Open forum

Recently published paper Remark on a Constrained Variational Principle for Heat Conduction, written by Zhao-Ling TAO and Guo-Hua CHEN, Thermal Science, 17 (2013), 3, pp. 951-952, initiated interesting contribution A Short Remark on He-Lee's Variational Principle for Heat Conduction, sent by Dong-Dong FEI, Fu-Juan LIU, Ping WANG, and Hong-Yan LIU that we are presenting in this issue.
We invited also Prof. Zhao-Ling TAO and Prof. Ji-Huan HE, to join the discussion. Now we present also, a short remark on the history of the semi-inverse method, sent by Prof. Ji-Huan HE, on the Semi-Inverse Method and Variational Principle, by Xue-Wei LI, Ya LI, and Ji-Huan HE.

A SHORT REMARK ON HE-LEE'S VARIATIONAL PRINCIPLE FOR HEAT CONDUCTION

by

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Tao and Chen claimed that they obtained a new constrained variational principle for heat conduction (Tao, Z. L., Chen, G. H., Thermal Science, 17 (2013), 3, pp. 951-952), however, this short note concludes that it is not new at all. It is equivalent to He-Lee 2009 variational principle.

Key words: *variational principle, equivalent variational principle, heat conduction*

Introduction

Recently Tao and Chen claimed that they obtained a novel variational principle for the 1-D heat equation [1], which reads:

$$J_{Tao\text{-}Chen}(\Phi,\Psi)=\int_0^t\int_a^b\left[\frac{\partial T}{\partial t}+\lambda T+\frac{1}{k^2}\frac{\partial^2 T}{\partial x^2}\right]\mathrm{d}x\mathrm{d}t \tag{1}$$

where T is the temperature, and k is a constant.

However, this variational principle is not new at all, it is equivalent to He-Lee 2009 variational principle [2], which reads:

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$$J_{He\text{-}Lee}(\Phi,\Psi)=\int_0^{t_0}\int_a^b\left[\frac{\partial^2 T}{\partial x^2}+k^2\frac{\partial T}{\partial t}+\lambda k^2 T\right]\mathrm{d}x\mathrm{d}t \tag{2}$$

Equivalent variational principles

To show the equivalence of the above two variational principles, we give a serious mathematical proof.

Consider a variational formulation in the form:

$$J(\Phi)=\iint L(\Phi,\Phi_t,\Phi_x,\Phi_{tt},\Phi_{xx})\mathrm{d}x\mathrm{d}t \tag{3}$$

where L is Lagrangian. Its stationary condition (Euler-Lagrange equation) is:

$$\frac{\partial L}{\partial \Phi}-\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \Phi_t}\right)-\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \Phi_x}\right)+\frac{\partial^2}{\partial t^2}\left(\frac{\partial L}{\partial \Phi_{tt}}\right)+\frac{\partial^2}{\partial x^2}\left(\frac{\partial L}{\partial \Phi_{xx}}\right)=0 \tag{4}$$

Remark

The following functional has the same Euler-Lagrange equation with eq. (3):

$$J_1(\Phi)=\iint\{\alpha L(\Phi)+\beta\}\mathrm{d}x\mathrm{d}t \tag{5}$$

where α and β are constants and $\alpha\neq 0$.

Proof

The Euler-Lagrange equation for eq. (5) is:

$$\begin{gathered}\frac{\partial[\alpha L(\Phi)+\beta]}{\partial \Phi}-\frac{\partial}{\partial t}\left\{\frac{\partial[\alpha L(\Phi)+\beta]}{\partial \Phi_t}\right\}-\frac{\partial}{\partial x}\left\{\frac{\partial[\alpha L(\Phi)+\beta]}{\partial \Phi_x}\right\}+\\+\frac{\partial^2}{\partial t^2}\left\{\frac{\partial[\alpha L(\Phi)+\beta]}{\partial \Phi_{tt}}\right\}+\frac{\partial^2}{\partial x^2}\left\{\frac{\partial[\alpha L(\Phi)+\beta]}{\partial \Phi_{xx}}\right\}=0\end{gathered} \tag{6}$$

or

$$\alpha\left[\frac{\partial L}{\partial \Phi}-\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \Phi_t}\right)-\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \Phi_x}\right)+\frac{\partial^2}{\partial t^2}\left(\frac{\partial L}{\partial \Phi_{tt}}\right)+\frac{\partial^2}{\partial x^2}\left(\frac{\partial L}{\partial \Phi_{xx}}\right)\right]=0 \tag{7}$$

It is obvious that eq. (7) is equivalent to eq. (4) when $\alpha\neq 0$.

Accordingly we can write down an equivalent one for He-Lee variational principle in the form:

$$\tilde{J}_{He\text{-}Lee}(\Phi,\ \Psi)=\int_0^{t_0}\int_a^b\left[\alpha\left(\frac{\partial^2 T}{\partial x^2}+k^2\frac{\partial T}{\partial t}+\lambda k^2 T\right)+\beta\right]\mathrm{d}x\mathrm{d}t \tag{8}$$

where α and β are non-zero constants.

Choosing $\alpha = 1/k^2$ and $\beta = 0$ in eq. (8), He-Lee's variational principle, eq. (8), turns out to be eq. (1).

Conclusions

A mathematical proof is given to elucidate that Tao-Chen 2013 variational principle is exactly the same with He-Lee's variational principle. Furthermore the derivation process is also the same with that in [2] using the semi-inverse method [3].

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