INTERCEPTION EFFICIENCY OF PARTICLE LADEN FLOW OVER A FINITE FLAT PLATE IN POTENTIAL FLOW REGIMES

by

Mingliang XIE[a], Qing HE[a], and Wenxi WANG[a,b]*

[a] The State Key Laboratory of Coal Combustion, Huazhong University of Science and Technology, Wuhan, China

[b] Henan Electric Power Survey and Design Institute, Zhengzhou, China

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The direct interception efficiency of particles from potential flows past flat plate is obtained based on the Zhukovsky conversion. The efficiency predictions depend on filter solidity, length of the flat plate, orientation of the cross-section relative to the incoming flow, and particle diameter. The expressions demonstrate that the efficiency for flat plate generally increase with increasing particles, decreasing the orientation of the cross-section relative to the incoming flow.

Key words: *interception efficiency, flat plate, Zhukovsky conversion*

Introduction

The particulate pollution has been a big problem to our environment for a long time. In the Romans times, the hazards to human health had been found, and people begin to use mask to protect the health of human being [1]. With the high speed development of the industrial and transportation, which make the particulate pollution become more serious. In order to capture the fine particles suspended in the air, the researchers develop the fibrous filter.

The fibrous filter is a complex object, which make the efficiency of the entire filter is so difficult to get. As the researchers' continue investigation, the single fiber element model is developed to study the efficiency of fibrous filter, which just consider one fiber of the fibrous filter and ignore the mutual influence between the fibers. The single fiber element model is the basis of the classic filtration theory of aerosol fiber separation in fibrous filters [1], which think filtration is mainly determined by three mechanisms: (1) inertial impaction, (2) direct interception, and (3) Brownian diffusion. All kinds of capture mechanisms are indicated in fig. 1.

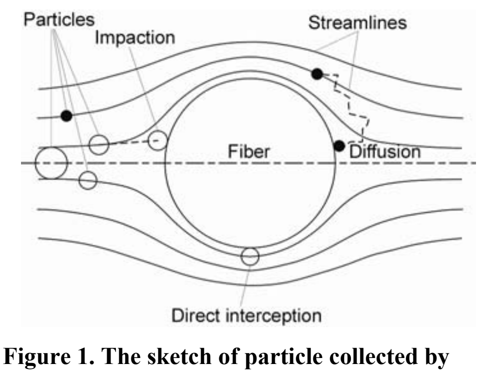

Figure 1. The sketch of particle collected by cylindrical fiber

The mathematical model of the fibers, the shape of the fiber, and the flow regimes over the fibers usually determine the collection efficiency of fibrous filter. There are already many researchers who studied different conditions, such as some researchers that have utilized analytical and numerical procedures to predict ve-

* Corresponding author; e-mail: wwxhust@sina.cn

locity fields and drag for fluid flow around fibers with square or rectangular cross-sections [2-5], Wang carried out numerical simulation to investigate filtration by fibers with elliptical cross [6, 7]. Wang have gotten the exact solution of interception efficiency over an elliptical fiber collector in the potential flow regime [8]. In the present study, starting with a solution for the velocity field around a finite flat plat based on the conformal transformation, which was first used to convert slender sharp tailed bodies into a circle by Zhukovsky [9], then the expressions for predicting the interception efficiency of a particle over a finite flat plate in the potential flow regime is developed.

Flow field computation

In the present study, the deduction of interception efficiency of flat plate is based on the theory of Zhukovsky conversion, which is defined as:

$$z=\frac{1}{2}\left(\zeta+\frac{b^2}{\zeta}\right) \tag{1}$$

The inverse transformation is:

$$\zeta=z+\sqrt{z^2-b^2} \tag{2}$$

The relationship between the co-ordinates in the $z(x, y)$ plane and $\zeta(\xi, \eta)$ plane is:

$$x=\frac{\xi(\xi^2+\eta^2+1)}{\xi^2+\eta^2};\quad y=\frac{\eta(\xi^2+\eta^2-1)}{\xi^2+\eta^2} \tag{3}$$

