A CONSTITUTIVE MODEL FOR PARTICULATE-REINFORCED TITANIUM MATRIX COMPOSITES SUBJECTED TO HIGH STRAIN RATES AND HIGH TEMPERATURES

by

Wei-Dong SONG[a*], Jian-Guo NING[a], Xiao-Nan MAO[b], and Jian-Qiao LI[a]

[a] State Key Laboratory of Explosion Science and Technology,
Beijing Institute of Technology, Beijing, China
[b] Northwest Institute for Non-Ferrous Metal Research, Xi'an, China

Original scientific paper
DOI: 10.2298/TSCI1305361S

Quasi-static and dynamic tension tests were conducted to study the mechanical properties of particulate-reinforced titanium matrix composites at strain rates ranging from 0.0001/s to 1000/s and at temperatures ranging from 20 °C to 650 °C Based on the experimental results, a constitutive model, which considers the effects of strain rate and temperature on hot deformation behavior, was proposed for particulate-reinforced titanium matrix composites subjected to high strain rates and high temperatures by using Zener-Hollomon equations including Arrhenius terms. All the material constants used in the model were identified by fitting Zener-Hollomon equations against the experimental results. By comparison of theoretical predictions presented by the model with experimental results, a good agreement was achieved, which indicates that this constitutive model can give an accurate and precise estimate for high temperature flow stress for the studied titanium matrix composites and can be used for numerical simulations of hot deformation behavior of the composites.

Key words: *constitutive model, titanium matrix composite, high strain rate, high temperature, split Hopkinson tensile bar*

Introduction

It is well recognized that strain rate and temperature are two important factors on the mechanical behavior of most metals, alloys, and composites. An understanding of the mechanical performance of these materials over a wide range of temperatures and strain rates is important in the design of structures in engineering applications. Recently, great efforts have been made to propose some phenomenologically and physically based constitutive models for different materials. Johnson-Cook (J-C) model is an empirical model and has been successfully incorporated into some commercial packages due to its simple multiplication form and few material constants [1]. In this model, the effects of strain hardening, strain-rate hardening, and thermal softening on the mechanical behavior are taken in account. Unlike J-C model, Zerilli-Armstrong (Z-A) model is a physically based constitutive model which considers the coupled effects of strain rate, temperature, and grain size on stress-strain relations [2]. Li *et al.* [3] conducted isothermal hot compression tests at three different strain rates and six different temperatures and proposed a constitutive model for high temperature flow stress prediction of Al-14Cu-7Fe alloy by using Zener-Hollomon equation and Arrhenius equations based on the

* Corresponding author; e-mail: swdgh@bit.edu.cn

experimental results. Marte *et al.* [4] investigated the elevated temperature deformation behavior of Al_3Ti– and near γ TiAl-matrix discontinuously-reinforced composites at temperatures of 1000, 1100, and 1200 °C using initial strain-rates of 10^{-3} and $10^{-4}s^{-1}$ and reported the flow behavior of the composites can be predicted by using a unified Zener-Holloman (temperature-compensated strain-rate) model to consider the effects of strain-rate and temperature on the resulting flow behavior of the composites.

The main objective of the present paper is to investigate the tensile properties of TiC particulate reinforced titanium matrix composites under both quasi-static and dynamic loadings, and to develop an empirical based constitutive model by using Zener-Hollomon constitutive equations and Arrhenius equations.

Constitutive model for hot deformation behavior of TiC/Ti composite

Generally, the flow stress σ of the materials under hot working conditions is related to the temperature T, the deformation temperature ε, the strain rate $\dot{\varepsilon}$, the chemical composition C, and microstructure S:

$$\sigma = f(\varepsilon, \dot{\varepsilon}, T, C, S) \tag{1}$$

During the deformation process, the chemical composition is not changed in a certain temperature range and the microstructure is constrained to the loading conditions, so an Arrhenius equation is proposed to describe and predict the deformation behavior by considering the effect of the activation energy and the temperature [5].

$$\dot{\varepsilon} = A\left[\sinh(\alpha\sigma)\right]^n \exp\left(-\frac{Q}{\mathrm{R}T}\right) \tag{2}$$

where R is the universal gas constant [R = 8.314 J/molK], Q – the activation energy, A – the structure constant, and α – the material constant.

