VISCOUS DISSIPATION EFFECT ON THE FLOW OF A THERMODEPENDENT HERSCHEL-BULKLEY FLUID

by

Nabila LABSI [a*], ***Youb Khaled BENKAHLA*** [a], ***M'barek FEDDAOUI*** [b*], ***and Abdelkader BOUTRA*** [a]

[a] Laboratory of Transfer Phenomena, Houari Boumediene University of Sciences and Technology, Algiers, Algeria
[b] Engineering Laboratory of Energy Materials and Systems, National School for Applied Sciences, University Ibn Zohr, Agadir, Morocco

Original scientific paper
DOI:10.2298/TSCI121106080L

The present study concerns the numerical analysis of both hydrodynamic and thermal properties of a Herschel-Bulkley fluid flow in a pipe. The flow, which involves forced heat transfer convection, is steady and takes place within a pipe of circular cross-section with uniform wall temperature. The Herschel-Bulkley model with the Papanastasiou regularization is used and flow index values of 1 and 1.5 are considered. The study focuses on the effect of neglecting both viscous dissipation and temperature dependence of the fluid consistency on its hydrodynamic and thermal properties. For that purpose, we investigate both wall heating (Br < 0) and wall cooling (Br > 0) as well as the exponential temperature dependence of the consistency. The results show that neglecting both of these parameters results in more than a 50% underestimation of the heat transfer due to the viscous nature of this kind of fluid.

Key words: *Herschel-Bulkley fluid, circular pipe, uniform wall temperature, finite volume method, temperature dependent consistency, viscous dissipation*

Introduction

The Herschel-Bulkley model describes the rheological behaviour of viscoplastic non-Newtonian fluids which are encountered in many industrial applications. These fluids are characterized by a yield stress which represents a finite stress required to achieve flow. The relation between the shear rate and the shear stress is non-linear once the latter exceed the value of the yield stress. However, below the yield stress, the material exhibits solid like characteristics. This characteristic is very important in process design and quality assessment for materials [1].

Since heat transfer is particularly important in many industrial applications dealing with viscoplastic fluids flow such as the cosmetic, food, petroleum, paint, and pharmaceutical industries, many researchers have investigated thermal convection in such viscoplastic flows [2-9]. In these studies, the fluid has constant rheological properties while viscous dissipation is either neglected or taken into account. In practice, the rheological properties vary with temperature, a dependency which adds a further complexity to the momentum and energy balance

* Corresponding authors; e-mails: m.feddaoui@uiz.ac.ma, nabilalabsi@yahoo.fr

equations. Therefore, numerical techniques are often needed to obtain a solution. Duvaut and Lions [10] studied analytically the velocity and temperature fields of a Bingham fluid taking into account the variation of the plastic viscosity with temperature. Vinay *et al.* [11] examined the waxy crude oil transportation into a pipeline, where the flowing oil was cooled down due to extreme external temperature conditions. The situation was simulated numerically by considering the transient non-isothermal flows of a Bingham fluid for which plastic viscosity and yield stress were assumed to be a function of temperature. Using a finite volume method, Soares *et al.* [12] studied the developing flow of a Herschel-Bulkley fluid inside a tube for both constant and temperature dependent rheological properties. In the same context, Nouar [13] investigated the combined forced and free convection heat transfer via a Herschel-Bulkley fluid in a horizontal duct heated uniformly with a constant heat flux density. Recently, Peixinho *et al.* [14] undertook an experimental study on the forced convection heat transfer for a thermodependent Carbopol aqueous solution by considering a transitional regime and neglecting viscous dissipation.

Inspection of the literature reveals that most of studies related to viscoplastic fluids consider constant rheological properties while few of them account for temperature dependency. However, viscous dissipation's effect, widely investigated for Newtonian and some non-Newtonian fluids [15-18], has not yet been deeply investigated for viscoplastic fluids with temperature dependent rheological properties, and especially for wall heating (corresponding to negative values of the Brinkman number). In addition, only few studies deal with the case of a shear thickening Herschel-Bulkley fluid ($n > 1$). This motivates the present numerical work, which concerns the developing flow of an incompressible Hershel-Bulkley fluid with a temperature dependent consistency, and two flow index values of 1 and 1.5.

Our purpose is to investigate the effect of both viscous dissipation and temperature dependent viscosity on both the hydrodynamic and the thermal properties of the laminar steady flow in a circular pipe with uniform wall temperature. The consistency of the fluid is assumed to vary exponentially with temperature and both heating and cooling cases are considered.

Mathematical modelling

Governing equations

Let's consider the steady laminar flow of an incompressible viscoplastic fluid obeying the rheological model of Herschel-Bulkley. The fluid has temperature-dependent consistency and flows within a circular pipe of length L and diameter D. By considering the following dimensionless variables for, respectively, the radial and axial co-ordinates, the axial and radial velocities, pressure, and temperature:

$$R = \frac{r}{D}, \qquad X = \frac{x}{D}, \qquad U = \frac{V_x}{V_0}, \qquad V = \frac{V_r}{V_0}, \qquad P^* = \frac{p^*}{\rho V_0^2}, \qquad \theta = \frac{T - T_\mathrm{w}}{T_0 - T_\mathrm{w}} \tag{1}$$

where x and r are the axial and the radial co-ordinates, respectively, V_x and V_r represent the axial and the radial velocity components, respectively, V_0 is the inlet velocity, p^* – the pressure, T_0 and T_w are the inlet and the wall temperatures, respectively, we obtain the dimensionless governing equations:

$$\frac{1}{R}\frac{\partial (RV)}{\partial R} + \frac{\partial U}{\partial X} = 0 \tag{2}$$

