PERIODIC SOLUTION TO GENERAL CONDUCTION PROBLEMS

by

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In this paper, we present a modified exp-function method, where hyperbolic cosine and cosine functions are used. The hyperbolic cosine functions are responsible for energy localization while cosine functions reveal the periodic effect. A general conduction problem is used as an example to illustrate the solution process.

Key words: *non-linear equation, exp-function method, solitary solution*

Introduction

Many kinds of soliton equations have been discovered up to now, for examples, non-linear Schrodinger equation, KdV equation, Sine-Gordon equation, and others. All of these equations can be transformed into bilinear forms by some special transformations [1] including the rational transformation, the logarithmic transformation, and the bi-logarithmic transformation. Once we get the bilinear forms of these equations, we can directly construct *N*-soliton solutions following the Hirota's basic assumptions. Furthermore, bilinear forms have some special intrinsic properties, which can bring us some free considerations. Own to these bilinear forms, Lou [2, 3] constructed many localized structure by the variable separation method, Hirota [1] obtained determinants and pfaffians solutions. Recently, Dai *et al.* [4] proposed the three-wave method for non-linear evolution equations (NEE), and He and Wu suggested the exp-function method for solitary solutions [5, 6]. Review on various methods is available in [7-9]. In this paper, we will suggest a modification of the exp-function method.

Consider a (2+1) dimensional non-linear evolution equation of the general form:

$$F(u, u_t, u_x, u_y ...) = 0 \tag{1}$$

where F is a polynomial of $u(x, y, t)$ and its derivatives.

We consider a bilinear equation of the form:

$$G(D_t, D_x, D_y, \cdots) f \cdot f = 0 \tag{2}$$

where G is a general polynomial in D_t, D_x, and D_y, where the D-operator is defined by:

$$D_x^m D_t^n F(x, y, t) \cdot G(x, y, t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n F(x, y, t) G(x', y', t') \Bigg|_{x'=x, y'=y, t'=t}$$

Traditionally, one obtains N soliton solutions using the assumption:

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$$f = \sum_{\mu=0,1} \exp\left(\sum_{i>j}^{N} A_{ij}\mu_i\mu_j + \sum_{i=1}^{n} \mu_i\xi_i \right) \tag{3}$$

According to the exp-function method [5-9], we assume that:

$$f = \sum_{i=1}^{m} a_i \left[\exp(\xi_i) + \exp(\xi_i)\right] + \sum_{j=1}^{n} b_i \left[\exp(\xi_j) + \exp(-i\xi_j)\right] \tag{4}$$

or equivalently:

$$f = 2\sum_{i=1}^{m} a_i \cosh(\xi_i) + 2\sum_{j=1}^{m} b_i \cos(\eta_j) \tag{5}$$

where $\xi_i = k_i x + l_i y + c_i t$ and $\eta_i = d_i x + e_i y + f_i t$.

In eq. (5), cosh functions are responsible for energy localization but trigonometric cos functions reveal periodic effect in real physical background.

Application to (2+1) dimensional NLEE equation

In this work, we study the following general conduction problem arising in fluid mechanics [5]:

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} = 0 \tag{6}$$

Bekir [10] has studied its Painleve property. By the dependent variable transformation $u = 2(\ln\phi)_{xx}$, then, eq. (6) is reduced to Hirota bilinear form:

$$(D_y D_t + D_x^3 D_y)\phi \cdot \phi = 0 \tag{7}$$

One soliton solution is assumed to have the form:

$$\phi = 1 + \mathrm{e}^{k_1 x + l_1 y + c_1 t} \tag{8}$$

Inserting eq. (8) into eq. (7), and after simple calculation, we obtain:

$$u(x,t) = \frac{2k_1 \mathrm{e}^{k_1 x + l_1 y - k_1^3 t}}{1 + \mathrm{e}^{k_1 x + l_1 y - k_1^3 t}} \tag{9}$$

Two soliton solutions can be constructed by substituting:

$$\phi = 1 + \mathrm{e}^{k_1 x + l_1 y + c_1 t} + \mathrm{e}^{k_2 x + l_2 y + c_2 t} + a_{12}\mathrm{e}^{k_1 x + l_1 y + c_1 t + k_2 x + l_2 y + c_2 t}$$

into eq. (7) and solving for the phase shift a_{12}, we find two-soliton solution in the form:

$$u(x,t) = \frac{2\left(k_1 \mathrm{e}^{k_1 x + l_1 y - k_1^3 t} + k_2 \mathrm{e}^{k_2 x + l_2 y - k_2^3 t} + a_{12}\mathrm{e}^{k_1 x + l_1 y - k_1^3 t + k_2 x + l_2 y - k_2^3 t}\right)}{1 + \mathrm{e}^{k_1 x + l_1 y - k_1^3 t} + \mathrm{e}^{k_2 x + l_2 y - k_2^3 t} + a_{12}\mathrm{e}^{k_1 x + l_1 y - k_1^3 t + k_2 x + l_2 y - k_2^3 t}}$$

Alternatively, we assume that:

$$\phi = \cosh(k_1 x + l_1 y + c_1 t) + \cos(k_2 x + l_2 y + c_2 t) + a_3 \cosh(k_3 x + l_3 y + c_3 t) \tag{10}$$

Substituting eq. (10) into eq. (7), we have the following relations:

$$c_1 = -k_3^3(-1+3l_3^2-6l_3^2a_3^2+3l_3^4a_3^2), \quad c_2 = k_3^3l_3(1-a_3^2)(l_3^2-2l_3^2a_3^2+a_3^4l_3^2-3),$$

$$c_3 = k_3^3(-1+3l_3^2-6l_3^2a_3^2+3a_3^4l_3^2), \quad k_1 = -k_3, \quad k_2 = l_3k_3(1-a_3^2), \quad l_2 = 1, l_1 = l_3$$

where l_3, a_3, and k_3 are free parameters. This case leads to a breath-kink solitary solution:

$$u(x,t) = \frac{2\left[k_1\sin(k_1x+l_1y+c_1t)-k_2\sin(k_2x+l_2y+c_2t)+a_3k_3\sin(k_3x+l_3y+c_3t)\right]}{\cosh(k_1x+l_1y+c_1t)+\cos(k_2x+l_2y+c_2t)+a_3\cosh(k_3x+l_3y+c_3t)}$$

This solution shows periodic breathing resulting from cosine function in above expressions.

Conclusions

Generally, *N*-soliton solution of non-linear evolution equation can be obtained by a similar manner illustrated. In this article, by the modified exp-function method, we obtain various solutions including the multiple kink solution and the breath-kink solitary solution. The method is proved to be an effective method to construct new exact solutions of non-linear evolution equation.

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