Equation (2) can be reduced to a power relationship at low stresses [6, 7]:

$$\dot{\varepsilon} = A_1\sigma^n \exp\left(-\frac{Q}{\mathrm{R}T}\right) \tag{3}$$

At high stresses, eq. 2 can be rewritten as [8, 9]:

$$\dot{\varepsilon} = A_2 \exp(\beta\sigma) \exp\left(-\frac{Q}{\mathrm{R}T}\right) \tag{4}$$

where A_1, A_2, n, and β are material constants independent of temperature, α and n are related by $\alpha = \beta/n$ [1], so that both of them can be evaluated from experimental results at high and low stresses.

Zener-Hollomon found that the flow stress dependents on strain rate and temperature, only through a single parameter Z (a temperature-compensated strain-rate parameter) [10]:

$$Z = \dot{\varepsilon} \exp\left(\frac{Q}{\mathrm{R}T}\right) \tag{5}$$

where Z is the Zener-Hollomon parameter. This relation assumed that the mechanical properties must depend only upon a dimensionless quantity, and, therefore, upon the ratio of the strain rate and some rate characteristic of the material itself.

Combining eqs. (5) and (2), we have:

$$Z = A[\sinh(\alpha\sigma)]^n \tag{6}$$

According to the definition of hyperbolic sine function, the flow stress relation can be obtained as a function of Z:

$$\sigma = \frac{1}{\alpha}\ln\left\{\left(\frac{Z}{A}\right)^{\frac{1}{n}} + \left[\left(\frac{Z}{A}\right)^{\frac{2}{n}} + 1\right]^{\frac{1}{2}}\right\} \tag{7}$$

Only when all these material constants are determined from experimental data, the flow stress of the TiC/Ti composites can be given by eq. (7).

Material constants

Taking the logarithms of both sides of eqs. (3) and (4), we have:

$$\ln\dot{\varepsilon} = \ln A_1 + n\ln\sigma \tag{8}$$

$$\ln\dot{\varepsilon} = \ln A_2 + \beta\sigma \tag{9}$$

The linear curves of $\ln\dot{\varepsilon}$ *vs.* $\ln\sigma$ and $\ln\dot{\varepsilon}$ *vs.* σ can be plotted by substituting the values of flow stress and strain rate at different temperatures into eqs. (8) and (9). n and β can be calculated from the slopes of the lines in $\ln\dot{\varepsilon}$ *vs.* $\ln\sigma$ and $\ln\dot{\varepsilon}$ *vs.* σ plots by a linear function.

From figs. 1 and 2, the values of n and β can be identified as 58.33 and 0.067 MPa^{-1}, respectively. Therefore, $\alpha = \beta/n$ = 0.001138 MPa^{-1}.

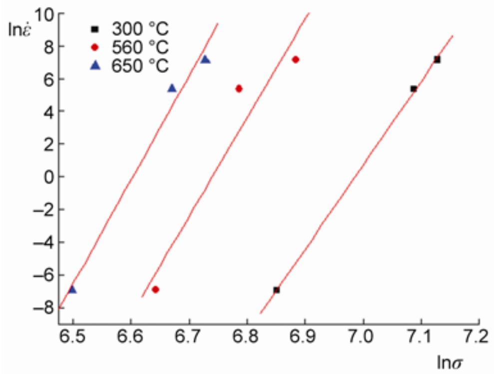


Figure 1. Relationship between ln $\dot{\varepsilon}$ and ln*σ*

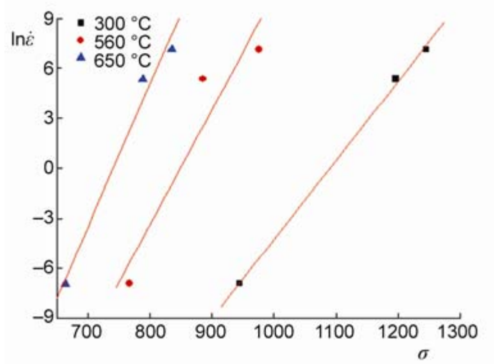


Figure 2. Relationship between ln $\dot{\varepsilon}$ and *σ*

Similarly, by taking a logarithm transformation, eq. (2) can be expressed:

$$\ln\dot{\varepsilon} = \ln A + n\ln[\sinh(\alpha\sigma)] - \frac{Q}{\mathrm{R}T} \tag{10}$$

