DESIGN OF A VERTICAL ANNULUS WITH MHD FLOW USING ENTROPY GENERATION ANALYSIS

by

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Optimal design of a heat exchanger is one of the concerns of energy conversion engineers. In the present work, the mixed convection flow between two vertical concentric pipes with constant heat flux at the boundaries and MHD flow effects is considered. To determine the optimal design for such a heat exchanger, at first, the momentum and energy equations are simplified and solved analytically. Next, using entropy generation analysis and cost analysis, the operational costs due to entropy generation are estimated. It is concluded that with an increase in the Hartmann number, the energy costs increase. In addition, for two small deviations from the base radius ratio (Π = 2) including Π = 19 and Π = 2.1, the changes in the energy cost are calculated. It is found that for Π = 1.9 the energy cost increases by 17.5% while for Π = 2.1 the energy cost is reduced by 13.6%.

Key words: *vertical annulus, magnetic field, mixed convection, entropy generation, cost analysis.*

Introduction

Mixed convection inside a vertical annulus in the presence of magnetic field has many practical applications. Some applications are found in industrial heat exchangers, micro electronic devices, cooling of nuclear reactors, petroleum equipment, and so forth. The optimal design of such equipments is necessary before the manufacturing process to reduce the operating and energy costs. Bejan's entropy generation minimization (EGM) method is a recognized approach to optimize the performance of thermal-fluid devices. This method also can be used to determine the optimum heat exchanger dimensions [1].

Generally, the entropy generation problems are non-linear. Therefore, a helpful way to study of entropy generation through a process with complex equations is the simplification of the governing equations with reasonable assumptions to obtain analytical solutions. In this context, Yilbas [2] examined the entropy generation for a rotating outer cylinder and differentially heated isothermal boundary condition with neglecting the irreversibility induced by viscous dissipation. The author [2] assumed a linear velocity profile in his work. Mahmud and Fraser [3, 4]

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solved analytically the dimensionless entropy generation equation for the flow and heat transfer between two rotating cylinders with isoflux and isothermal boundary conditions. Tasnim and Mahmud [5] considered the fully developed laminar and mixed convection flow in a vertical annulus with circular cross-section and obtained analytical expressions for entropy generation in the annulus. They obtained the optimum radius ratio, at which the entropy generation is minimized. Tasnim and Mahmud [6] considered a fluid with temperature-dependent viscosity and developed their previous work [5]. Mirzazadeh *et al.* [7] studied the entropy generation due to flow and heat conduction of a non-linear viscoelastic fluid between concentric rotating cylinders. Mahian *et al.* [8] investigated analytically the entropy generation between two rotating cylinders using nanofluids with different volume fractions and isoflux boundary conditions. In another work, Mahian *et al.* [9] presented an analytical solution of the second law analysis between two rotating cylinders using nanofluids. They studied the effects of uncertainties in the models presented for thermophysical properties of nanofluids on entropy generation.

The study of flow and heat transfer in a closed cavity or a channel in the presence of MHD flow is important because of engineering applications such as MHD micropumps, micro electronic devices, electronic packages, cooling of nuclear reactors, and MHD marine propulsion [10]. Here, several works in which the effects of MHD flow on entropy generation for various flows and geometries are investigated, briefly have been reviewed. Salas *et al.* [11] and Ibanez *et al.* [12] analysed the second law for MHD induction devices, such as electromagnetic pumps, and electrical generators. Mahmud *et al.* [13] examined the entropy generation due to mixed convection in a channel made of two parallel plates. Later on, Tasnim *et al*. [14] solved the same problem using porous media. Mahmud and Fraser [15] analytically investigated the entropy generation due to mixed convection-radiation interaction in a vertical porous channel in the presence of MHD flow. In another paper, Mahmud and Fraser [16] studied the problem of entropy generation in a porous cavity with laminar natural convection and MHD flow. Mahmud and Fraser [17] presented a general equation for entropy generation for a single-plate thermoacoustic system, which is subjected to a constant magnetic field. Ibanez and Cuevas [18] considered a stationary buoyant MHD flow of a liquid metal immersed in a MHD flow through a vertical rectangular duct. They obtained the optimum conductance ratio of the wall in which the entropy generation is minimized. The effects of slip and Joule dissipation on the entropy generation in a single rotating disk in the presence of MHD flow are investigated by Arikoglu *et al.* [19]. Aiboud and Saouli [20] applied the analysis of entropy generation for a viscoelastic MHD flow over a stretching surface. Recently, Mahian *et al.* [21] investigated the entropy generation between two isothermal rotating cylinders in the presence of magnetic field. They revealed that the entropy generation increases with an increase in MHD flow. Mahian *et al.* [22] studied the effects of nanofluids on entropy generation between two cylinders in the presence of MHD flow. They obtained the conditions in which using nanofluids results in a decrease in entropy generation.