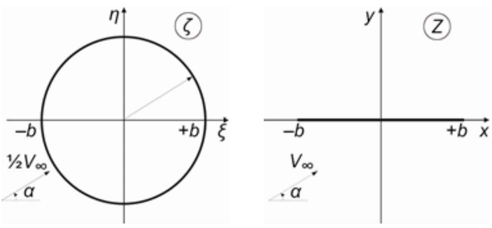

Figure 2. Transformation from circle to finite flat plate

In order to obtain the transformation from circle to finite flat plate with arbitrary orientation of incoming fluid flow (α), a sketch is shown in fig. 2. We take the procedure as follows. First, the equation of a circle with radius b in the $\zeta(\xi, \eta)$ plane is:

$$\xi^2+\eta^2=b^2 \tag{4}$$

The corresponding flat plate equation in $z(x, y)$ plane based on the transformation is:

$$\begin{cases} x=b\cos\varepsilon \\ y=0 \end{cases},\ \varepsilon\in(0, 2\pi) \tag{5}$$

Secondly, the velocity components of incoming flow at the infinity in the (ξ, η) plane is:

$$\left(\frac{\mathrm{d}\chi}{\mathrm{d}\zeta}\right)_{\zeta=\infty}=\left(\frac{\mathrm{d}\chi}{\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}\zeta}\right)_{\zeta=\infty}=\frac{1}{2}v_\infty \mathrm{e}^{-i\alpha} \tag{6}$$

And the complex potential function for the fluid passing through a circle can be written as:

$$\chi(\zeta)=\frac{1}{2}v_\infty\left(\mathrm{e}^{-i\alpha}\zeta+\frac{b^2}{\zeta\mathrm{e}^{-i\alpha}}\right) \tag{7}$$

Based on the inverse transformation eq. (2), the complex potential function in $z(x, y)$ plane is:

$$\chi(z)=\frac{1}{2}v_\infty\left(z+\sqrt{z^2-b^2}\right)\mathrm{e}^{-i\alpha}+\frac{1}{2}v_\infty\frac{b^2}{\left(z+\sqrt{z^2-b^2}\right)\mathrm{e}^{-i\alpha}} \tag{8}$$

According to the definition of complex potential function, the potential function (real part) and stream function (imaginary part) in $z(x, y)$ plane can be written, respectively:

$$\varphi=\frac{1}{2}v_\infty\left(\xi\cos\alpha+\eta\sin\alpha\right)\left(1+\frac{b^2}{\xi^2+\eta^2}\right);\quad \psi=\frac{1}{2}v_\infty\left(\eta\cos\alpha-\xi\sin\alpha\right)\left(1-\frac{b^2}{\xi^2+\eta^2}\right) \tag{9}$$

Interception efficiency

The interception efficiency is the theoretical collection efficiency by a fiber for spherical particles under the assumption that both the particle inertia relative to the flow and the Brownian diffusion are negligible so that they follow air streamlines around the fiber. If the center of particle reaches one particle radius from the surface of a fiber, it is considered to having been collected by the fibers. The diagram is shown in fig. 3. For the flat plate, the define of the interception efficiency is:

$$\eta_R=\frac{L_\infty}{L} \tag{10}$$

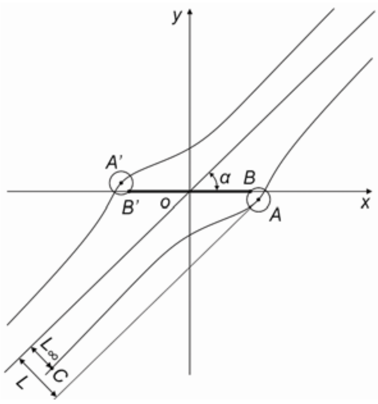

Figure 3. The sketch of fluid flow over a flat plate

where L_∞ is the distance between the streamline that passes through point B, and the line that passes through zero point with same direction (that means the slope of line is tgα) of incoming flow at infinity. L is the distance between the line through zero point with slope tanα and the parallel line that passes through tangent point A on the plate. And d_p is the diameter of the small particle. The corresponding calculation method for the co-ordination for A and B, and the length for L, L_∞ are shown in the following.

