of TE module on the copper inlay

Table 1. Characteristics of materials used in problem

Material	Characteristics of materials		
	$\rho\left[kgm^{-3}\right]$	$\lambda\left[Wm^{-1}K^{-1}\right]$	$C_p\left[Jkg^{-1}K^{-1}\right]$
Cutting tip P20	11600	40	523
Turning tool	7830	46	500
Copper inlay	8960	386	385

Cooling of the hot module side is done by the heat exchanger based on the fluid MELCOR LI201CL [18]. Exchanger dimensions are bigger than the very module so a foam isolation based on aluminum-trioxide (Al_20_3) is set on the side around the module. Whole construction of the exchanger is connected with the cutting tool by bolts. For the proper function of TE module, its proper installation is very important.

Regulating of the temperature experimental values T_h (30, 50 and 70 ºC) on the hot module side is done by automatic regulator with digital temperature setting.

Comparative analysis of experimental and numerical results

Using mathematical model implemented in the program package PAK-T, results describing TE module operation in different processing regimes are obtained. Direction coefficient k_1 and heat flux which draws TE module on the unit of surface q_{co} are calculated according to the data from [17]. These values are used in the calculation and depend on the temperature on the hot module side as given in the tab. 2.

Experimental verification of numerical model is done by temperature measuring in seven points along the length of the turning tool body using the system 2300A Temperature Scanner, product of Dutch company “Fluke”. Measuring was done in the vertical plane passing through the cutting tip at the distance of 10 mm from the top surface of the turning tool as presented in fig. 13. In the drilled opens of 1.6 mm in diameter, thermocouples are placed. Analogical signals with thermocouples enter the scanner through the port 002 (T/C’s) and them are sent towards 2190A thermometer through cables. After analogical-digital conversion and linearization, the data on the temperatures are sent back to 2300A Temperature Scanner, which has the outputs for the printer, 1120A-IEEE converter or RS-232-C device.

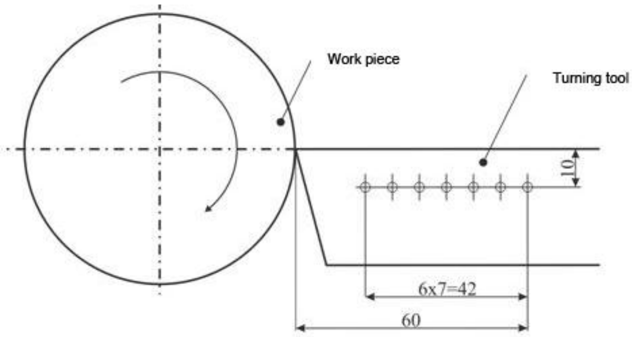

Figure 13. Schematic presentation of experimental temperature measuring at the top of turning tool

Table 2. Characteristics of TE module on different temperatures T_h

$T_h = 30^\circ C$			$T_h = 50^\circ C$			$T_h = 70^\circ C$		
U [V]	k_1 [$W/mm^2\,^oC$]	q_{co} [W/mm^2]	U [V]	k_1 [$W/mm^2\,^oC$]	q_{co} [W/mm^2]	U [V]	k_1 [$W/mm^2\,^oC$]	q_{co} [W/mm^2]
23.9	0.00128	0.08125	26.2	0.00125	0.0875	28.4	0.00122	0.095
18.3	0.00128	0.0775	20	0.00125	0.08375	21.7	0.00122	0.09
13.8	0.00128	0.0675	15	0.00125	0.0725	16.3	0.00122	0.0775
9.2	0.00128	0.05125	10	0.00125	0.05625	10.9	0.00122	0.06

Comparative analysis of numerical and experimental results is given in figs. 14-15. Experimental researches are done for the temperatures on the hot module side $T_h=30\ ^oC$ and $T_h=70\ ^oC$. Diagram shows that for $T_h=30\ ^oC$ there is a good match of experimental results with numerical ones for the processing regimes with voltage of 9.2V, 13.8V and 23,9V. For $T_h=70\ ^oC$ experimental results match for the processing regimes with voltage of 10.9V and 28.4V.

Deviation from the calculating curves at the tip of the tool should be regarded because of the small division of finite element mesh. It was necessary due to the manner of flux assignment on the front and back clearance considering that they are the surfaces of extremely small dimensions where fluxes change according to complex curve laws.