$$\frac{1}{R}\frac{\partial(RVU)}{\partial R}+\frac{\partial(UU)}{\partial X}=-\frac{\partial P^*}{\partial X}+\frac{1}{\mathrm{Re}}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\eta_{\mathrm{eff}}R\frac{\partial U}{\partial R}\right)+\frac{\partial}{\partial X}\left(\eta_{\mathrm{eff}}\frac{\partial U}{\partial X}\right)\right]+ \\ +\frac{1}{\mathrm{Re}}\left[\frac{\partial\eta_{\mathrm{eff}}}{\partial R}\frac{\partial V}{\partial X}+\frac{\partial\eta_{\mathrm{eff}}}{\partial X}\frac{\partial U}{\partial X}\right] \tag{3}$$

$$\frac{1}{R}\frac{\partial(RVV)}{\partial R}+\frac{\partial(UV)}{\partial X}=-\frac{\partial P^*}{\partial R}+\frac{1}{\mathrm{Re}}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\eta_{\mathrm{eff}}R\frac{\partial V}{\partial R}\right)+\frac{\partial}{\partial X}\left(\eta_{\mathrm{eff}}\frac{\partial V}{\partial X}\right)\right]+ \\ +\frac{1}{\mathrm{Re}}\left[\frac{V}{R}\frac{\partial\eta_{\mathrm{eff}}}{\partial R}-\eta_{\mathrm{eff}}\frac{V}{R^2}+\frac{\partial\eta_{\mathrm{eff}}}{\partial X}\frac{\partial U}{\partial R}+R\frac{\partial\eta_{\mathrm{eff}}}{\partial R}\frac{\partial}{\partial R}\frac{V}{R}\right] \tag{4}$$

Taking into account viscous dissipation and assuming that the physical properties of the fluid (ρ, C_p, and k) are constant, the dimensionless energy equation can be written:

$$\frac{1}{R}\frac{\partial(RV\theta)}{\partial R}+\frac{\partial(U\theta)}{\partial X}=\frac{1}{\mathrm{Pr\,Re}}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)+\frac{\partial^2\theta}{\partial X^2}\right]+ \\ +\frac{\mathrm{Br}}{\mathrm{Pr\,Re}}\eta_{\mathrm{eff}}\left[2\left\{\left(\frac{\partial V}{\partial R}\right)^2+\left(\frac{\partial U}{\partial X}\right)^2+\left(\frac{V}{R}\right)^2\right\}+\left(\frac{\partial U}{\partial R}+\frac{\partial V}{\partial X}\right)^2\right] \tag{5}$$

The eqs. (2)-(5) result from the basic concepts, *i. e.*, conservation of mass, conservation of momentum which arises from the second law of Newton, and conservation of energy considered from the first law of thermodynamics [19, 20].

Note that the chosen dimensional analysis, for the case of a pipe maintained at uniform wall temperature, generates the following dimensionless numbers: $\mathrm{Re} = \rho V_0^{2-n}D^n/K_0$ (Reynolds number), $\mathrm{Pr} = K_0C_pV_0^{2-n}/kD^{n-1}$ (Prandtl number), and $\mathrm{Br} = K_0V_0^2/k(T_0 - T_w)$ (Brinkman number).

The dimensionless Brinkman number compares the dissipation term with the conduction term in the energy equation. It represents the viscous dissipation function. A negative value of the Brinkman number means that the fluid is heated (heating case) whereas a positive value indicates that the fluid is cooled down (cooling case).

The general model of Herschel-Bulkley fluid is given by the following rheological law which relies the shear stress τ to the shear rate $\dot{\gamma}$:

$$\begin{cases}\tau = K\dot{\gamma}^n+\tau_0 & \tau\ge\tau_0\\ \dot{\gamma}=0 & \tau<\tau_0\end{cases} \tag{6}$$

where K is the consistency of the fluid, n – the flow index, and τ_0 – the yield stress.

This model is appropriate for many fluids and is very suitable since Newtonian ($n = 1$, $\tau_0 = 0$), power law ($0 < n < \infty$, $\tau_0 = 0$), and Bingham ($n = 1$, $\tau_0 \neq 0$) behaviour may be considered as special cases. The model provides a better fit to some experimental data [1, 21].

In order to avoid numerical instabilities in the low shear rate region, some authors [7, 22, 23] recommend the use of the following constitutive law, which was proposed by Papanastasiou [24]:

$$\eta_{\mathrm{eff}}=\dot{\gamma}^{*n-1}+\frac{HB}{\dot{\gamma}^*}[1-\exp(-M\dot{\gamma}^*)] \tag{7}$$

where η_{eff} is the dimensionless effective viscosity, $\dot{\gamma}$ – the dimensionless shear rate, and $M = mV_0/D$ represents the dimensionless exponential growth parameter, while m is the Papanastasiou regularization parameter. To approximate the ideal viscoplastic model, the value of m should be large. Typically, a value of m = 1000 s is considered [22, 23].

Equation (7) highlights another dimensionless number called the Herschel-Bulkley number, which represents the ratio of the yield stress to the nominal shear stress. It is defined by:

$$HB = \frac{\tau_0 D^n}{K_0 V_0^n} \tag{8}$$

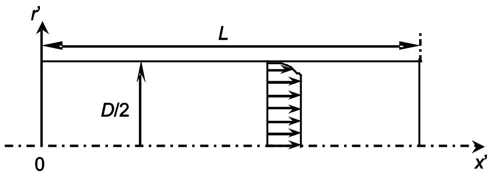

Figure 1. Flow domain geometry

Flow geometry and boundary conditions

The flow domain is a 2-D axisymmetric pipe geometry described by cylindrical co-ordinates as shown in fig. 1. The orthoradial component of the velocity is assumed to be zero. The boundary conditions for the flow domain are:

– at the inlet

Fully developed Dirichlet conditions are prescribed for the radial component of the velocity and for the uniform axial velocity and temperature.

$$U = \theta = 1, \quad V = 0 \tag{9}$$

– at the wall

A no-slip condition is set on the velocity, and Dirichlet condition is imposed for temperature.