By differentiating eq. (10), we get:

$$n = \left\{\frac{\partial\ln\dot{\varepsilon}}{\partial\ln[\sinh(\alpha\sigma)]}\right\}_T \quad \text{and} \quad \frac{Q}{n\mathrm{R}} = \left\{\frac{\partial\ln[\sinh(\alpha\sigma)]}{\partial T}\right\}_{\dot{\varepsilon}} \tag{11}$$

$$Q = \mathrm{R}n\frac{Q}{R} = \mathrm{R}\left\{\frac{\partial \ln \dot{\varepsilon}}{\partial \ln[\sinh(\alpha\sigma)]}\right\}_T \left\{\frac{\partial \ln[\sinh(\alpha\sigma)]}{\partial \frac{1}{T}}\right\}_{\dot{\varepsilon}} \tag{12}$$

The curves of $\ln\dot{\varepsilon} - \ln[\sinh(\alpha\sigma)]$ and $\ln[\sinh(\alpha\sigma)] - 1000/T$ are plotted by substituting the experimental data into eq. (12) and then fitted by a linear function and are shown in figs. 3 and 4, respectively. Q can be obtained from the products of the two slopes of the fitted lines as 337.133 kJ/mol. Taking the logarithm of both sides of eq. (6) gives,

$$\ln Z = \ln A + n\ln[\sinh(\alpha\sigma)] \tag{13}$$

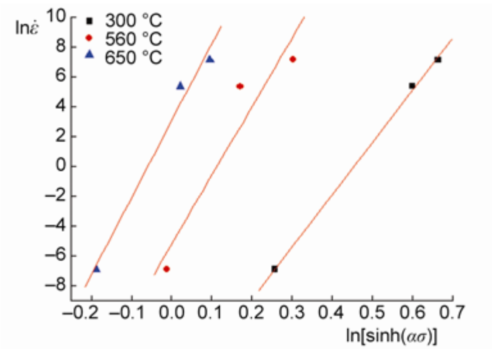


Figure 3. Relationship between ln $\dot{\varepsilon}$ and ln[sinh($\alpha\sigma$)]

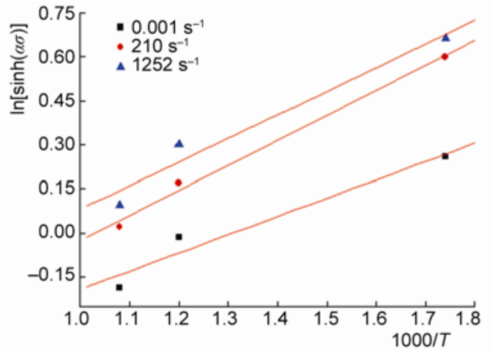


Figure 4. Relationship between ln[sinh($\alpha\sigma$)] and 1000/T

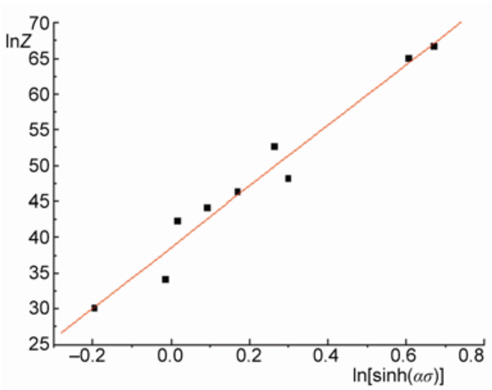


Figure 5. Relationship between ln$Z$ and ln[sinh($\alpha\sigma$)]

Figure 5 shows the relationship between ln$Z$ and ln[sinh($\alpha\sigma$)] and the corresponding fitting line. Then, the values of ln$A$ and n can be derived from the intercept and the slope of the fitting line as 46.908 and 49.93, respectively.

