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DETERMINATION OF GAS TEMPERATURE IN THE PLASMATRON CHANNEL ACCORDING TO THE KNOWN DISTRIBUTION OF ELECTRONIC TEMPERATURE

by

Alexander V. GERASIMOV*, Alexander P. KIRPICHNIKOV, and Leonid A. RACHEVSKY

Kazan National Research Technological University, Kazan, Russia

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An analytical method to calculate the temperature distribution of heavy particles in the channel of the plasma torch on the known distribution of the electronic temperature has been proposed. The results can be useful for a number of model calculations in determining the most effective conditions of gas blowing through the plasma torch with the purpose of heating the heavy component. This approach allows us to understand full details about the heating of cold gas, inpouring the plasma, and to estimate correctly the distribution of the gas temperature inside the channel.

Key words: *analytical method, gas temperature, electronic temperature, plasma, channel*

Introduction

The two-temperature plasma model is currently the most suitable to describe the physical and chemical processes occurring in the channel of electric arc or high-frequency torches in those cases where the radiation can still be ignored. Conditions, ensuring the effective heating of the largest possible amount of feed gas, and namely, of the ion-atom component, having the greatest thermal capacity and heating capacity, are of primary interest. The full solution of optimization task of the electric-arc (and high frequence) gas heaters can be obtained only numerically, thus a number of simplified models allowing to evaluate correctly the conditions of the best heating of the gas being fed is of interest for engineering calculations.

One of the most important problems in two-temperature model of plasma is the determination of gas temperature according to the T_e distribution, specified or known from the experiment. Urgency of such a statement is related to the fact that the current methods of direct measurement of the temperature of the atoms and ions are poorly developed, while measurement of electronic temperature (or the temperature close to it) by methods of spectral analysis does not represent difficulties. There is a great deal of data on the T_e distribution in the arc and radio frequency plasma torches, while the main characteristic of the plasma – the temperature of the heavy particles – still remains little explored. In this paper, we present a simple analytical method allowing to calculate in first approximation the temperature of the atoms and ions, using the extensive data of the measured values of T_e.

* Corresponding author; e-mail: gerasimov@kstu.ru

Initial equations

The equation of energy balance of the atoms and ions in a given field $T_e(r, z)$ is:

$$\rho \upsilon c_p \frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right) + \frac{3}{2}\kappa\delta v n_e (T_e - T) \tag{1}$$

An assumption is used that the primary heat removal in the axial direction is by convection, and the cylindrical channel of the plasma torch with the radius R is considered. The coefficients ρ, υ, c_p, λ, κ, δ, v, and n_e in eq. (1) are taken equal to their average over the cross-section and length of the plasma torch values.

Equation (1) using the notations $a = \rho\upsilon c_p$, $b = 3/2\kappa\delta v n_e$ looks as:

$$a\frac{\partial T}{\partial z} - \lambda \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - b(T_e - T) = 0 \tag{2}$$

Solution method

We solve the eq. (2) with the boundary conditions:

$$\frac{\partial T}{\partial r}_{|r=0} = T_{|r=R} = T_{|z=0} = 0 \tag{3}$$

using the method of Kantorovich [1]. That is the solution of eqs. (2), and (3) is sought in the form of $T(r,z) = \sum_{i=1}^{n} f_i(z) g_i(r)$, where $g_i(r)$, $i = 1, 2, ..., n$ – is a system of independent functions, introduced by us, that satisfy the boundary conditions $g_i'(0) = g_i(R) = 0$. Given the nature of the differential operator – the left side of eq. (2), as $g_i(r)$, $i = 1, 2, ..., n$ it is convenient to take the family of zero order Bessel functions of the first kind $J_0(\mu_i r/R)$, $i = 1, 2 ... n$. $J_0(x) = 0$. Obviously, the functions $J_0(\mu_i r/R)$, $i = 1, 2 ... n$. $J_0(x) = 0$ satisfy the necessary boundary conditions.

To reduce the boundary task (2), (3) to a system of ordinary differential equations, let's multiply the left side of eq. (2) in a scalar form by $J_0(\mu_i r/R)$, $i = 1, 2 ... n$, requiring the equality to zero for the received scalar products. The scalar product represents:

$$(F, G) = \int_0^R F(r) G(r) r \mathrm{d}r \tag{4}$$

Due to the above choice of approximating functions, *i. e.*, to the orthogonality of $J_0(\mu_i r/R)$ in the Hilbert space with the scalar product (4), the system of ordinary differential equations from the method of Kantorovich represent n independently solved linear differential equations:

$$\frac{R^2}{2} J_1^2(\mu_j)\left\{ a f_j')z) + \left[b + \lambda \left(\frac{\mu_j}{R}\right)^2 \right] f_j(z) \right\} - b\int_0^R T_e(r) r J_0\left(\mu_j \frac{r}{R}\right) \mathrm{d}r = 0 \quad j = 1, 2, \dots n \tag{5}$$

Solving the eq. (5) with condition $f_j(0) = 0$, we get:

$$f_j(z) = \frac{b A_j}{b + \lambda\left(\frac{\mu_j}{R}\right)^2}\left(1 - \exp\left\{ -\left[\frac{b}{a} + \frac{\lambda}{a}\left(\frac{\mu_j}{R}\right)^2 \right] z \right\} \right)$$

Here

$$A_j = \frac{2}{R^2 J_1^2(\mu_j)} \int_0^R T_e(r) r J_0\left(\frac{\mu_j r}{R}\right) \mathrm{d}r \tag{6}$$

Finally, we have:

$$T(r,z)=\sum_{j=1}^{n}\frac{bA_j}{b=\lambda\left(\frac{\mu_j}{R}\right)^2}\left(1-\exp\left\{-\left[\frac{b}{a}+\frac{\lambda}{a}\left(\frac{\mu_j}{R}\right)^2\right]z\right\}\right)J_0\left(\frac{\mu_j r}{R}\right) \tag{7}$$

Equation (7) solves the problem of the gas temperature distribution in the channel of the plasma torch in the approximation being considered.