Conclusion

Heat sources during metal processing by cutting produce inhomogeneous transient temperature field spreading approximately concentrically compared to the location of the heat sources. For three-dimensional numerical modeling and analysis of this temperature field, finite element method is most appropriate. The developed mathematical model can be applied in the temperature fields analysis in the cutting tool with a TE module when a higher quality surface finish is required, and when the use of CLM is not possible, which leads to intense heating of the cutting tip.

Using the cooling systems based on the TE module decreasing the temperature in the cutting zone, thus the drop of mechanical properties of hard metal is prevented as well as its intensive wear. As the results obtained in numerical calculation correspond to the experimentally obtained results, it can be concluded that the developed mathematical model adequately describes the observed cooling process of the cutting tool by TE module. The developed mathematical model can be successfully applied to other objects cooled by TE module.

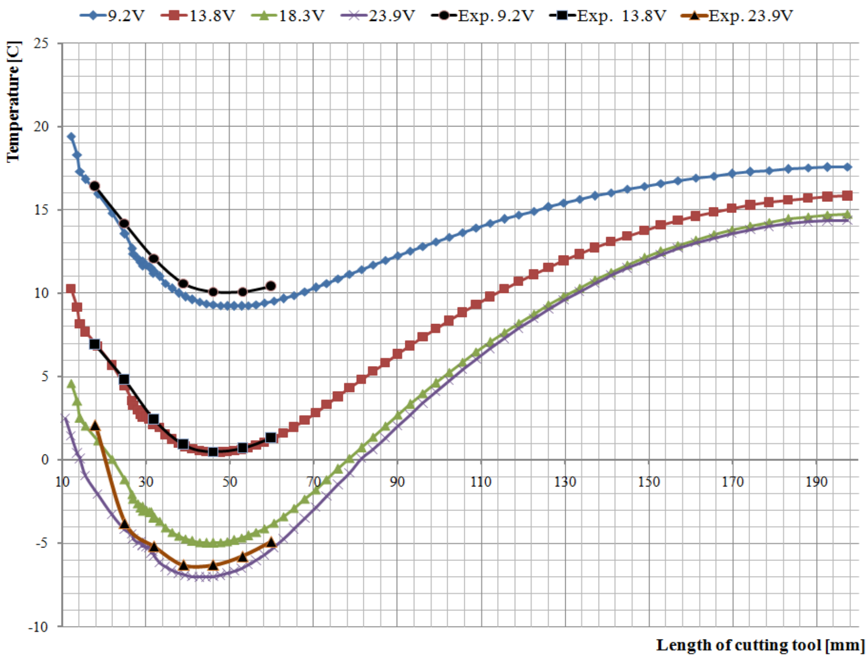

Figure 14. Comparative presentation of temperature distribution along the cutting tool for T_h= 30 °C

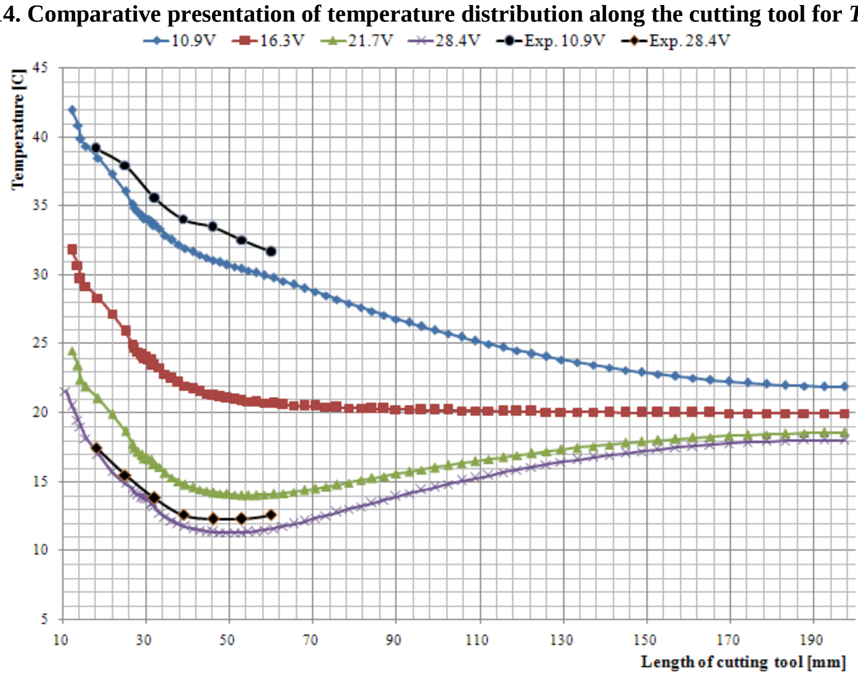

Figure 15. Comparative presentation of temperature distribution along the cutting tool for T_h= 70 °C

