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EXTENDING THE BEJAN NUMBER TO A GENERAL FORM

by

Mohamed M. AWAD [a*] ***and Jose L. LAGE*** [b]

[a] Mechanical Power Engineering Department, Faculty of Engineering, Mansoura University, Mansoura, Egypt
[b] Mechanical Engineering Department, Bobby B. Lyle School of Engineering, Southern Methodist University, Dallas, Tex.,USA

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A modified form of the Bejan number, originally proposed by Bhattacharjee and Grosshandler for momentum processes, is obtained by replacing the dynamic viscosity appearing in the original proposition with the equivalent product of the fluid density and the momentum diffusivity of the fluid. This modified form is not only more akin to the physics it represents but it also has the advantage of being dependent on only one viscosity coefficient. Moreover, this simple modification allows for a much simpler extension of Bejan number to other diffusion processes, such as a heat or a species transfer process, by simply replacing the diffusivity coefficient. Consequently, a general Bejan number representation for any process involving pressure-drop and diffusion becomes possible. It is shown that this general representation yields analogous results for any process satisfying the Reynolds analogy (i. e., when Pr = Sc = 1), in which case the momentum, energy, and species concentration representations of Bejan number turn out to be the same.

Key words: *Bejan number, momentum diffusivity, thermal diffusivity, mass diffusivity*

The Bejan number (Be), named after the renowned Duke University Professor Adrian Bejan, seems to have been first introduced by Bhattacharjee and Grosshandler [1] who defined Be as equal to the following dimensionless group:

$$\mathrm{Be} = \frac{\Delta P L^2}{\mu \nu} \tag{1}$$

where ΔP, L, μ, and ν are, the pressure difference, the flow length, the dynamic viscosity, and the kinematic viscosity of the fluid, respectively. They named this dimensionless group "the Bejan number" in view of Bejan's ground breaking contributions to convection heat transfer and the study of scale analysis.

A few years later, Petrescu [2] observed the similarity between another non-dimensional group appearing in heat transfer and eq. (1). Hence, he proposed an extension to the Be for heat transfer, in which case eq. (1) would be written as:

$$\mathrm{Be} = \frac{\Delta P L^2}{\mu \alpha} \tag{2}$$

* Corresponding author; e-mail: m_m_awad@mans.edu.eg

Notice the only difference between the equivalent Be in eqs. (1) and (2) is the replacement of the kinematic viscosity (ν) in eq. (1) by the thermal diffusivity (α) in eq. (2). More recently, Awad [3] observe yet another possible application of Be in the case of species concentration transfer problems, in which case the kinetic viscosity of eq. (1) is replaced by the mass diffusivity (D), yielding yet another form of the Be, namely:

$$\mathrm{Be} = \frac{\Delta P L^2}{\mu D} \tag{3}$$

When considering all three previous forms of Be, eqs. (1)-(3), the unusual aspect of eq. (1) bringing the dynamic and the kinematic viscosities side-by-side in the denominator seems noteworthy. Alternatively, one could express the dynamic viscosity (μ) as a product of the fluid density (ρ) and the kinematic viscosity of the fluid (ν). In this case, the original Be of eq. (1), written for momentum transport problems (Be_ν), would become:

$$\mathrm{Be}_\nu = \frac{\Delta P L^2}{\rho \nu^2} \tag{4}$$

This new form is advantageous for not only having one single viscosity coefficient in it, *i. e.*, the momentum diffusivity, but also for being more akin to the physics it represents – it gives the correct emphasis to the momentum diffusivity in its relationship with the pressure-drop, at same time in which it isolates the pressure-drop effect from the dynamic viscosity effect.

Considering now eq. (4), the extension of Be_ν to heat transfer (Be_α) is easily obtained by replacing the momentum diffusivity (ν) by the thermal diffusivity (α), as:

$$\mathrm{Be}_\alpha = \frac{\Delta P L^2}{\rho \alpha^2} \tag{5}$$

Interestingly, the ratio of these two new Be forms is exactly the Prandtl number (Pr):

$$\frac{\mathrm{Be}_\nu}{\mathrm{Be}_\alpha} = \frac{\alpha^2}{\nu^2} = \left(\frac{1}{\mathrm{Pr}}\right)^2 \tag{6}$$

Now, considering eq. (4) again, the new Be form for species concentration transfer problems (Be_D) would be:

$$\mathrm{Be}_D = \frac{\Delta P L^2}{\rho D^2} \tag{7}$$

And again, a relation between Be_ν and Be_D is shown to be proportional to the Schmidt number (Sc):

$$\frac{\mathrm{Be}_\nu}{\mathrm{Be}_D} = \frac{D^2}{\nu^2} = \left(\frac{1}{\mathrm{Sc}}\right)^2 \tag{8}$$

For the case of Reynolds analogy being valid (Pr = Sc = 1), it is clear that all these three new forms of the Be, namely eqs. (4), (5), and (7), reduce to exactly the same quantity.

Therefore, it would be more natural and broad to define Be in general, simply as:

$$\mathrm{Be}_\delta = \frac{\Delta P L^2}{\rho \delta^2} \tag{9}$$

where δ would be the corresponding diffusivity of the process in consideration.

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