In order to include the strain effect on the flow stress, different deformation strains ranging from 0.03 to 0.06 at an interval of 0.0025 are chosen to identify all the material constants by using the above method. Figures 6-9 show the relationship between the deformation strain and the material constants, α, Q, n, and ln$A$, respectively. By using a 2nd order polynomial fit of the above curves, we get the following polynomials for the four material constants:

$$\begin{aligned} \alpha &= 0.0012 + 0.00044\varepsilon + 0.00418\varepsilon^2 \\ Q &= 397.42697 - 1688.37079\varepsilon + 9571.20433\varepsilon^2 \\ n &= 57.24996 - 145.72091\varepsilon + 90.47193\varepsilon^2 \\ \ln A &= 49.67737 + 34.25679\varepsilon - 1670.11229\varepsilon^2 \end{aligned} \tag{14}$$

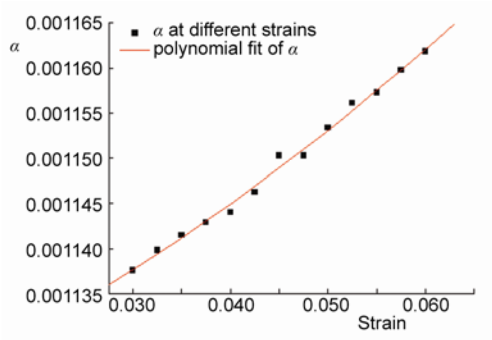


Figure 6. *α* at different deformation strains

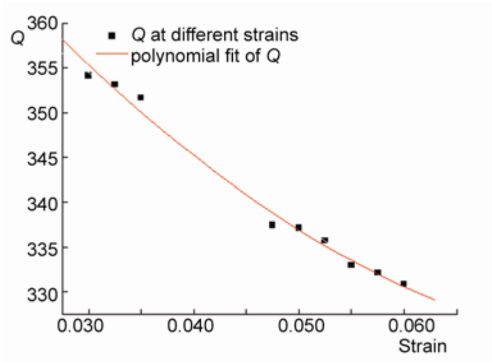


Figure 7. *Q* at different deformation strains

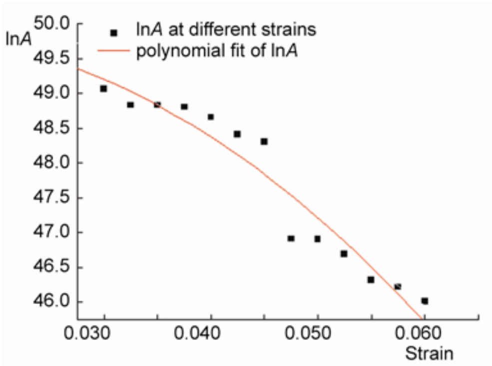


Figure 8. ln*A* at different deformation strains

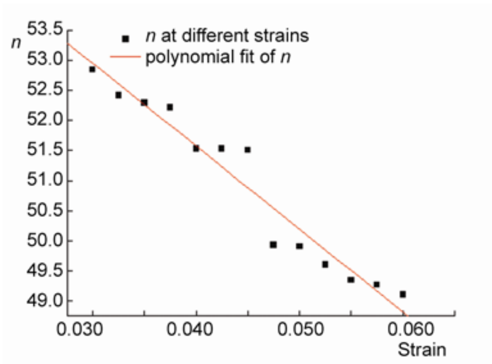


Figure 9. *n* at different deformation strains

Combining eqs. (5), (7) and (14), a phenomenological-based constitutive model is established to predict the deformation behavior of TiC/Ti composites under high temperatures and high strain rates. The flow stress can be written as the formula containing Zener-Hollomon parameters. Combining eq. (6) with eq. (5), the constitutive equations of the thermal deformation of the material can be expressed as:

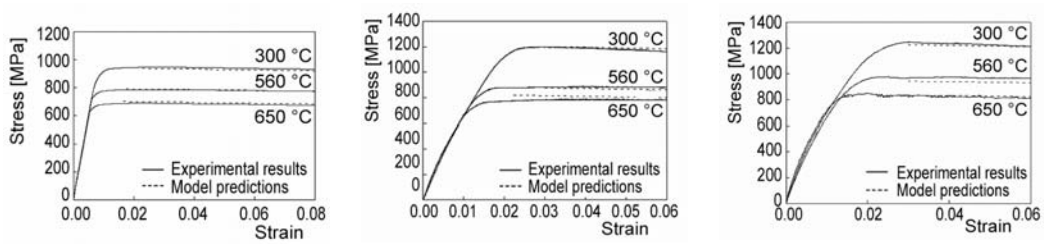
$$\begin{cases} \dot{\varepsilon} = A[\sinh(\alpha\sigma)^n]\exp\left(-\dfrac{Q}{RT}\right) \\ Z = \dot{\varepsilon}\exp\left(\dfrac{Q}{RT}\right) \end{cases} \tag{15}$$