$$U = \theta = V = 0 \tag{10}$$

– along the axis of symmetry

$$V = 0, \qquad \frac{\partial \theta}{\partial R} = 0, \qquad \tau_{RX}^{*} = 0 \tag{11}$$

– at the outlet

Fully developed Neumann boundary conditions are set for the temperature, whereas Dirichlet conditions are imposed on the radial velocity component and axial extra-stress tensor:

$$V = 0, \qquad \frac{\partial \theta}{\partial X} = 0, \qquad \tau_{XX}^{*} = 0 \tag{12}$$

Due to the temperature dependent viscosity and the viscous dissipation, the governing equations quoted previously are highly coupled. The set of equations is solved numerically using the finite volume method proposed by Patankar [25]. They are discretized and put in the form of a set of algebraic equations, which is solved using a code based on the SIMPLER algorithm. The grid is non-uniform and consists of 250 nodes in the X direction and 50 nodes in the R direction. The convergence criterion which is based on the residual is set to 10^{-5} for both the velocity components and the temperature and 10^{-6} for pressure.

Results and discussion

Numerical simulations with the Herschel-Bulkley model have been carried out with a parametric study by an analysis of the flow with a temperature dependent fluid viscosity and negligible viscous dissipation. The second part consists of the study of the effect of viscous dissipation on both hydrodynamic and thermal properties of the flow when considering both constant and temperature dependent consistency. The following set of conditions are selected in the computation: HB = 2, Pr = 50, and Re = 20, *i. e.* Pe = 1000 which corresponds to a moderate value (Pe > 100 in order to neglect axial diffusion [26]).

Effect of consistency temperature dependence

In practice, viscoplastic fluids are highly viscous and most of the time, their apparent viscosity is highly temperature dependent. This results in a strong coupling between the momentum and energy equations.

To analyze the effect of a temperature dependent apparent viscosity on hydrodynamic and thermal behaviour of the flow, we consider a temperature dependent consistency $K(T)$ described by the following exponential function used by many authors [12, 13, 27]:

$$K(T) = K(T_0)\exp[-a(T - T_0)] \tag{13}$$

where T_0 is a reference temperature (chosen as the inlet temperature), and a – the temperature coefficient.

Since the present study deals with dimensionless parameters, we substitute the dimensionless temperature of eq. (1) into eq. (13). Thus, the latter can be written as:

$$K^*(\theta) = \exp[-a^*(1-\theta)] \tag{14}$$

where $a^* = a\Delta T$ is the dimensionless temperature-viscosity coefficient and $K^*(\theta) = K(T)/K(T_0)$ – the dimensionless fluid consistency.

The effect of the consistency temperature dependence on the fully developed velocity profile is shown in fig. 2 for both heating ($a^* > 0$) and cooling ($a^* < 0$). Two values of the flow index, n = 1, fig. 2(a), and n = 1.5, fig. 2(b), are considered.

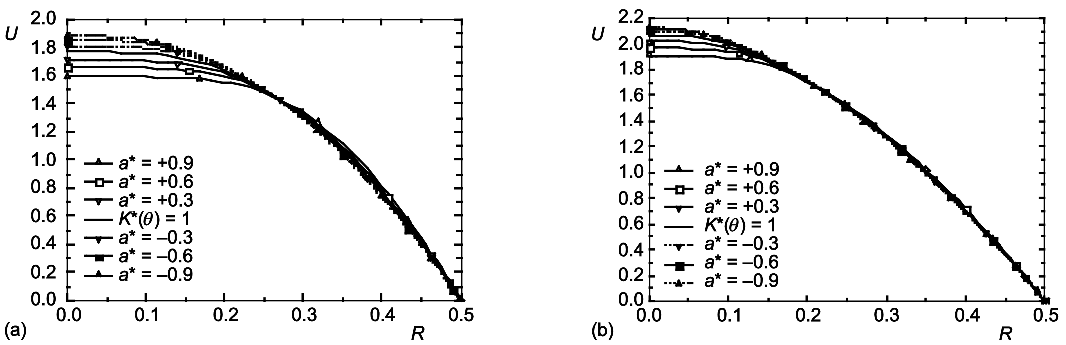

Figure 2. Velocity profiles for different values of the temperature coefficient a^*; Re = 20, Pr = 50, HB = 2, Br = 0; (a) n = 1, (b) n = 1.5

We note that the consistency temperature dependence results in a distortion of the profile in both cases. In fact, a heating (or a cooling) of the wall fluid layers causes a wall lubrication (or a slowing down) of the adjacent fluid layers. As a consequence, a contraction (or a stretching) of the velocity profile takes place in order to conserve the flow rate. In addition, an increase of the dimensionless temperature-viscosity coefficient a^*, from +0.3 to +0.9, leads to an increase in the extent of the unyielded region (plug flow region). This happens since the shear stress at the wall increases due to an increase in the wall velocity. Conversely, when a^* goes from –0.3 to –0.9 for the cooling case, the fluid is yielded and the unyielded region becomes smaller.

A similar behaviour is observed in figs. 3(a) and 3(b), which display the axial distribution of the centreline velocity for $n = 1$ and $n = 1.5$, respectively. For the heating case, the velocity increases from the inlet value $U_C = 1$, reaches a maximum value, then decreases towards its fully developed flow value. The same trend has been reported in [11]. On the other hand, for the cooling case, the centreline velocity increases gradually until it riches it's fully developed flow value. We notice that an increase in the value of a^* leads to a decrease of the centreline velocity. A similar behaviour has been observed by Metivier and Nouar [28], who considered the heating case in a thermodependent Herschel-Bulkley fluid flow in a horizontal plane channel.