The co-ordinates for the tangent point of plate A is $(b, 0)$, then the point B (x_0, y_0) for particles with diameter d_p just pass by the plate can be expressed as:

$$x_0=b+\frac{d_p}{2}\sin\alpha;\quad y_0=-\frac{d_p}{2}\cos\alpha \tag{11}$$

The distance between the point A and the line through the origin with a slope tan (α) is:

$$L-d_p/2=\frac{\left|\mathrm{tg}\alpha\cdot x_1-y_1\right|}{\sqrt{1+\mathrm{tg}^2\alpha}}=b\sin\alpha \tag{12}$$

Any point C (x_∞, y_∞) located at streamline at the infinity will be a line with the slope tg α, and the line can be written:

$$y_\infty=\mathrm{tg}\alpha\cdot x_\infty+K \tag{13}$$

where the K is the intercept. The distance between line through point C (x_∞, y_∞) and line through the origin with the same slope α can be written as:

$$L_\infty = \left|K\cos\alpha\right| = \left|x_\infty \sin\alpha - y_\infty \cos\alpha\right| \tag{14}$$

The streamline at point C is the same as that of point B:

$$\psi(x_\infty, y_\infty) = \psi(x_0, y_0) \tag{15}$$

The co-ordination (x_∞, y_∞) can be represent with (x_0, y_0), then the L_∞ is gained:

$$L_\infty = \left|\frac{\psi(x_0, y_0)}{v_\infty}\right| = \frac{1}{2}\left|(\eta_0\cos\alpha - \xi_0\sin\alpha)\left(1 - \frac{b^2}{\xi_0{}^2 + \eta_0{}^2}\right)\right| \tag{16}$$

where the co-ordinates in the $\zeta(\xi, \eta)$ plane can be calculated based on that of the corresponding $z(x, y)$ plane in the fourth quadrant as shown in fig. 3 as:

$$\begin{aligned} \xi_0 &= x_0 + \sqrt{\frac{\left(x_0{}^2 - y_0{}^2 - b^2\right) + \sqrt{\left(x_0{}^2 - y_0{}^2 - b^2\right)^2 + 4x_0{}^2 y_0{}^2}}{2}} \\ \eta_0 &= y_0 - \sqrt{\frac{-\left(x_0{}^2 - y_0{}^2 - b^2\right) + \sqrt{\left(x_0{}^2 - y_0{}^2 - b^2\right)^2 + 4x_0{}^2 y_0{}^2}}{2}} \end{aligned} \tag{17}$$

The interception efficiency becomes:

$$\eta_R = \frac{L_\infty}{L} = \frac{\left|(\eta_0\cos\alpha - \xi_0\sin\alpha)\left(1 - \dfrac{b^2}{\xi_0{}^2 + \eta_0{}^2}\right)\right|}{2b\sin\alpha + d_p} \tag{18}$$

The formula for interception efficiency is only related to the parameter (b) which determines the length shape of plate, particle size (d_p) and the orientation angle (α). In order to discuss it conveniently, the interception efficiency can be considered as the function including the parameters above with dimensionless formation, it can be obtained:

$$\eta_R = f\left(\frac{d_p}{b}, \alpha\right) \tag{19}$$

Results and discussions

Figure 4 shows the velocity field under five different orientations of the incoming flow (α). The streamlines are divides into two parts which passing around different sides of flat plate. The upstream and downstream branches of this streamline are consequently given by eq. (9) for different quadrant, and the hyperbolae which are orthogonal to potential line linked with the flat plate and which asymptote to the line $y = x\,\mathrm{tg}\alpha$. Moreover, these branches of the dividing streamline are the same for all members of the family of elliptic boundaries.