In the present work, the energy costs of a vertical annulus heat exchanger with MHD flow and constant heat flux at the boundaries in different conditions are determined by using Bejan' EGM method.

Mathematical modelling

Analysis of the First law of thermodynamics

A steady, laminar and fully developed mixed convection flow of a Newtonian, incompressible fluid is considered where the fluid enters a vertical annulus with length of L and inlet velocity U_0 and an inlet temperature T_0. In practical situations, a differential pressure transmitter can be installed to measure the pressure losses through the heat exchanger. A constant heat flux is imposed on the boundary of the inner pipe. The cooling fluid flows inside the annulus and cools the surface of the inner pipe and a portion of the heat is dissipated to the surrounding based on the energy balance. The heat exchanger is subjected to a transverse magnetic field with constant strength of B_0. The schematic of problem is indicated in fig. 1.

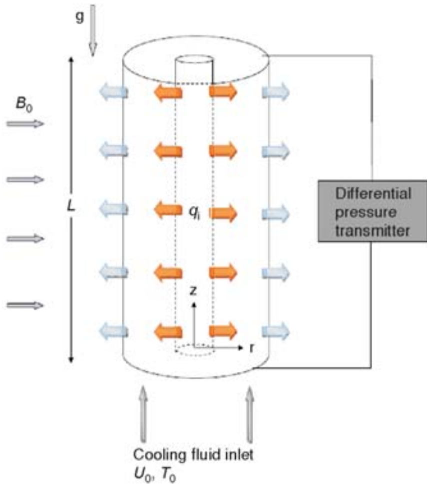

Figure 1. Schematic of heat exchanger considered in the study

The governing equations for 2-D flow in the heat exchanger can be written in general as:

– continuity

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z} = 0 \tag{1}$$

– r-momentum

$$\rho\left(V_r \frac{\partial V_r}{\partial r} + V_z \frac{\partial V_r}{\partial z}\right) = -\frac{\partial P}{\partial r} + \mu\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} - \frac{V_r}{r^2} + \frac{\partial^2 V_r}{\partial z^2}\right) \tag{2}$$

– z-momentum

$$\rho\left(V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r}\frac{\partial V_z}{\partial r} + \frac{\partial^2 V_z}{\partial z^2}\right) - \rho g - \sigma B_0^2 V_z \tag{3}$$

– energy

$$\rho C_p\left(V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] + \sigma B_0^2 V_z^2 \tag{4}$$

As mentioned before, the flow is assumed as hydrodynamically and thermally fully developed; hence, using the order of magnitude the radial velocity (V_r) can be neglected in comparison with the axial velocity (V_z). With this assumption, the continuity equation reduces to $\partial V_z/\partial z = 0$. Therefore, using the Boussinesq approximation in the buoyancy term, and neglecting the axial effects (see refs. [5, 6]), the simplified dimensionless equations can be expressed as:

$$\frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} = -\frac{\text{Gr}}{\text{Re}}\theta + M^2 U + \frac{\partial P^*}{\partial Z} \tag{5}$$

$$\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R}\frac{\partial \theta}{\partial R} = 0 \tag{6}$$

where the dimensionless parameters are defined as:

$$R = \frac{r}{r_o}, \quad U = \frac{V_z}{U_0}, \quad \text{Re} = \frac{U_0 r_o}{\upsilon}, \quad \lambda = \frac{r_i}{r_o}, \quad P^* = \frac{P}{U_0\mu} r_o, \quad Z = \frac{z}{r_o}, \quad M = \sqrt{\frac{\sigma}{\mu}} B_0 r_0 \tag{7}$$

In the above relations M is the Hartmann number, and σ – the electrical conductivity of fluid. The subscripts of i and o indicate the characteristics of inner and outer pipes, respectively. The Grashof number and dimensionless temperature are defined as:

$$\mathrm{Gr} = \frac{g\beta q_\mathrm{i} r_o^4}{k\upsilon^2}, \quad \theta = \frac{T - T_0}{\dfrac{q_\mathrm{i} r_\mathrm{o}}{k}} \tag{8}$$