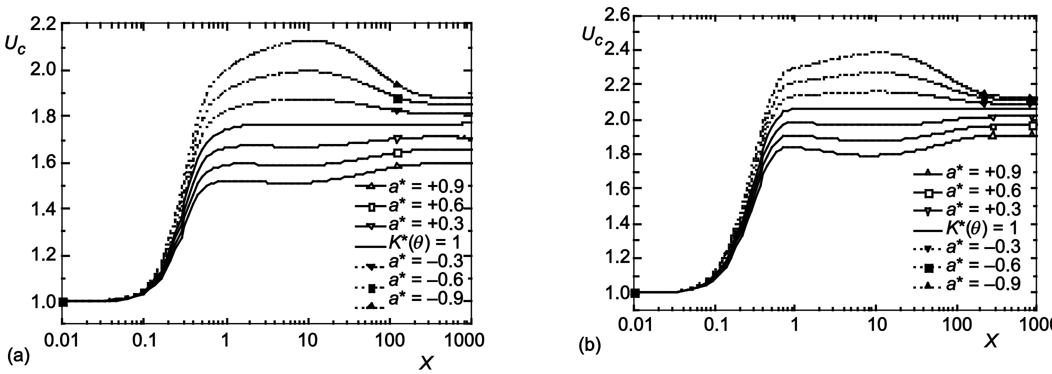

Figure 3. Axial evolution of the centreline velocity for different values of temperature coefficient a^*; Re = 20, Pr = 50, *HB* = 2, Br = 0; (a) *n* = 1, (b) *n* = 1.5

Since a change in apparent viscosity affects the development of the axial velocity profile, it also alters the heat transfer. To enhance our understanding on heat transfer process, the axial evolution of the Nusselt number along the pipe is shown in figs. 4(a) and 4(b) for both $n = 1$ and $n = 1.5$, respectively.

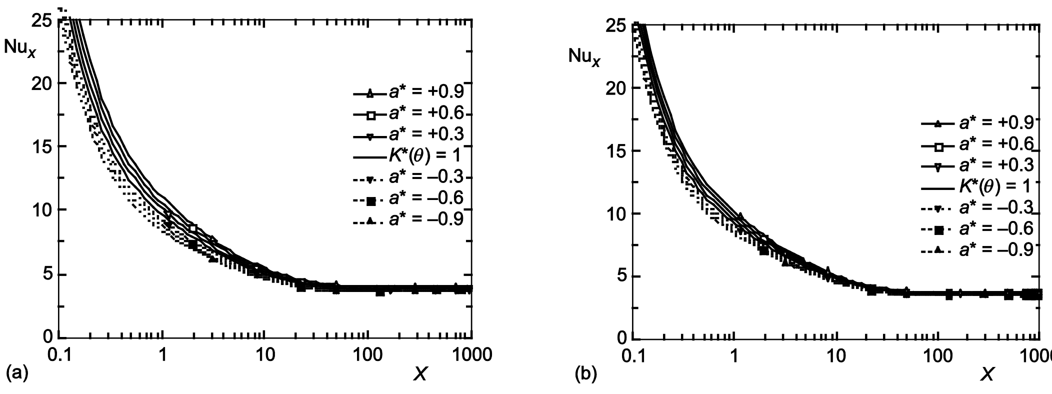

Figure 4. Axial evolution of the Nusselt number for different values of temperature coefficient a^*; Re = 20, Pr = 50, *HB* = 2, Br = 0; (a) *n* = 1, (b) *n* = 1.5

Increasing a^* for the heating case leads to an increase in the Nusselt number values. This may be explained by the fact that the apparent viscosity close to the wall is reduced when a^* rises resulting in an increase of the wall velocity gradient. A similar behaviour has been observed by Nouar *et al.* [29] for the case of a shear thinning Herschel-Bulkley fluid (n = 0.5).

On the other hand, for the cooling case, since the apparent viscosity near the wall increases when the wall is cooled down, the fluid velocity near the wall decreases and the heat transfer is less important. It is also interesting to note that the effect of the consistency temperature dependence is more pronounced at the inlet than in the fully developed region.

Neglecting the temperature dependence of consistency would underestimate the heat transfer by about the values given in tabs. 1 and 2 for n = 1 and n = 1.5 respectively. This deviation is more important for the greatest values of a^*. Indeed, it can reach 15.86% for a^* = +1.8, and 10.06% for a^* = –1.8, for n = 1 and n = 1.5, respectively.

It is of interest to note that the deviation of the Nusselt number calculated along the pipe, from its constant consistency value is more noticeable for n = 1 (Bingham fluid).

Table 1. Deviations of the Nusselt number from its constant consistency value for both heating and cooling cases, for *n* = 1, Br = 0, and for several values of a^*

	Deviation from the case of a constant consistency [%]	
$\|a^*\|$	Heating case (a^* > 0)	Cooling case (a^* < 0)
0.3	5.34	5.75
0.6	6.67	6.41
0.9	9.44	7.04
1.2	9.57	5.36
1.8	15.86	7.91

Table 2. Deviations of the Nusselt number from its constant consistency value for both heating and cooling cases, for *n* = 1.5, Br = 0, and for several values of a^*

	Deviation from the case of a constant consistency [%]	
$\|a^*\|$	Heating case (a^* > 0)	Cooling case (a^* < 0)
0.3	5.97	6.74
0.6	4.91	7.31
0.9	4.90	8.03
1.2	3.97	8.69
1.8	5.30	10.06

Effect of viscous dissipation

Figure 5 shows the effect of the Brinkman number on the axial evolution of the Nusselt number for both n = 1 and n = 1.5 and for both heating (Br < 0) and cooling (Br > 0). For the Bingham fluid case, fig. 5(a), the results are compared to those in [7]. The agreement is good**,** whether or not the viscous dissipation is neglected or is taken into account**,** since the deviation between the two studies does not exceed 3%.

