NEW METHODS TO COPE WITH TEMPERATURE ELEVATIONS IN HEATED SEGMENTS OF FLAT PLATES COOLED BY BOUNDARY LAYER FLOW

by

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This paper documents two reliable methods to cope with the rising temperature in an array of heated segments with a known overall heat load and exposed to forced convective boundary layer flow. Minimization of the hot spots (peak temperatures) in the array of heated segments constitutes the primary goal that sets the platform to develop the methods. The two proposed methods consist of: (1) Designing an array of unequal heaters so that each heater has a different size and generates heat at different rates, and (2) Distancing the unequal heaters from each other using an insulated spacing. Multi-scale design based on constructal theory is applied to estimate the optimal insulated spacing, heaters size, and heat generation rates, such that the minimum hot spots temperature is achieved when subject to space constraint and fixed overall heat load. It is demonstrated that the two methods can considerably reduce the hot spot temperatures and consequently, both can be utilized with confidence in industry to achieve optimized heat transfer.

Key words: *constructal design, thermal performance, peak temperature, optimization*

Introduction

The important role played by convection heat transfer in industry has attracted multiple activities for maximizing this heat transfer mode subject to global constraints. In the last few decades, researchers have addressed a large number of problems possessing multi-scale components [1] and have indicated that the effort for maximizing the heat transfer density leads to the design of systems with the best possible configuration. This is indeed the essential concept of constructal theory [2]. It stipulates that for a finite-size system to persist in time, it must evolve in such a way that it provides easier access to the imposed currents that flow through it. In such activities, the main objective is to endow the flow configuration with a certain freedom to change, such that it provides easier and greater access to its currents [3, 4]. One of the factors that lower the access of the heat sources to the convective heat transfer current revolves around

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developing the thermal boundary layer along the wall when the flow sweeps the heat generating surfaces. This produces descend in the convective heat transfer coefficient in the flow direction. It has been demonstrated that this problem is more sensitive in laminar flows where the heat transfer coefficient, h, decreases rapidly along the surface when compared to that in turbulent flows or even with natural convection flows. For example, for a laminar flow with forced convection heat transfer along a flat plate, the convection heat transfer coefficient h, decreases as $x^{-1/2}$ while for natural convection it decreases as $x^{-1/4}$ where x is the distance along the flow direction over a plate measured from the leading edge. Many methods have been proposed in the past to control the development of the boundary layer, for example imposing suction.

It is clear that the consequence of the described descend in the heat transfer coefficient is the rising temperature of the heated segments in the flow direction, which obviously reduces the thermal performance. To cope with the forgoing, one must manipulate the location, distribution or the geometry of the heaters. For example, some researchers have reported the effect of non-uniform placement of discrete heat sources on the thermal performance, experimentally [5-9]. Bhowmik *et al*. [6], for example, performed steady-state experiments to study general convective heat transfer patterns from an in-line four simulated electronic chips in a vertical rectangular channel using water as the working fluid. Other researchers have analyzed the same problems numerically [10-19]. Wang *et al.* [10], for example, performed a numerical simulation to investigate the laminar natural convection air cooling of a vertical plate with five wall-attached protruding, discretely heated integrated circuit packages. Steady, natural convection from a discrete flush-mounted rectangular heat source on the bottom of a horizontal enclosure was studied numerically by Sezai and Mohamad [13]. They found that the rate of heat transfer is not so sensitive to the vertical wall boundary conditions. Sudhakar *et al.* [16] reported the results of a numerical investigation of the problem of finding the optimum configuration for five discrete heat sources, mounted on a wall of a 3-D vertical duct under mixed convection heat transfer, using artificial neural networks. Recently, an analytical analysis was carried out by Hajmohammadi *et al.* [20, 21] to reveal that by placing an insulated spacing placed between two heat sources cooled by in-tube [20] or external [21] laminar forced convection, the rising temperature of the heat sources in the flow direction can be attenuated significantly. Using constructal theory, these authors also quantified the optimal insulated spacing that minimizes the peak temperature to cope with the rising temperature of the heat sources in the flow direction. Although, Hajmohammadi *et al*. [21] have managed to lower the peak temperature of heat sources with unequal length but equal rate of heat generation cooled by laminar forced convection, the present paper proposes an alternative opportunity by designing an array of unequal heaters so that each heater has a different size and generates heat at different rates. The previous method is subsequently coupled with the method proposed in [21] to examine the opportunity of designing unequal heaters (unequal length and unequal heat generation) with insulated spacing to each other.