The velocity boundary conditions with no slip assumption are written as:

$$\begin{aligned} R = \lambda \Rightarrow U = 0 \\ R = 1 \Rightarrow U = 0 \end{aligned} \tag{9}$$

The thermal boundary conditions are given by:

$$\begin{aligned} R = \lambda \Rightarrow \frac{\partial\theta}{\partial R} = -1 \\ R = 1 \Rightarrow \frac{\partial\theta}{\partial R} = f(q_\mathrm{i}) \end{aligned} \tag{10}$$

Analysis of the Second law of thermodynamics

Entropy generation rate in the presence of MHD flow can be written as:

$$\dot{S}_{gen}^{m} = \frac{k}{T_0^2}[\nabla T]^2 + \frac{\mu}{T_0}\phi + \frac{1}{T_0}[(J - QV)(E + V \times B)] \tag{11}$$

where

$$J = \sigma(E + V \times B) \tag{12}$$

In the above equations, ϕ is viscous dissipation, J – the electric current, Q – the electric charge density, V – the velocity vector, E – the electric field, and B – the magnetic induction. Neglecting QV in comparison with J and disregarding E in comparison with $V \times B$, the relation (11) reduces to the equation:

$$\dot{S}_{gen}''' = \frac{k}{T_0^2}\left(\frac{\partial T}{\partial r}\right)^2 + \frac{\mu}{T_0}\left(\frac{\partial V_\mathrm{z}}{\partial r}\right)^2 + \frac{\sigma B_0^2}{T_0}V_z^2 \tag{13}$$

The dimensionless entropy generation rate is determined as:

$$N_\mathrm{S} = \frac{\dot{S}_{gen}'''}{\dfrac{\mu U_0^2}{T_0 r_o^2}} = \frac{\Omega}{\mathrm{Br}}\left(\frac{\partial\theta}{\partial R}\right)^2 + \left(\frac{\partial U}{\partial R}\right)^2 + M^2 U^2 = N_\mathrm{H} + N_\mathrm{F} + N_\mathrm{M} \tag{14}$$

where N_H, N_F, and N_M on the right hand of the equation are the irreversibilities due to heat transfer, fluid friction, and magnetic field, respectively. Also, the Brinkman number (Br) and the parameter Ω are defined as:

$$\mathrm{Br} = \frac{U_0^2 \mu}{q_\mathrm{i} r_\mathrm{o}}, \quad \Omega = \frac{q_\mathrm{i} r_\mathrm{o}}{k T_0} \tag{15}$$

The irreversibility distribution can be obtained using the definition of Bejan number (Be) that is the ratio of entropy generation due to heat transfer to the overall entropy generation as [23]:

$$\mathrm{Be} = \frac{N_\mathrm{H}}{N_\mathrm{H} + N_\mathrm{F} + N_\mathrm{M}} \tag{16}$$

The average volumetric entropy generation number is given by:

$$N_{\mathrm{S,\,ave}} = \frac{1}{\forall}\int_{\forall} N_{\mathrm{S}} \mathrm{d}\forall \tag{17}$$

Solution of the problem

Velocity and temperature fields

The eqs. (5) and (6) are coupled *via* the buoyancy term in the momentum equation.

Therefore, first, the energy equation must be solved. The solution of the energy equation is:

$$\theta(R) = -\lambda \ln R + C \tag{18}$$

where C is the constant of integration. In this stage, the constant C cannot be obtained because of the isoflux boundary conditions. The velocity field is obtained using the temperature distribution and solving the related differential equation as:

$$U(R) = \Gamma_5 K_0(MR) + \Gamma_6 I_0(MR) - \frac{\left(\dfrac{\partial P^*}{\partial Z}\right) + \left(\dfrac{\mathrm{Gr}}{\mathrm{Re}}\right)\lambda \ln R}{M^2} + \frac{\left(\dfrac{\mathrm{Gr}}{\mathrm{Re}}\right)}{M^2} C \tag{19}$$

where the constants Γ_5 and Γ_6 are equal to:

$$\begin{aligned} \Gamma_5 &= -\frac{\dfrac{\mathrm{Gr}}{\mathrm{Re}} C[I_0(M\lambda) - I_0(M)] - \dfrac{\partial P^*}{\partial Z}[I_0(M\lambda) - I_0(M)] + \dfrac{\mathrm{Gr}}{\mathrm{Re}} \lambda \ln \lambda I_0(M)}{M^2[K_0(M) I_0(M\lambda) - K_0(M\lambda) I_0(M)]} \\ \Gamma_6 &= -\frac{\dfrac{\mathrm{Gr}}{\mathrm{Re}} C[K_0(M\lambda) - K_0(M)] - \dfrac{\partial P^*}{\partial Z}[K_0(M\lambda) - K_0(M)] + \dfrac{\mathrm{Gr}}{\mathrm{Re}} \lambda \ln K_0(M)}{M^2[K_0(M) I_0(M\lambda) - K_0(M\lambda) I_0(M)]} \end{aligned} \tag{20}$$

The unknown constant C can be determined using the continuity equation that is:

$$\int_{r_i}^{r_o} \rho V_z(z) 2\pi r \mathrm{d}r = \rho U_0[\pi(r_0^2 - r_i^2)] \tag{21}$$

which in dimensionless form is:

$$2\int_{\lambda}^{1} U(R) R \mathrm{d}R = 1 - \lambda^2 \tag{22}$$

By substituting the velocity profile from eq. (19) into the above relation and solving the equation, the constant C is calculated. The expression obtained for C is long, therefore to save the space, the constant C is not presented here, but it is a function of involving parameters in the problem as:

$$C = h\left(M, \lambda, \frac{\mathrm{Gr}}{\mathrm{Re}}, \frac{\partial P^*}{\partial Z}\right) \tag{23}$$

Entropy generation

The local entropy generation is obtained using eq. (14) and the velocity and temperature distributions as:

$$N_S = \frac{\Omega}{\text{Br}}\left(\frac{-\lambda}{R}\right)^2 + \left(-\Gamma_5 M K_1(MR) + \Gamma_6 M I_1(MR) - \frac{\lambda\left(\frac{\text{Gr}}{\text{Re}}\right)^2}{M^2 R}\right) + \\ +M^2\left[\Gamma_5 K_0(MR) + \Gamma_6 I_0(MR) - \frac{\frac{\partial P^*}{\partial Z} + \frac{\text{Gr}}{\text{Re}}\lambda \ln(R)}{M^2} + \frac{\frac{\text{Gr}}{\text{Re}}}{M^2}C\right]^2 \quad (24)$$

The average volumetric entropy generation number is calculated based on eq. (17) as:

$$N_{\text{S, ave}} = \frac{2}{1-\lambda^2}\int_{\lambda}^{1} N_{\text{S}} R \text{d}R \quad (25)$$

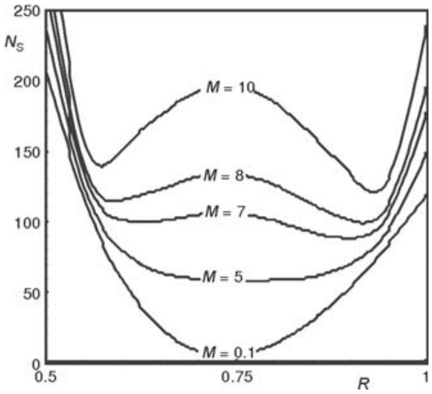

Figure 2. Effects of MHD flow on entropy generation number for Gr/Re = 10 and Ω/Br = 10

Results and discussion

Local entropy generation

The effects of Hartmann number on local entropy generation for Gr/Re =10, dP^*/dZ = –0.1 and Ω/Br = 10 are shown in fig. 2. It is seen that an increase in the Hartmann number leads to an increase in entropy generation number. The entropy generation number is greater near the walls (especially near the inner wall) due to higher gradients of temperature and velocity. The effects of parameter Gr/Re on the entropy generation and Bejan numbers for M = 1 and Ω/Br = 20 are displayed in fig. 3. It is observed that the entropy generation number has an unpredictable trend in