The curves display a sharp decrease of the Nusselt number near the inlet and then it reaches an asymptotic value which corresponds to the fully developed flow. It is interesting to note that for both heating and cooling, this asymptotic value is independent of Brinkman number. The asymptotic value is equal to 10.66 and 9.40 for n = 1 and n = 1.5, respectively when Br ≠ 0. In addition, the heat transfer increases with Brinkman number. As a result, neglecting viscous dissipation leads to underestimate the heat transfer by about 180% and 164% for n = 1 and n = 1.5, respectively. It is also interesting to note that in the case of heating (Br < 0), the curves present a discontinuity. Moreover, due to the change in heat direction [30], negative values of the Nusselt number appear.

In the case of constant apparent viscosity, viscous dissipation affects only the heat transfer. This also has been reported in [7, 9]. However, for a temperature dependent apparent

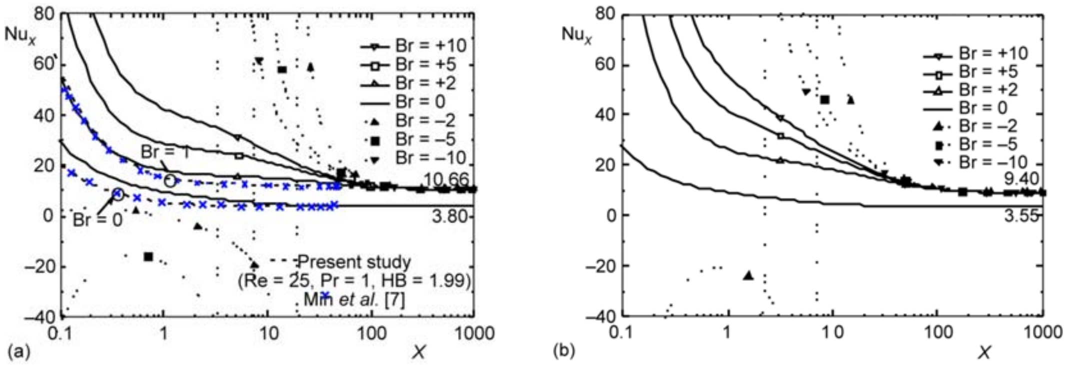

Figure 5. Axial evolution of the Nusselt number for different values of the Brinkman number; Re = 20, Pr = 50, HB = 2, $K^*(\theta)$ = 1; (a) n = 1, (b) n = 1.5

viscosity, viscous dissipation affects both hydrodynamic and thermal properties. This is due to the fact that the energy and momentum equations become strongly coupled.

When both apparent viscosity temperature dependence and viscous dissipation are considered, some authors use the Nahme number instead of the Brinkman number in order to characterize how much viscous dissipation affects the temperature dependent viscosity. This choice depends on the temperature characteristic scale. Indeed, for these authors, the temperature is dimensionlessed against the heat flux. However, in our case, the temperature is dimensionlessed against the temperature difference ($T_0 - T_w$).

In contrast to the case with constant consistency $K^*(\theta)$ = 1, taking into account viscous dissipation when dealing with the case of a temperature dependent viscosity leads to a significant change in the centreline velocity. In fact, as shown in figs. 6(a) and 6(b) for n = 1 and n = 1.5, respectively, an increase of Br leads to a decrease of the centreline velocity for the cooling case.

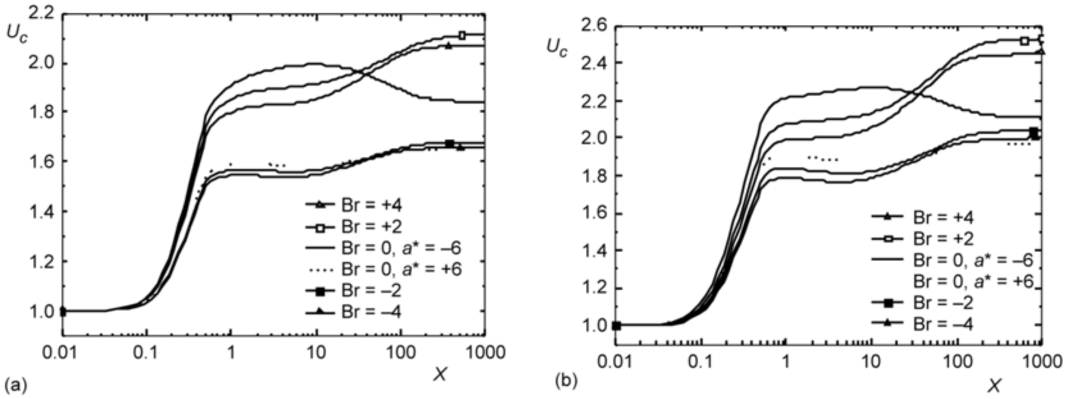

Figure 6. Axial evolution of the centreline velocity for different values of the Brinkman number; Re = 20, Pr = 50, HB = 2, a^* = ±0.6; (a) n = 1, (b) n = 1.5

This behaviour can be explained by the fact that when viscous dissipation is introduced for the cooling case, it intensifies the cooling of the fluid near the wall, especially when the Brinkman number increases (fig. 7 for n = 1 for instance). As a result, the fluid is cooled down at the vicinity of the wall and its velocity decreases since the fluid apparent viscosity increases.

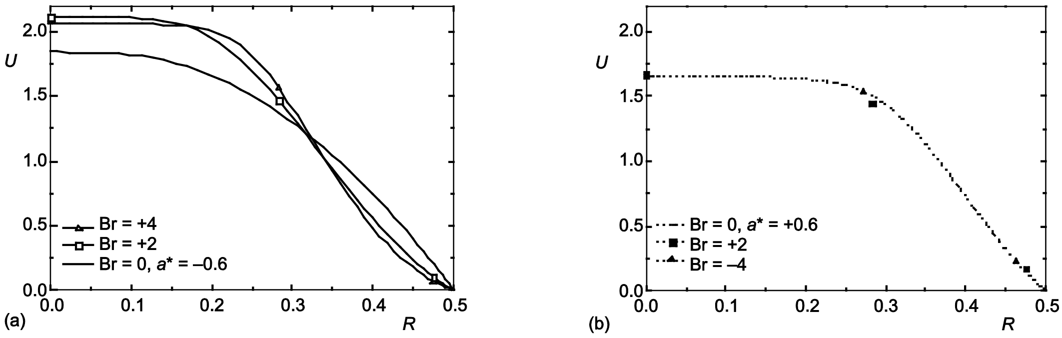

Figure 7. Velocity profile for different values of the Brinkman number; Re = 20, Pr = 50, *HB* = 2, a^* = ±0.6, *n* = 1

At the centre of the pipe where the fluid temperature reaches its maximum values, according to the Brinkman number (fig. 8), the fluid consistency decreases and then the extend of the unyielded region (the core region) increases.