Physical problem and mathematical formulation

In the majority of thermal engineering systems, the amount of heat transferred from a hot body to a cold fluid is known a priori. In the context of electronics cooling, the primary objective is to control the wall (heaters) temperature, such that the highest temperature in the package (the hot spot) does not exceed a specified allowable value. This issue is more sensible in laminar boundary layer flows over heated segments on plates, where the temperature of the heaters surface rises in the flow direction, as the convection heat transfer coefficient h, decreases in that direction. According to the constructal theory, in order to minimize the peak temperature,

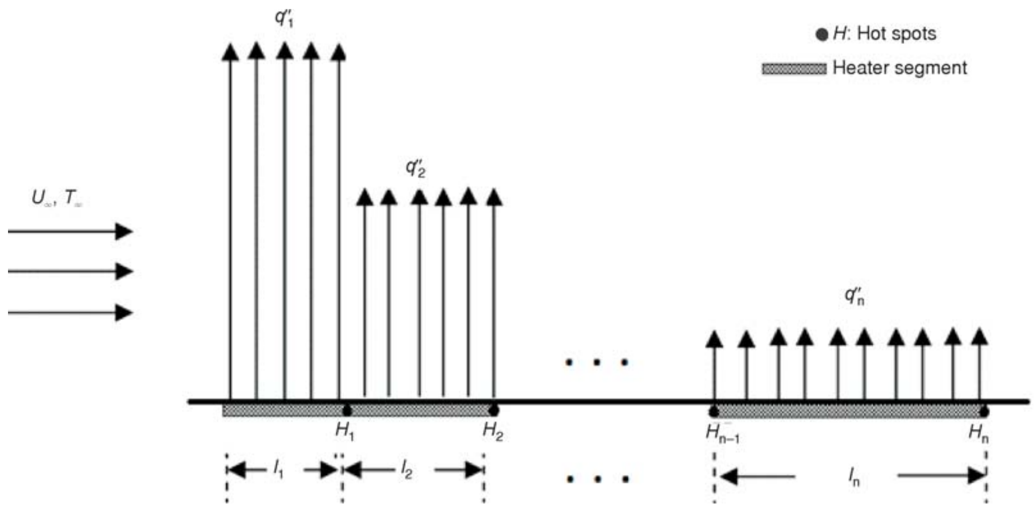

Figure 1. Multi-scale stepwise distribution of heat flux which corresponds to the heated segments with variable rates of heat generation

the entire plate must operate at the same temperature [22, 23]. Based on the so-called strategy, one method might be to manipulate the uniform heat flux distribution over the plate, $q''(x)$, by designing the heat sources (heaters) with different sizes (l_i) and different rates of heat generation ($\dot{q}_\mathrm{i}$). Applying this method, the heater with the highest rate of heat generation is placed at the leading edge of the plate, where h is highest, while the other heat sources are located at the successive downstream regions of the plate, where h diminishes as $h(x) \sim x^{-1/2}$.

Next, consider n unequal heater segments of lengths $l_1, l_2, \ldots$ and l_n mounted on a plate to be heated by a forced convection boundary layer flow as observable in fig. 1. Again, the flow is assumed laminar and 2-D with free stream velocity U_∞ and free stream temperature T_∞. The heaters generate uniform heat rate per unit length at unequal rates of $\dot{q}_\mathrm{i}$, which results in a stepwise distribution of heat flux, $q''(x)$ as shown in fig. 1. The foregoing description can be expressed:

$$q''(x) = \dot{q}_1 m_i, \quad x_i - l_i < x < x_i, \quad m_i = \frac{\dot{q}_i}{\dot{q}_1}, \quad m_i = 1, \; i = 1, 2, \ldots, n \tag{1}$$

where m_i represents the ratio of the heat generation rate of the i-th heater to that of the first heater and x_i is the location of the hot spots as marked with H in fig. 1. Besides, it is assumed that under the circumstances of different heat flux distributions, the total heat removal rate from the plate is fixed, and can be expressed:

$$\sum_{i=1}^{n} \tilde{l}_i \dot{q}_i = \dot{q}_m = \text{constant} \tag{2}$$

where $\dot{q}_m$ is the mean value of heat generation rates, $\dot{q}_i$. In the case of non-uniform heat flux, the temperature of the plate can be simply obtained by the classical relation [4]:

$$T_s(x) - T_\infty = \frac{0.623}{k} \mathrm{Pr}^{-1/3} \mathrm{Re}_x^{-1/2} \int_{\xi=0}^{\xi=x} \left[1 - \left(\frac{\xi}{x}\right)^{3/4}\right]^{-2/3} q'(\xi)\, \mathrm{d}\xi \tag{3}$$

The temperature of the hot spots cab be obtained by using eq. (1) and substituting x by the location of hot spots. In non-dimensional equation form, this corresponds to:

$$\tilde{T}_i = \frac{1}{\sum_{i=1}^{n} m_i \bar{l}_i} \tilde{x}_i^{1/2} \sum_{k=1}^{i} m_k \left\{ I_{1/3,4/3}\left[1-\left(\frac{\hat{x}_{k-1}}{\tilde{x}_i}\right)^{3/4}\right] - I_{1/3,4/3}\left[1-\left(\frac{\hat{x}_k}{\tilde{x}_i}\right)^{3/4}\right]\right\} \tag{4}$$

where $I_{m,n}(x)$ is regularized incomplete beta function defined by:

$$I_{m,n}(x) = \frac{\int_0^x u^{m-1}(1-u)^{n-1}\mathrm{d}u}{\int_0^1 u^{m-1}(1-u)^{n-1}\mathrm{d}u} \tag{5}$$

and the dimensionless variables are given by:

$$(\tilde{\xi}, \tilde{x}, \tilde{s}, \tilde{l}) = \frac{(\xi, x, s, l)}{L}, \quad \tilde{T} = \frac{T_s - T_\infty}{T_L - T_\infty} \tag{6}$$

where L denotes the total length of the heaters, namely the length of the plate when the heaters are flush mounted one after the other onto the plate and no insulated spacing is placed between the heaters.

Optimization procedure

Consider two heaters with length ratios of $\tilde{l}_1$ and $\tilde{l}_2 = 1 - \tilde{l}_1$, and the heat generation rate ratio of $m_2 = \dot{q}_2/\dot{q}_1$. In this case, two hot spots are marked with H_1 and H_2 in fig. 1 related to peak temperatures $\tilde{T}_1$ and $\tilde{T}_2$, respectively. Under these circumstances, the goal is to find the optimal value of $\tilde{l}_1$ and m_2, in such a way that $\tilde{T}_{\max} = \max(\tilde{T}_1, \tilde{T}_2)$ is minimized. Accordingly, the effect of the two parameters on $\tilde{T}_1$ and $\tilde{T}_2$ is next studied. Figure 2 depicts the effect of m_2 on $\tilde{T}_1$ and $\tilde{T}_2$ for a constant value of $\tilde{l}_1$ in harmony with eq. (4). As observed here, with increments in m_2, the temperature $\tilde{T}_1$ decreases and the temperature $\tilde{T}_2$ elevates. This peculiar behavior is due to the fact that by increasing m_2, the amount of $\dot{q}_1$ is reduced, the temperature grows slowly along the first heater and therefore, the level of $\tilde{T}_1$ is reduced. In the same way, the direct consequence of increasing m_2 is augmentation of $\tilde{T}_2$. Shown in fig. 3 is the influence of $\tilde{l}_1$ on $\tilde{T}_1$ and $\tilde{T}_2$ for a con-