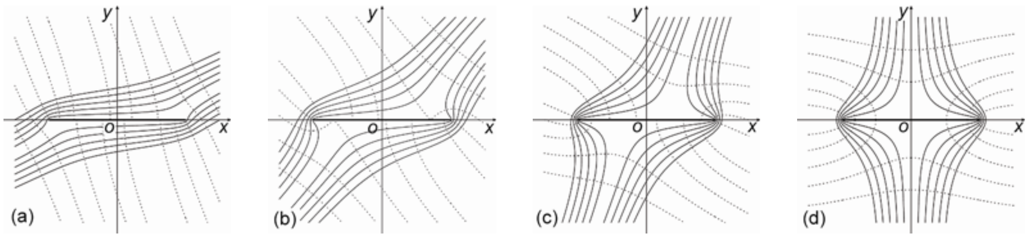

Figure 4. The contours of stream function and potential function for different angle of incoming flow, (a) α = π/8; (b) α = π/4; (c) α = 3π/8; (d) α = π/2

Figure 5 demonstrates the relationship between the interception efficiency η_R and the orientation angle α, which show that the efficiency almost decrease as the orientation of the cross-section relative to the incoming flow angle α. If the orientation of the cross-section relative to the incoming flow $\alpha = 0$, then $\eta_R = 1$, which is a constant! If the orientation of the cross-section relative to the incoming flow $\alpha = \pi/2$, then:

$$\eta_R = \sqrt{1-\left(\frac{1}{1+d_p/2b}\right)^2} \tag{20}$$

It is only related to the ratio of particle size and the length of flat plate, and it is smallest interception efficiency for given finite flat plate in the potential flow regime.

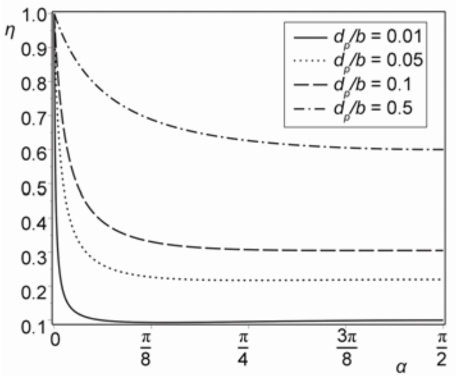

Figure 5. The effect of orientation of incoming flow on the interception efficiency

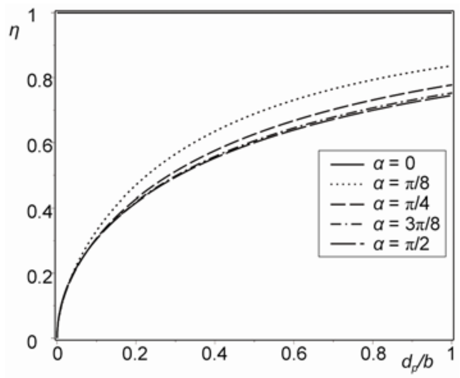

Figure 6. The effect of particle diameter d_p/b on the interception efficiency

Particle size is the crucial factor to determine the interception efficiency. Figure 6 demonstrates the relationship between the interception efficiency η_R and the particle diameter d_p/b. It can be seen that the interception efficiency of flow with bigger size particles is much larger than that of flow with smaller size particles. It can be also found that the interception efficiency increase as the particle diameter. For small particles, the calculated interception efficiency is very low for different orientation of the cross-section relative to the incoming flow orientation (α). And the filtration is not mainly determined by the interception mechanisms.

Conclusions

Based on the Zhukovsky conversion, we get a solution for the velocity field around flat plate cross-sections, and then the expressions for predicting efficiency for particle collection by the interception mechanism is developed for flat plate in the potential flow regime, which is only related to the orientation of incoming flow (α) and the ratio between particle size and the length of finite flat plate (d_p/b). The results show that the direct interception efficiency (η_R) increases monotonically with the increase of the particle diameter (d_p/b), and decreases as the incoming flow orientation angle α increases. The smallest interception efficiency for given particle and flat plate occurs at the orientation of incoming flow is perpendicular with the flat plate.

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