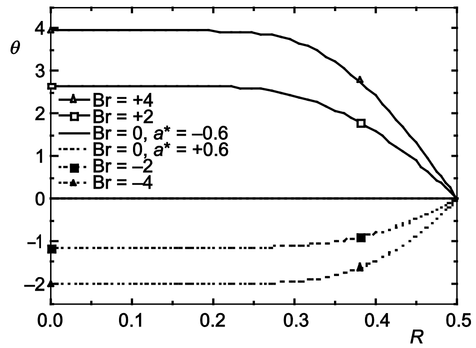

Figure 8. Temperature profiles for different values of the Brinkman number; Re = 20, Pr = 50, *HB* = 2, a^* = ±0.6, *n* = 1

This provides the decrease of centreline velocity. In order to conserve the flow rate, a stretching of the velocity profile is observed between the two regions.

It is interesting to note that when neglecting viscous dissipation (Br = 0), the fluid undergoes only a wall heating or cooling. The intensity of cooling is then, less noticeable comparing to the case when viscous dissipation is taken into account. The fluid velocity near the wall is thus, greater than the one obtained when viscous dissipation is taken into account. Moreover, a similar behaviour is observed for the heating case. In addition, the great values of the wall velocity gradient are observed for the heating case.

The effect of both temperature dependent consistency and viscous dissipation on the thermal properties is illustrated in figs. 9(a) and 9(b). We notice a pronounced decrease of the

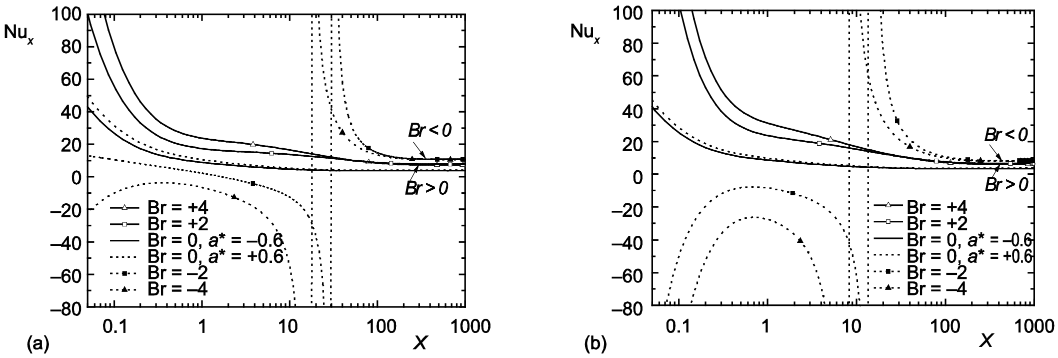

Figure 9. Axial evolution of the Nusselt number for different values of the Brinkman number; Re = 20, Pr = 50, *HB* = 2, a^* = ±0.6; (a) *n* = 1, (b) *n* = 1.5

Nusselt number near the inlet followed by a convergence to an asymptotic value for each value of the coefficient a^*, which is independent of Brinkman number. The asymptotic value corresponds to the case of a fully developed thermal flow and is equal to 10.531 and 8.246 for $n = 1$ and $n = 1.5$, respectively, for the heating case ($a^* = \pm 0.6$, Br < 0) and to 7.5 and 6.5 for $n = 1$ and $n = 1.5$, respectively, for the cooling case ($a^* = -0.6$, Br > 0). It is to note that this result is different from:

– the one obtained for the case of a temperature dependent consistency when viscous dissipation is neglected (fig. 4) and for which we noticed almost no effect of the temperature coefficient a^* in the fully developed region. The asymptotic value of the Nusselt number approaches in this case that obtained for a constant consistency ($K^*(\theta) = 1$) when viscous dissipation is neglected (Br = 0), and
– the one obtained when the temperature dependent consistency is neglected (fig. 5). For this case, the literature [7, 31] reveals only a single asymptotic value of the Nusselt number whatever the value of Br (Br > 0 or Br < 0), which is greater than the one obtained for Br = 0.

In addition, it is interesting to note that heat transfer is improved when Br increases for both heating or cooling; this is more noticeable in the heating case ($a^* > 0$, Br < 0).

Industrial equipment design requires the knowledge of both Nusselt number and pressure drop. These two important parameters are closely related to both the heat transfer coefficient and the wall velocity gradient. Therefore, neglecting either the viscous dissipation or the temperature dependence of the fluid consistency may result in significant errors in equipment design, a fact which is more important when dealing with very viscous fluids such as viscoplastic fluids. The present study reveals that heat transfer is underestimated for about more than 50% when neglecting both the temperature dependence of consistency and viscous dissipation (tab. 3). The deviation reaches 166.41% for the heating case of a Bingham fluid ($a^* = +0.6$ and Br = –2). It is interesting to note that this deviation is more important comparing to the case when only the temperature dependent consistency is taken into account as previously shown in tabs. 1 and 2.