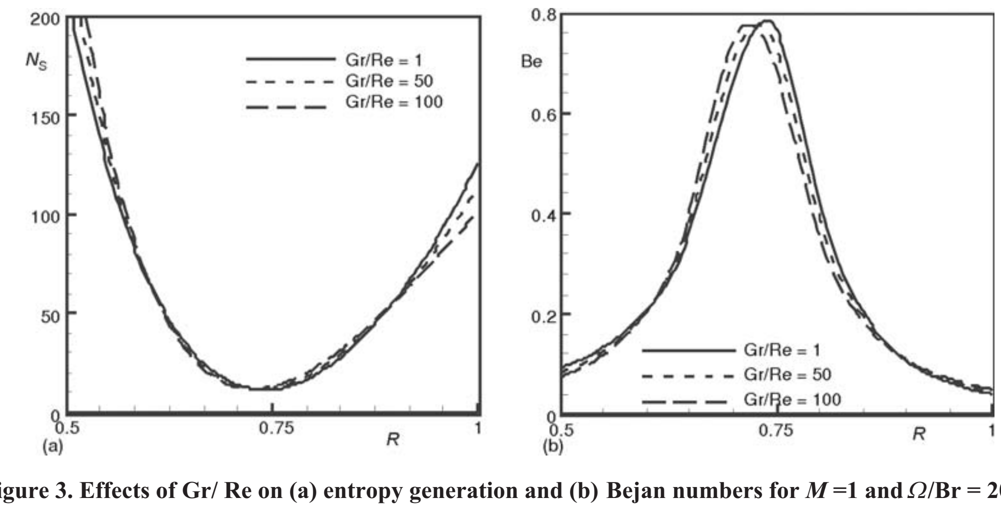

Figure 3. Effects of Gr/ Re on (a) entropy generation and (b) Bejan numbers for M =1 and Ω/Br = 20

the gap. The interactions among the viscous effects near the walls, the reduction of heat flux along the radial distance and buoyancy forces make this irregular behaviour. The effects of Gr/Re on the Bejan number are drawn in fig. 3(b). It is found that the Bejan number is approximately maximized in the middle of the annulus. At this point, the contribution of viscous effects to entropy generation is lowest. This maximum point moves towards the outer cylinder with a decrease in the force of natural convection and heat flux.

Design and cost analysis

In this section, by using the Bejan's EGM method and cost analysis the operational costs due to entropy generation in the heat exchanger at different conditions are determined. The operational cost due to entropy generation is reduced by optimal design and hence leads to the most saving in energy consumption cost.

Sahin *et al.* [24] calculated the total cost of irreversibility for the flow in a pipe in the absence of magnetic field as:

$$\dot{C} = C_{\rm H} T_0 (\dot{S}_{\rm gen})_{\rm H} + C_{\rm F} T_0 (\dot{S}_{\rm gen})_{\rm F} \tag{26}$$

In the above relation, $\dot{C}$ is the total cost of irreversibility [$ per day], $C_{\rm H}$ – the unit cost of irreversibility due to heat transfer, $C_{\rm F}$ – the unit cost of irreversibility due to fluid friction, $\dot{S}_{\rm gen}$ – the entropy generation. In this work, eq. (26) is developed to the following relation in the presence of MHD flow:

$$\dot{C} = C_{\rm H} T_0 (\dot{S}_{\rm gen})_{\rm H} + C_{\rm F} T_0 (\dot{S}_{\rm gen})_{\rm F} + C_{\rm M} T_0 (\dot{S}_{\rm gen})_{\rm M} \tag{27}$$

Here, it is assumed that $C_{\rm E} = C_{\rm H} = C_{\rm F} = C_{\rm M} = 0.2$, where $C_{\rm E}$ is the unit cost of irreversibility in the system. Therefore, the above relation for the heat exchanger with length of 1 m can be rewritten as:

$$\dot{C} = C_{\rm E} T_0 \left(\frac{\mu U_0^2}{T_0 r_0^2} \right) \forall N_{\rm S,\,ave} = \pi C_{\rm E} \mu U_0^2 (1 - \lambda^2) N_{\rm S,ave} \tag{28}$$

The above relation in terms of mass flow rate and (inverse of λ) can be expressed as:

$$\dot{C} = \frac{C_{\rm E} \dot{m}^2 \upsilon \Pi^2}{\pi \rho r_o^4 (\Pi^2 - 1)} N_{\rm S,\,ave} \tag{29}$$

where ρ and υ are density and kinematic viscosity of the working fluid at the inlet temperature. If the base radius ratio is 2 (Π = 2) , using the relation (30) one can calculate the amount of changes in energy cost, $\dot{C}_{\rm r}$, in comparison with another radius ratio:

$$\dot{C}_{\rm r} = \frac{\dot{C})_{\Pi=2} - \dot{C})_{\Pi}}{\dot{C})_{\Pi=2}} \cdot 100 \tag{30}$$

Now as a numerical example, the engine oil with mass flow rate of 0.85 kg/s is considered as the cooling fluid in the heat exchanger where the outer radius is 1. 5 cm.