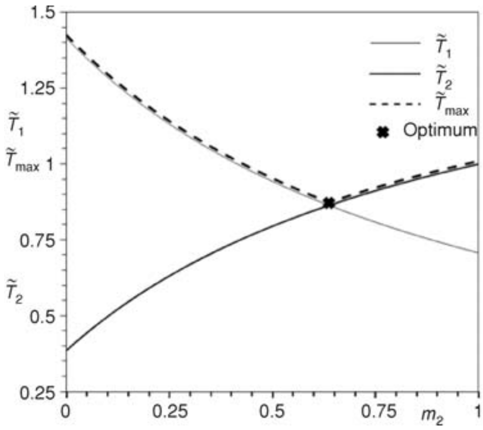

Figure 2. The effect of m_2 on $\tilde{T}_1$ and $\tilde{T}_2$ for a constant value of $\tilde{l}_1 = 0.5$ when two heat sources are designed with the ratio of heat generation rate, m_2

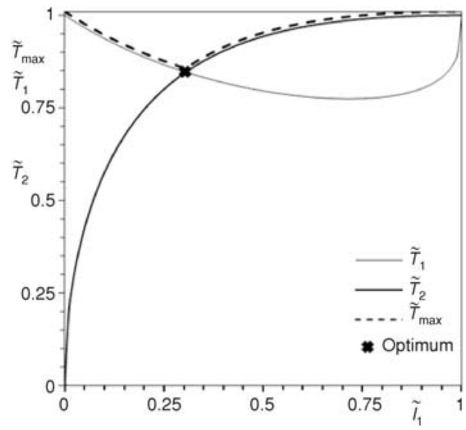

Figure 3. The effect of $\tilde{l}_1$ on $\tilde{T}_1$ and $\tilde{T}_2$ for a constant value of $m_2 = 0.5$ when two heat sources are designed with the length ratio of $\tilde{l}_1$

stant value of m_2 linked to eq. (4). As observed here, with increments in $\tilde{l}_1$, the temperature $\tilde{T}_1$ augments, while $\tilde{T}_2$ exhibits a descendant-ascendant trend. The rise in $\tilde{T}_1$ is clear since the temperature climbs over a longer heater. The reason behind the decreasing-increasing behavior of $\tilde{T}_2$ is that for low values of $\tilde{l}_1$, as a result of increasing $\tilde{l}_1$, a higher proportion of the total heat load is transferred to the fluid near the leading edge of the plate where h is more vigorous. On the contrary, when $\tilde{l}_1$ exceeds a specified value, elevations in $\tilde{T}_1$, which also have a bearing on $\tilde{T}_2$, compensates the so-called reducing trend, and eventually leads to the augmentation of $\tilde{T}_2$.

Switching the attention to the diagrams in the tandem of figs. 2 and 3, one can conclude that the optimal point is reached by intersecting the $\tilde{T}_1$ curve with the $\tilde{T}_2$ curve. Thereby, the governing equation for optimization in this section is $\tilde{T}_2 = \tilde{T}_1$, which in conformity with eq. (4) delivers the following relation:

$$\tilde{l}_1^{\,1/2} = 1 + (m_2 - 1) I_{1/3.4/3} (1 - \tilde{l}_1^{\,3/4}) \tag{7}$$

The optimization process delineated here can be generalized to a larger number of heaters attached to a plate because the extension requires solving a Lagrange system of equations. The resulting system could be solved using a numerical approach of Newton's. It is also clear that, in the case of distancing the heaters with insulated spacing, m_i must be substituted by zero.

Results and discussion

In this section, the numerical results of the proposed optimization procedure that led to the minimum level of the peak temperature are presented and the direct impact of the controlling parameters is investigated. In this regard, the outcome of utilizing the proposed method utilized alone and utilized in combination with the method of distancing the heaters by insulated spacing. It must be mentioned that the insulating segment is in fact an approximation for the existence in such applications of a very low thermal conductivity substrate

Method 1: Unequal heaters without insulated spacing

Figure 4 contains the optimized values for the ratio of the heat generation rate of the two heaters, $m_{2,\mathrm{opt}}$, which minimizes the peak temperature, for various values of the length ratio of the two heated segments, $\tilde{l}_1$. The results mapped in fig. 4, predict larger $m_{2,\mathrm{opt}}$ for higher values of $\tilde{l}_1$. An alternative facet of the optimization is also evident from the decreasing-increasing variation of $\tilde{T}_{\max}$ with respect to $\tilde{l}_1$, *i. e.*, there is an optimal value of $\tilde{l}_1$ that maximizes the efficacy of Method 1 for minimizing the peak temperature, $\tilde{T}_{\max}$. When the number of unequal heaters is increased to three, the optimal values which minimize the peak temperature for various values of $\tilde{l}_1$ are found in fig. 5. The optimization results of figs. 4 and 5 are summarized in tab. 1 when two, three, and infinite number of unequal heaters are utilized under the platform of Method 1. Upon

