FRACTAL APPROACH TO HEAT TRANSFER IN SILKWORM COCOON HIERARCHY

by

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Silkworm cocoon has a complex hierarchic structure with discontinuity. In this paper, heat transfer through the silkworm cocoon is studied using fractal theory. The fractal approach has been successfully applied to explain the fascinating phenomenon of cocoon survival under extreme temperature environment. A better understanding of heat transfer mechanisms for the cocoon could be beneficial to the design of biomimetic clothes for special applications.

Key words: *silkworm cocoon, heat transfer, hierarchy*

Introduction

Silkworm cocoons have evolved over millions of years by a process of natural selection to nurture and protect moths and butterflies in a wide range of different environments and exposed to many different threats and predators [1, 2]. A cocoon is a natural protein polymer composite shell with special hierarchical porous microstructures [3], which enable the cocoon to have outstanding mechanical properties of strength and toughness [4]. Silkworm cocoon is called to be "breathing", which has a superior heat and moisture transport capability comparing with other natural or man-made fibers. Recently, some researchers found that the cocoon does not have any impact on the oxygen and water vapor transportation, which shows superior gas permeability and moisture-penetrability [5-8]. In addition, wild silkworm chrysalis can survive in extreme temperature environment, at –40 °C or +40 °C, maybe due to some special functional performance and the configuration of the cocoon. Therefore, it is imperative to investigate mechanism of heat transfer in such a natural porous media composed by fibers. There will be helpful for developing new clothing with multifunction.

In this paper, the fractal model for heat transfer in hierarchic cocoons was adopted for the fascinating phenomenon of cocoon survival under extreme temperature environment. The fractal heat transfer of cocoon was analysed with assistance of the fractal derivative model.

Fractal model for heat transfer in hierarchic porous media

In continuous media, heat transfer of an isotropic medium without inner heat source follows Fourier's law, expressed as:

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$$\frac{\partial T}{\partial t}+\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=0 \tag{1}$$

When the temperature does not change with time, heat transfer can be regarded as one-dimensional steady case. Equation (1) becomes:

$$q=k\frac{\partial T}{\partial x} \tag{2}$$

where T is the temperature, q – the heat flux, and k – the heat conductivity. Equation (2) indicates that heat flux is constant.

The solution of eq. (2) is:

$$T_o=\frac{q}{k}x+T_i \tag{3}$$

where T_o and T_i are outer and inner temperature of the porous media, respectively. Equation (3) implies a linear temperature variation, see the line *a* in fig. 1.

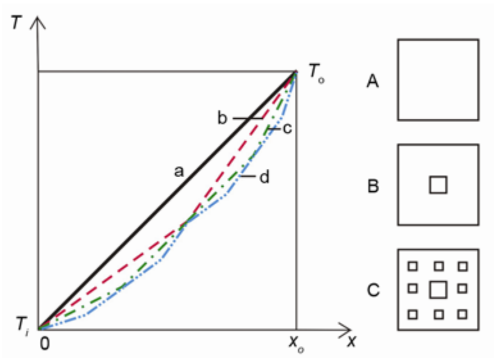

Figure 1. The schematic of heat transfer in hierarchic porous media. Curves a, b, c, and d are: continuous media, fractal media with one, two, and three iterations, respectively. On the right, squares A, B, and C correspond to curves a, b, and c, respectively *(for color image see journal web site)*

The analysis is approximately valid for a large scale case (*e. g.* A in fig. 1). When the scale becomes smaller, discontinuity occurs (see B and C in fig. 1). To deal with such complex heat transfer problem in fractal media, the fractal derivative [9] is a simple method and has been used to model heat transfer [10-12]. According to fractal theory, the actual length of heat transfer is:

$$\mathrm{d}s=k_1x^{\alpha} \tag{4}$$

where α is the fractal dimension and k_1 – the constant. Based on fractal derivative method [9], the heat flux in a discontinuous media reads:

$$\tilde{q}=\tilde{k}\frac{\mathrm{D}T}{\mathrm{D}x^{\alpha}}=\frac{\mathrm{d}T}{\mathrm{d}s} \tag{5}$$

The solution of eq. (5) changes:

$$\tilde{T}_o=\frac{\tilde{q}}{\tilde{k}}x^{\alpha}+\tilde{T}_i \tag{6}$$

where $\tilde{T}_o$ and $\tilde{T}_i$ are outer and inner temperature separately, $\tilde{q}$ is the heat flux, and $\tilde{k}$ – the constant for the discontinuous hierarchic media. On a smaller scale (B in fig. 1), eq. (6) implies the curve b in fig. 1, and the curve c is for an even smaller scale (C in fig. 1).

Discussion

To our knowledge, silkworm cocoon has special hierarchical microstructures and excellent protective functions. A pupa in the cocoon has superior ability to survive in harsh environment, extreme temperature like +40 °C and –40 °C. In this paper, it is found that the unique hierarchical structure of the cocoon plays an important role.

As shown in fig. 1, the continuous media can not withstand extreme environment very well due to linear temperature variation. However, the hierarchical silkworm cocoon has fractal heat transfer property. The slope of temperature change ΔT tends to as a small value as possible with the increase of iterations. So a thin thickness of cocoon can achieve almost no change of temperature at the inner wall. Therefore, the pupa has been protected from harsh environment and has enough space to live and transport oxygen and water vapor from the outside.

Conclusions

In summary, the fractal model for heat transfer in hierarchic cocoons has been proposed for the first time to explain the fascinating phenomenon. The fractal derivative is an efficient method to handle these complicated heat transfer problems of porous media. The establishment of heat transfer mechanisms for the cocoon could be beneficial to the design of biomimetic clothes in many special fields of applications.

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