Thus, $Z = A[\sinh(\alpha\sigma)^n]$, and $\sinh(\alpha\sigma) = (Z/A)^{1/n}$. Because:

$$\operatorname{arcsin h}(x) = \ln\left[x + \sqrt{x^2+1}\right], \quad \sigma = \frac{1}{\alpha}\ln\left\{\left(\frac{Z}{A}\right)^{\frac{1}{n}} + \left[\left(\frac{Z}{A}\right)^{\frac{2}{n}} + 1\right]^{\frac{1}{2}}\right\}$$

$$\begin{cases} \sigma = \dfrac{1}{\alpha}\ln\left\{\left(\dfrac{Z}{A}\right)^{\frac{1}{n}} + \left[\left(\dfrac{Z}{A}\right)^{\frac{2}{n}} + 1\right]^{\frac{1}{2}}\right\} \\ Z = \dot{\varepsilon}\exp\left(\dfrac{Q}{RT}\right) \end{cases} \tag{16}$$

In fig. 10, a good agreement between the model predictions and the experimental results at different strain-rates is obtained, which indicates that this model can be used to describe the flow behavior of TP-650 at different strain rates and temperatures.



quasi-static strain-rate: 210s^{-1} strain-rate: 1252s^{-1}

Figure 10. Comparison of model predictions with experimental results at different strain rates

Conclusion

Based on the corrected Arrhenius relation, the Zener-Hollomon parameters are fitted successfully by the test data. The results show that the function in the form of hyperbolic logarithmic expressed by Zener-Hollomon parameters can be used to describe the flow stress equation of the TP-650 high-temperature alloy very well when the alloy deforms at high temperature. Thus, the Z parameters of TP-650 high-temperature alloy and the corresponding constitutive model are given.

Acknowledgment

This paper is supported by the National Nature Science Foundation of China (91016013, 11221202), the Opening Research Fund of State Key Laboratory of Porous Metal Materials (PMM-SKL-1-2012), the Project of State Key Laboratory of Explosion Science and Technology (SKLEST-13-07) and the Program for New Century Excellent Talents in University.

Reference

[1] Davenport, S. B., *et al*., Development of Constitutive Equations for Modeling of Hot Rolling, *Materials Science and Technology, 16* (2000), 5, pp. 539-546
[2] Zhang, H. J., *et al*., A Modified Zerilli-Armstrong Model for IC10 Alloy over a Wide Range of Temperature and Strain Rate, *Material Science and Engineering A, 527* (2009), 1-2, pp. 328-333
[3] Li, W., *et al*., Constitutive Equations of High Temperature Flow Stress Prediction of Al-14Cu-7Ce Alloy, *Material Science and Engineering A, 528* (2011), 12, pp. 4098-4103
[4] Marte, J. S., *et al*., Elevated Temperature Deformation Behavior of Discontinuous-Reinforced Titanium Aluminide Matrix Composites, *Material Science and Engineering A, 346* (2003), l-2, pp. 292-301
[5] Li, W. X., *et al*., Hot Simulation of High Temperature Deformation Al-Fe-V-Si Alloy, *Journal of Central South University of Technology (Natural Science), 31* (2000), l, pp. 56-59

[6] Wongi, W. A., Jonas, J. J., Aluminum Extrusion as a Thermally Activated Process, *Transactions of the metallurgical Society of AIME, 242* (1968), 11, pp. 2271-2280
[7] Banic, M. S., *et al*., Prediction of Heat Generation in Rubber or Rubber-Metal Springs, *Thermal Science, 16* (2012), Suppl. 2, pp. S527-S539
[8] Doric, J. Z., Klinar, I. J., Efficiency Characteristics of a New Quasi-Constant Volume Combustion Spark Ignition Engine, *Thermal Science, 17* (2012), 1, pp. 119-133
[9] Zhang, Z. Q., *et al*., A Simulated Study on the Performance of Diesel Engine with Ethanol Diesel Blend Fuel, *Thermal Science, 17* (2013), 1, pp. 205-216
[10] Zener, C., Hollomon, J. H., Effect of Strain Rate Upon Plastic Flow of Steels, *Journal of Applied Physics, 15* (1944), 1, pp. 22-32

Paper submitted: February 22, 2013
Paper revised: April 20, 2013
Paper accepted: April 25, 2013