The total cost due to irreversibility, $\dot{C}$, is presented in fig. 4 for Gr/Re = 0.1 and Gr/Re = 50 , three different Hartmann numbers and the radius ratio between 1.5 and 6, where it is assumed the system works 10 hours in a day. It is observed that the total cost of irreversibility for the system in one day is approximately 0.5-10 $ depends on the radius ratio of the heat exchanger. The total cost of irreversibility increases with increases in the Hartmann number because the average volumetric entropy generation number increases with the increase of *M*. It is also seen that with an increase in the radius ratio, the total costs due to entropy generation de-

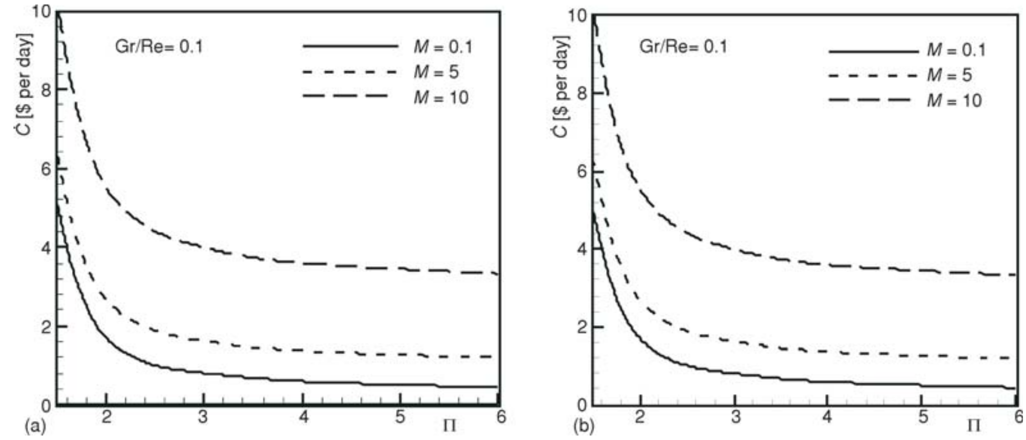

Figure 4. Effects of MHD flow and Gr/Re on total cost of irreversibility; (a) Gr/Re = 0.1, (b) Gr/Re = 50

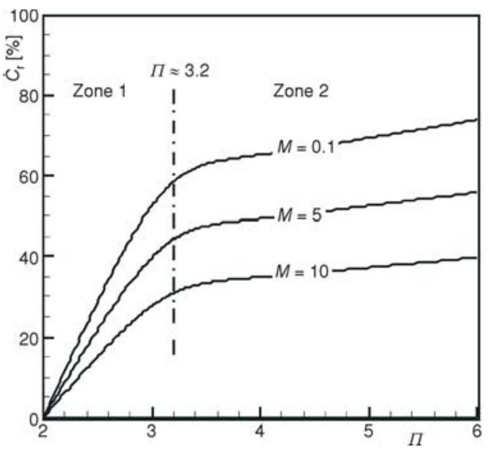

Figure 5. Effects of Hartmann number and Gr/Re on savings in operational costs

crease. This happens because the gradients of temperature and especially velocity with increasing the radius ratio are increased. From fig. 4, it is also observed that the parameter of Gr/Re has no visible effect on the total costs. Of course, with the increase of Gr/Re from 0.1 to 50, the total costs increase slightly.

Figure 5 shows the amount of changes in energy cost, $\dot{C}_\mathrm{r}$, for different values of Hartmann number, $2 \leq \Pi \leq 6$ and Gr/Re = 50. The graph can be divided into two zones as indicated in the figure. In the first zone ($\Pi \leq 3.2$), the amount of $\dot{C}_\mathrm{r}$ increases rapidly, whereas for $\Pi > 3.2$ the rate of increases in $\dot{C}_\mathrm{r}$ is reduced. As shown, $\dot{C}_\mathrm{r}$ is higher in magnitude for smaller magnetic fields. It is also concluded that where the radius ratio increases from 2 to 6, the savings in energy cost are approximately 74%, 56%, and 40% for the Hartmann numbers of 0.1, 5, and 10, respectively.