Table 1. Optimization results demonstrating the optimal size and optimal rate of heat generation associated with two, three, and countless number of heat sources

n	$\tilde{l}_{1,\mathrm{opt}}$	$m_{2,\mathrm{opt}}$	$\tilde{l}_{2,\mathrm{opt}}$	$m_{3,\mathrm{opt}}$	$\tilde{l}_{3,\mathrm{opt}}$	$\tilde{T}_{\max}$	Peak temperature reduction [%]
2	0.269	0.471	0.731	–	–	0.8457	15.43
3	0.11	0.4556	0.3281	0.2856	0.5619	0.7898	21.02
∞	–	–	–	–	–	0.6846	31.54

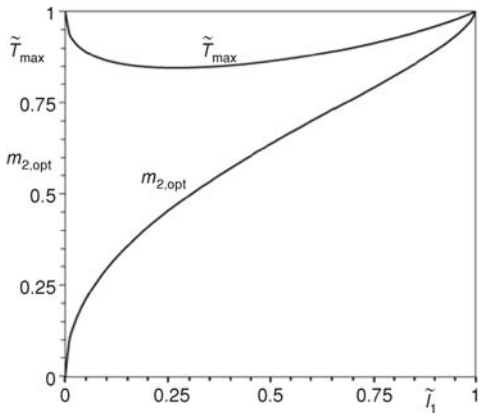

Figure 4. Optimal values for ratio of the heat generation rate of the two heaters with respect to the variation of $\tilde{l}_1$

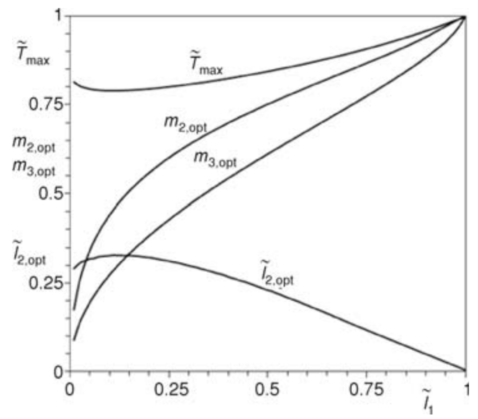

Figure 5. Optimal values which minimize the peak temperature of three heat sources with respect to the variation of $\tilde{l}_1$

comparing the results listed in this table, the advantage of increasing the number of unequal heaters is recognizable. It is also observed that the upper bound for the efficacy of Method 1 for minimizing the peak temperature in the limiting case of $n = \infty$ where the heat flux distribution is nearly continuous, ($q'' \sim x^{-1/2}$), produces a 32% reduction in the peak temperature.

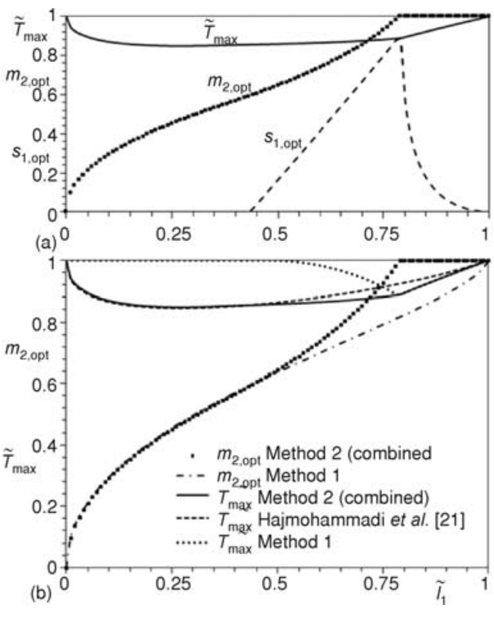

Figure 6. (a) Variations of optimum values of $\tilde{s}_1$, m_2 and their corresponding maximum temperature according to variation of $\tilde{l}_1$ (b) comparison of optimum values of m_2 and maximum temperature in different cases; the results of [21] correspond to the case of distancing the heaters with unequal length, but having the equal rate of heat generation