Results and discussion

Here are a few typical examples of the use of the formula (7).

Quite often [2] radial distributions of the electronic temperature determined experimentally with the order of accuracy sufficient for engineering assessments can be approximated as:

$$T_e(r)=T_e(0)J_0\left(\frac{ar}{R}\right) \tag{8}$$

Here, $T_e(0)$ is the value of the electronic temperature on the axis, and J_0 – the zero order Bessel function of the first kind. The parameter a appearing in the formula (8) is determined from the equation:

$$J_0(a)=\frac{T_e(R)}{T_e(0)}$$

Here, $T_e(R)$ is the value of the electronic temperature on the channel wall. Condition of the electronic temperature non-negativity means that the a value of the parameter will range from 0 to μ_1, *i. e.*, the first root of the Bessel function $J_0(x)$. In this case:

$$A_j=2\frac{T_e\mu_j J_0(a)}{(\mu_j^2-a^2)J_1(\mu_j)} \tag{9}$$

which implies:

$$T(r,z)=\sum_{j=1}^{n}\frac{2bT_e\mu_j J_0(a)J_0\left(\frac{\mu_j r}{R}\right)}{(\mu_j^2-a^2)J_1(\mu_j)\left[b+\lambda\left(\frac{\mu_j}{R}\right)^2\right]}\left(1-\exp\left\{-\left[\frac{b}{a}+\frac{\lambda}{a}\left(\frac{\mu_j}{R}\right)^2\right]z\right\}\right)$$

Very useful [3] is also a case where the radial dependence $T_e(r)$ in all sections by z has a power- law trend:

$$T_e(r)=T_e(0)\left[1-a\left(\frac{r}{R}\right)^m\right]$$

where m = 1, 2.

In this case, the integrals in the expression for A_j are also reduced to tabular [4], so that after some simple calculations, we obtain:

$$A_j=\frac{2}{\mu_j J_1(\mu_j)}-\frac{2a}{(m+2)J_1^2(\mu_j)}\,{}_1F_2\left(\frac{m+2}{2};\ \frac{m+4}{2},\ 1;\ -\frac{\mu_j^2}{4}\right)$$

whence

$$T(r,z)=\sum_{j=1}^{n}\frac{bJ_0\left(\frac{\mu_j r}{R}\right)}{\left[b+\lambda\left(\frac{\mu_j r}{R}\right)^2\right]}\left[\frac{2}{\mu_j J_1(\mu_j)}-\frac{2a}{(m+2)J_1^2(\mu_j)}\cdot\right.$$

$$\left.\cdot{}_1F_2\left(\frac{m+2}{2};\frac{m+4}{2},1;-\frac{\mu_j^2}{4}\right)\right]\left(1-\exp\left\{-\left[\frac{b}{a}+\frac{\lambda}{a}\left(\frac{\mu_j}{R}\right)^2\right]z\right\}\right) \tag{10}$$

Here ${}_1F_2$ is the generalized hypergeometric function [4]. It is easy to see that in the simplest ($a=0$) case T_e(r) = const., the formula (9) and (10) provide a known expression first obtained in [5]:

$$T(r,z)=T_e\left[1-\frac{I_0\left(r\sqrt{\frac{b}{\lambda}}\right)}{I_0\left(R\sqrt{\frac{b}{\lambda}}\right)}\right]-2T_e\sum_{n=1}^{\infty}\frac{J_0(r,K_n)}{J_1(R,K_n)}\exp\left(-\xi_n\frac{\lambda}{a}z\right)$$

Conclusion

In conclusion, we point out that the fast convergence of the series in formulas (9) and (10) (as is easily shown) can greatly simplify the application of the stated procedure for calculation of low-temperature plasma flows in cylindrical channels.

Nomenclature

- c_p – heat capacity, [Jkg^{-1}K^{-1}]
- I_0 – modified Bessel function of order zero
- J_0 – the Bessel function of order zero
- J_1 – the Bessel function of order one
- k – Boltzmann constant, [JK^{-1}]
- n_e – concentration of electrons in the plasma, [m^{-3}]
- R – radius of the cylindrical channel, [m]
- r – radial co-ordinate, [m]
- T – gas temperature, [K]
- T_e – electronic temperature, [K]
- v – gas velocity, [ms^{-1}]
- z – axial co-ordinate, [m]

Greek symbols

- δ – fraction of the energy transmitted by the electrons to the heavy particles
- λ – coefficient of thermal conductivity, [Wm^{-1}K^{-1}]
- μ_i – j-th root of equation
- ν – frequency of collisions of electrons with atoms and ions, [s^{-1}]
- ρ – density, [kgm^{-3}]

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