Finally, the effects of a small deviation in the radius ratio are investigated. This deviation may be produced in the manufacturing process. If the base radius ratio is 2, the effects of a deviation of ±0.1 in the radius ratio on the amount of $\dot{C}_\mathrm{r}$ are investigated. It is found, for $\Pi = 1.9$ and $M = 0.1$, the amount of $\dot{C}_\mathrm{r}$ is –17.5%. The negative sign means that where the radius ratio decreases from 2 to 1.9, the total costs due to entropy generation increase by 17.5%. On the other hand, for $\Pi = 2.1$, it is found that the costs decrease between 6.3-13.6% depend on the Hartmann number. It should be noted that the costs required to produce the pipes in different sizes and the changes in costs due to heat transfer enhancements are not considered in this study.

Conclusions

An entropy generation analysis is performed for flow and heat transfer between two vertical cylinders subjected to constant heat flux and MHD flow. The equations of momentum and energy in cylindrical co-ordinates are simplified and solved analytically. The results are presented for different values of Hartmann number and a flow parameter Gr/Re. The Bejan's EGM (entropy generation minimization) method and cost analysis are used to find the effects of different radius ratios on the total costs due to irreversibilities in the system where engine oil is considered as the working fluid. It is perceived that with an increase in the Hartmann number, the energy costs increase while the total cost does not change with increases in Gr/Re. In addition, for two small deviations from the base radius ratio ($\Pi = 2$) including $\Pi = 1.9$ and $\Pi = 2.1$, the changes in the energy cost are estimated. It is found that for $\Pi = 1.9$ the energy cost increases by 17.5% while for $\Pi = 2.1$the energy consumption cost reduces by 13.6%.

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Nomenclature

Be – Bejan number (= $N_H/N_H + N_F + N_M$), [–]
Br – Brinkman number (= $U_0^2\mu/q_ir_0$) , [–]
B_0 – constant magnetic flux density, [T]
C – constant of integration in energy equation, [–]
C_E – unit cost of irreversibility, [$\$W^{-1}h^{-1}$]
C_H – unit cost of irreversibility due to heat transfer, [$\$W^{-1}h^{-1}$]
C_F – unit cost of irreversibility due to fluid friction, [$\$W^{-1}h^{-1}$]
C_M – unit cost of irreversibility due to MHD flow, [$\$W^{-1}h^{-1}$]
$\dot{C}$ – total cost of irreversibility, [$ per day]
$\dot{C}_r$ – amount of changes in energy cost, [%]
E – electric field , [Vm^{-1}]
g – constant of gravity, [m^2s^{-1}]
Gr – Grashof number, (= $g\beta q_i r_o^4/k\upsilon^2$), [–]
J – electric current , [A]
k – thermal conductivity, [$Wm^{-1}K^{-1}$]
M – Hartmann number, (= $(\sigma/\mu)^{1/2}B_0r_0$), [–]
N_F – entropy generation number, fluid friction, [–]
N_H – entropy generation number, heat transfer, [–]
N_M – entropy generation number, MHD flow, [–]
N_S – entropy generation number, total, [–]
P – pressure [Pa]
P^* – dimensionless pressure (= $Pr_0/\mu U_0$), [–]
Q – electric charge density, [Asm^{-3}]
R – dimensionless radius (= r/r_0), [–]
r – radius,[m]
S_G – entropy generation rate, [$Wm^{-3}K^{-1}$]
T – temperature, [K]
T_0 – inlet temperature, [K]
U – dimensionless velocity, [ms^{-1}]
U_0 – velocity inlet, [ms^{-1}]
$\forall$ – volume, [m^3]

Greek symbols

β – thermal expansion coefficient,[K^{-1}]
Γn – constants, n = 1, 2,….
θ – dimensionless temperature, [–]
λ – radius ratio (= r_i/r_o)
μ – dynamic viscosity, [$kgm^{-1}s^{-1}$]
υ – kinematic viscosity, [m^2s^{-1}]
Π – radius ratio, (= r_o/r_i)
ρ – density, [kgm^{-3}]
Ω – dimensionless temperature difference, [–]

Subscript

i – value at the inner cylinder
o – value at the outer cylinder

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