Table 3. Nusselt number deviations in comparison with the case of both constant consistency and negligible viscous dissipation

Br		–2	+2
a^*		+0.6	–0.6
$\Delta \mathrm{Nu}_x = \left(\dfrac{\mathrm{Nu}_{a^*,\mathrm{Br}} - \mathrm{Nu}_{\mathrm{K}(\theta)=1,\mathrm{Br}=0}}{\mathrm{Nu}_{\mathrm{K}(\theta)=1,\mathrm{Br}=0}} \right) 100$	$n = 1$	166.41	52.56
	$n = 1.5$	58.69	51.20

Conclusions

A numerical study based on the finite volume method has been carried on the problem of a laminar forced convection flow of an incompressible Herschel-Bulkley fluid in a circular pipe maintained at uniform wall temperature. The main results can be summarized as follows.

- As expected, viscous dissipation improves significantly the heat transfer since the asymptotic value of the Nusselt number is much larger than the one corresponding to the case where viscous dissipation is neglected, by about 180% and 164% for $n = 1$ and $n = 1.5$, respectively. This effect is noticeable only on the thermal properties of the flow when the consistency is independent of temperature.
- When viscous dissipation is taken into account in the energy equation, the governing equations become coupled and the temperature dependence of the consistency, and consequently the apparent viscosity, intensifies the changes observed on both the hydrodynamic and thermal properties.

In summary, the results show that in order to achieve accurate design of industrial equipments involving non-Newtonian fluids in general and viscoplastic fluids in particular, it is necessary to take into account both viscous dissipation and temperature dependence of the apparent viscosity in the computations since the heat transfer might be underestimated of about more than 50% for both heating and cooling and may reach 166% for $n = 1$ according to the present results.

Nomenclature

a – temperature coefficient, $[\mathrm{K}^{-1}]$
a^* – dimensionless temperature coefficient, $(= a\Delta T)$
Br – Brinkman number, $[= K_0V_0^2/k(T_0 - T_\mathrm{w})]$
C_p – specific heat at constant pressure, $[\mathrm{Jkg^{-1}K^{-1}}]$
D – pipe diameter, [m]
h – heat transfer coefficient, $[\mathrm{Wm^{-2}K^{-1}}]$
K – fluid consistency, $[\mathrm{Pa{\cdot}s^n}]$
K^* – dimensionless fluid consistency
K_0 – fluid consistency at reference temperature, $[\mathrm{Pa{\cdot}s^n}]$
k – fluid thermal conductivity, $[\mathrm{Wm^{-1}K^{-1}}]$
L – pipe length, [m]
m – exponential growth parameter, [s]
Na – Nahme number, $(= aK_0V_0^2/k)$
Nu – Nusselt number, $[= hD/k = (-1/\theta_m)(\partial\theta/\partial R)|_{R=0.5}]$
Pe – Peclet number, (= Re Pr)
Pr – Prandtl number, $(= K_0C_pV_0^{n-1}/kD^{n-1})$
P^* – dimensionless pressure, $(= p^*/\rho V_0^2)$
p^* – pressure, [Pa]
R – dimensionless radial co-ordinate, r/D
Re – Reynolds number, $(= \rho V_0^{2-n}D^n/K_0)$
r – radial co-ordinate, [m]
T – temperature, [K]
T_0 – inlet and reference temperature, [K]
T_w – wall temperature, [K]
ΔT – temperature difference, [K]
U – dimensionless x-component velocity, $(= V_X/V_0)$
V – dimensionless r-component velocity, $(= V_r/V_0)$
V_0 – mean velocity, $[\mathrm{ms^{-1}}]$
V_x – x-component velocity, $[\mathrm{ms^{-1}}]$
V_r – r-component velocity, $[\mathrm{ms^{-1}}]$
X – dimensionless axial co-ordinate, x/D
x – axial co-ordinate, [m]

Greek symbols

Δ – deviation
$\dot\gamma$ – shear rate, $[\mathrm{s^{-1}}]$
$\dot\gamma^*$ – dimensionless shear rate, $\dot\gamma D/V_0$
θ – dimensionless temperature
θ_m – dimensionless mean temperature
η – effective viscosity, $[\mathrm{Pa\ s^n}]$
η_eff – dimensionless effective viscosity, η/K_0
ρ – density of the fluid, $[\mathrm{kg\ m^{-3}}]$
τ – shear stress, [Pa]
τ^*_{XX} – dimensionless axial component of the extra-stress tensor
τ^*_{RX} – dimensionless radial component of extra-stress tensor
τ_0 – yield stress, [Pa]
τ_w – wall shear stress, [Pa]

References

[1] Steffe, J. F., *Rheological Methods in Food Process Engineering*, 2nd ed., Freeman Press, East Lansing Mich., USA, 1992
[2] Piau, J. M., Carbopol Gels: Elastoviscoplastic and Slippery Glasses Made of Individual Swollen Sponges Meso- and Macroscopic Properties, Constitutive Equations and Scaling Laws, *J. Non-Newtonian Fluid Mech.*, *144* (2007), 1, pp. 1-29
[3] Khatyr, R., *et al*., Analytical Approach of Bingham Fluids Forced Convection within a Pipe (in French), *C. R. Mecanique*, *330* (2002), 2, pp. 69-75
[4] Khatyr, R., *et al*., Viscous Dissipation Effects on the Asymptotic Behaviour of Laminar Forced Convection for Bingham Plastics in Circular Ducts, *Int. J. Heat Mass Transfer*, *46* (2003), 4, pp. 589-598
[5] Soares, E. J., *et al*., Heat Transfer to Viscoplastic Materials Flowing Axially through Concentric Annuli, *Int. J. Heat Fluid Flow*, *24* (2003), 5, pp. 762-773
[6] Nascimento, U. C. S., *et al*., Thermal Entry Region Analysis through the Finite Integral Transform Technique in Laminar Flow of Bingham Fluids within Concentric Annular Ducts, *Int. J. Heat Mass Transfer*, *45* (2002), 4, pp. 923-929
[7] Min, T., *et al*., Laminar Convective Heat Transfer of a Bingham Plastic in a Circular Pipe II. Numerical Approach Hydrodynamically Developing Flow and Simultaneously Developing Flow, *Int. J. Heat Mass Transfer, 40* (1997), 15, pp. 3689-3701