Method 2: Unequal heaters with insulated spacing

In this section, it is assumed that two unequal heaters with the ratios of $\tilde{l}_1$ and $\tilde{l}_2 = 1 - \tilde{l}_1$ are spaced with an insulated spacing $\tilde{s}_1$ to each other. In addition, the ratio of the heat generation rate at the two heaters corresponds to m_2. In fact, the main goal in this section is to examine whether the combination of the present method (designing the unequal heaters) and the method proposed in [21] (distancing the heaters of unequal length but equal rate of heat generation) is superior to each of the independent methods. To do this, a multi-parametric optimization is performed by relaxing the flux ratio and the insulated spacing. Figure 6(a) shows the variation of the optimal values of $\tilde{s}_1$ and m_2 which minimize $\tilde{T}_{max}$ for a wide range of $\tilde{l}_1$. For further clarification, a one-to-one comparison is made in fig. 6(b) between the optimization results associated with the case of the two isolated methods and the case of combined methods. The comparison over the optimal values of m_2 reveals that; in the case of Method 2, a larger optimal ratio of the heat generation rate at the heaters is predicted in comparison with the case of Method 1 is utilized alone. The comparison over the minimized peak temperature in each cases reveals that the Method 2 is superior than the Method 1 and the method used in [21] for minimizing the peak temperature when $0.45 < \tilde{l}_1 < 0.78$. In addition, it can be also realized that when $\tilde{l}_1 < 0.75$ usage of Method 1 is superior than the method proposed in [21] in lowering the level of peak temperature. In contrast, when $\tilde{l}_1 < 0.75$ the method used in [21] is more efficient in reducing the peak temperature in comparison with Method 1.

Possible future extensions

Two possible extensions can be envisioned for the future. One deals with considering the effect of axial conduction and conjugate heat transfer in the heaters. The other pertains to applying the proposed methods to other types of fluid flows crossing over the heated segments, such as internal flows or cross flows over the heated walls.

Conclusions

The main conclusions that may be drawn from this work are the implementation of two reliable methods to cope with the temperature elevations of heated segments (heaters) under a laminar boundary layer flow. In the first method, it is proposed to design the unequal heaters with different heat generation rates, such that the heater with the highest rate of heat generation is placed at the leading edge of the plate, while the others are placed at downstream regions of the plated. The optimal length ratio of the heaters and the optimal ratio of the heat generation at the heaters which minimize the peak temperature are obtained. According to the optimized results, it is shown that utilizing this method reduces the peak temperature up to 15% when two heaters are utilized, while the peak temperature reduction is reported up to 21% when the method is applied to three heaters. The second scheme consists in distancing the previous unequal heaters with an insulated spacing. It is shown that when the unequal heaters are distanced by insulated spacing, the scheme provides higher efficiency on reducing the peak temperature for certain range of length ratio of the two heaters.

Nomenclature

c_p – specific heat, [$JK^{-1}kg^{-1}$]
h – convection heat transfer coefficient, – [$Wm^{-2}K^{-1}$]
$I_{m,n}$ – incomplete beta function
k – thermal conductivity, [$Wm^{-1}K^{-1}$]
l – heater size, [m]
L – sum of heater lengths, [m]
m – ratio of the heat generation rate
n – number of heaters
Pr – Prandtl number, [$c_p\rho\nu/K^{-1}$]
q', q'' – heat flux, [Wm^{-2}]
$\dot{q}$ – heat transfer per unit of length, [Wm^{-2}]
Re_x – Reynolds number (= $U_\infty x/\nu$)
s – insulated spacing between heaters, [m]
T_i – terminal point temperature of heater number i, [K]
T_{max} – plate maximum temperature, [K]
T_s – plate temperature, [K]
T_∞ – free stream temperature, [K]
U_∞ – free stream velocity, [ms^{-1}]
x – Cartesian co-ordinate, [m]

Greek symbols

ν – fluid kinematic viscosity, [m^2s^{-1}]
ρ – density, [kgm^{-3}]

Subscripts

$i = 0,1,2,\ldots,n$ – heaters index
(~) – dimensionless variables

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