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Nomenclature

C_p – specific heat, $[Jkg^{-1}K^{-1}]$
I – electric current, $[A]$
K_1 – direction coefficient, $[WK^{-1}]$
Q – heat energy, $[W]$
Q_c – heat quantity on the cold joint, $[W]$
Q_h – heat quantity on the hot joint, $[W]$
Q_{pc} – quantity of Peltier's heat on the cold joint,[W]
Q_{ph} – quantity of Peltier's heat on the hot joint, [W]
Q_j – Joule's heat, [W]
Q_λ – heat between two joints with different temperatures, [W]
q – heat source intensity, [Wm^{-3}]
R– total thermo element electric resistance [Ω]
S– surface, [m^2]
t – time, [s]
T – temperature, [K]
T_c – temperature on the cold module side, [K]
T_h – temperature on the hot module side, [K]
T_{amb} – environment temperature, [K]
T_s – temperature of the cutting tool surface, [K]
ΔT – temperature difference, [K]
U – internal energy, [J]
V – volume, [m^3]

Greek letters

α_i – convection coefficient, [$Wm^{-2}K^{-1}$]
α_{pn} – differential Seebeck's coefficient, [VK^{-1}]
λ– specific heat conduction, [WK^{-1}]
ρ - material density, [$kg\ m^{-3}$]

References

[1] Snyder J. G., et al., Hot Spot Cooling using Embedded Thermoelectric Coolers, SEMI-THERM 21th Symposium Proceedings, IEEE, Dallas, TX, USA,2006.
[2] Fairbanks. J., Chair's Overview of High Efficiency Thermoelectrics and Potential Applications, High Efficiency Thermoelectrics Workshop, San Diego, California, 2004.
[3] Beňo J., et al., Technologické a materiálové činitele obrábania, Vienala, Košice, 2004.
[4] Nikolić R., et al., Model of Temperature Field in the Cutting Tool During Dry Machining as a Basis for Researching the New Cooling Systems, 10th International Conference, RaDMI, Donji Milanovac, 2010.
[5] Vukelja D., Termodinamika rezanja, monografija, Građevinska knjiga, Beograd, 1990.
[6] Müller B., Thermische Analyse des Zerspanens metallischer Werkstoffe bei hohen Schnittgeschwindungkeiten, Ph. D. thesis, Fakultät für Maschinenwesen, Achen, 2004.
[7] Kojić M., et al., PAK-T – Program for FE Heat Transfer Analysis, Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, Serbia, 2003.
[8] M. Berkovic, S. Maksimovic and A. Sedmak, "Analysis of Welded Joints by Applying the Finite Element Method," *Structural Integrity and Life*, vol. 4, no. 2, pp. 75-83, 2004.
[9] Kojić M., et al., Finite Element Method I – Linear Analysis (in Serbian), Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, Serbia, 2003.
[10] Živković M., et al., Non-linear Transient Heat Conduction Analysis of Insulation Wall of Thank for Transportation of Liquid Aluminium, *Thermal Science, 14* (2010), Issue Supplement, pp. S299-S312
[11] Lazić, V., et al., Theoretical-Experimental Determining of Cooling Time ($t_{8/5}$) in Hard Facing of Steels for Forging Dies, *Thermal Science, 14* (2010), Issue 1, pp. 235-246
[12] Kraftmakher, Y., Simple experiments with a thermoelectric module, European Journal of Physics, Volume 26 (2005), pp. 959–967
[13] Melcor Thermoelectric Handbook, Melcor Thermoelectrics, Trenton, New Jersey, USA, 2009.
[14] Thermoelectric Cooling, Part C: Electronics Cooling Methods in Industry, Mechanical Power Engineering Department, Faculty of Engineering, Cairo University, Egypt, 2011.
[15] FAQ About Peltier Cooling And Heating Technology, Tellurex, Traverse City, MI, USA, 2011.
[16] Lau, P.G., et al, Temperature and time dependent finite-element model of a thermoelectric couple, TE Technology, Traverse City, MI, USA, 1996
[17] HP-199-1.4-0.8, Thermoelectric Module (Peltier Module) Specifications, TE Technology, Traverse City, MI, USA, 2011.
[18] Melcor Liquid Heat Exchanger LI201CL, Product Information, Melcor Thermoelectrics, Trenton, New Jersey, USA, 2009.

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