[8] Hammad, K. J., Vradis, G. C., Viscous Dissipation and Heat Transfer in Pulsatile Flows of a Yield Stress Fluid, *Int. Comm. Heat Mass Transfer*, *23* (1996), 5, pp. 599-612
[9] Vradis, G. C., *et al*., Entrance Pipe Flow and Heat Transfer for a Bingham Plastic, *J. Heat Mass Transfer*, *36* (1993), 3, pp. 543-552
[10] Duvaut, G., Lions, J. L. Heat Transfer in a Bingham Fluid of a Temperature Dependent Viscosity (in French), *J. Funct. Anal.*, *11* (1972), 1, pp. 93-110
[11] Vinay, G., *et al*., Numerical Simulation of Non-Isothermal Viscoplastic Waxy Crude Oil Flows, *J. Non-Newtonian Fluid Mech.*, *128* (2005), 2-3, pp. 144-162
[12] Soares, M., *et al*., Heat Transfer to Viscoplastic Materials Flowing Laminarly in the Entrance Region of Tubes, *Int. J. Heat Fluid Flow*, *20* (1999), 1, pp. 60-67
[13] Nouar, C., Thermal Convection for a Thermo-Dependent Yield Stress Fluid in an Axisymmetric Horizontal Duct, *Int. J. Heat Mass Transfer*, *48* (2005), 25-26, pp. 5520-5535
[14] Peixinho, J., *et al*., Heat Transfer of a Non-Newtonian Fluid (Carbopol Aqueous Solution) in Transitional Pipe Flow, *Int. J. Heat Mass Transfer*, *51* (2008), 1-2, pp. 198-209
[15] Salah El-Din, M. M., Laminar Fully Developed Mixed Convection with Viscous Dissipation in a Uniformly Heated Vertical Double-Passage Channel, *Thermal Science*, *11* (2007), 1, pp. 27-41
[16] Sharma, P. R., Singh, G., Effects of Ohmic Heating and Viscous Dissipation on Steady MHD Flow Near a Stagnation Point on an Isothermal Stretching Sheet, *Thermal Science*, *13* (2009), 1, pp. 5-12
[17] Etemad, S. G., Mujumdar, A. S., Effects of Variable Viscosity and Viscous Dissipation on Laminar Convection Heat Transfer of a Power Law Fluid in the Entrance Region of a Semicircular Duct, *Int. J. Heat Mass Transfer*, *38* (1995), 12, pp. 2225-2238
[18] Jambal, O., *et al*., Effects of Viscous Dissipation and Fluid Axial Heat Conduction on Heat Transfer for Non Newtonian Fluids in Ducts with Uniform Wall Temperature – Part I: Parallel and Circular Duct, *Int. J. Heat Mass Transfer*, *32* (2005), 9, pp. 1165-1173
[19] Yamaguchi, H., *Engineering and Fluid Mechanics*, Springer, Rotterdam, The Netherlands, 2008
[20] Kay, J. M., Nedderman, R. M., *Fluid mechanics and transfer process*, Cambridge University Press, Cambridge, UK, 1985
[21] Chhabra, R. P., Richardson, J. F., *Non-Newtonian Flow in the Process Industries – Fundamentals and Engineering Applications*, Butterworth Heinemann, Oxfort, UK, 1999
[22] Mitsoulis, E., On Creeping Drag Flow of a Viscoplastic Fluid past a Circular Cylinder: Wall Effects, *Chem. Eng. Sci.*, *59* (2004), 4, pp. 789-800
[23] Mitsoulis, E., Galazoulas, S., Simulation of Viscoplastic Flow past Cylinders in Tubes, *J. Non-Newtonian Fluid Mech.*, *158* (2009), 1-3, pp. 132-141
[24] Papanastasiou, T. C., Flow of Materials with Yield, *J. Rheol.*, *31* (1987), 5, pp. 385-404
[25] Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*, Hemisphere Publishing Co., N. Y., USA, 1980
[26] Nouar, C., *et al*., Laminar Mixed Convection in a Horizontal Annular Duct. Case of Thermodependent Non-Newtonian Fluid, *Eur. J. Mech. B – Fluids*, *19* (2000), 3, pp. 423-452
[27] Javaherdeh, K., Devienne, R., Heat Transfer for Yield Fluids Flow within Cylindrical Pipes: Cooling Case with a Constant Heat Exchange Coefficient (in French), *Int. J. Heat Mass Transfer*, *42* (1999), 20, pp. 3861-3871
[28] Metivier, C., Nouar, C., Linear Stability of the Rayleigh-Bénard Poiseuille Flow for Thermodependent Viscoplastic Fluids, *J. Non-Newtonian Fluid Mech.*, *163* (2009), 1-3, pp. 1-8
[29] Nouar, C., *et al*., Thermal Convection for a Herschel-Bulkley Fluid in the Entrance Region of a Pipe (in French), *Int. J. Heat Mass Transfer*, *37* (1994), 1, pp. 1-12
[30] Aydin, O., Effects of Viscous Dissipation on the Heat Transfer in Forced Pipe Flow. Part 1: both Hydrodynamically and Thermally Fully Developed Flow, *Energ. Convert. and Manag.*, *46* (2005), 5, pp. 757-769
[31] Labsi, N., *et al*., *Hydrodynamic and Thermal Characterization of the Flow of a Herschel-Bulkley Fluid in a Pipe*, Book series Computer-Aided Chemical Engineering, Elsevier, ISBN 978-0-444-53569-6, 28 (2010), pp. 1411-1416

Paper submitted: September 27, 2012
Paper revised: March 25, 2013
Paper accepted: March 31, 2013