THE EFFECT OF THERMAL RADIATION, HEAT GENERATION AND SUCTION/INJECTION ON THE MECHANICAL PROPERTIES OF UNSTEADY CONTINUOUS MOVING CYLINDER IN A NANOFLUID

by

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Original scientific paper
DOI:10.2298/TSCI121007111E

The effect of thermal radiation, heat generation, suction/injection, nanoparticles type, and nanoparticle volume fraction on heat transfer characteristics and the mechanical properties of unsteady moving cylinder embedded into cooling medium consist of water with Cu, Ag, or Al_2O_3 particles are studied. The governing time dependent boundary layer equations are transformed to ordinary differential equations containing unsteadiness parameter, thermal radiation parameter, heat source parameter, suction/injection parameter, curvature parameter, nanoparticle volume fraction, and Prandtl number. These equations are solved numerically. The velocity and temperature profiles within the boundary layer are plotted and discussed in details for various values of the different parameters. Also the effects of the cooling medium and the external thermal forces on the mechanical properties of the cylinder are investigated.

Key words: *thermal radiation, heat generation, suction/injection, moving cylinder, nanofluid*

Introduction

The process of heat treating is the method by which metals are heated and cooled in a series of specific operations that never allow the metal to reach the molten state. The purpose of heat treating is to make a metal more useful by changing or restoring its mechanical properties. Through heat treating, we can make a metal harder, stronger, and more resistant to impact. Heat treating can also make a metal softer and more ductile. The one disadvantage is that no heat-treating procedure can produce all of these characteristics in one operation. Some properties are improved at the expense of others; for example, hardening a metal may make it brittle. So we are interested to study the effect of the heat treating process with new cooling medium (nanofluid) on the mechanical properties.

Sakiadis [1] introduced the boundary layer flow induced by a moving plate in a quiescent ambient fluid. Tsou *et al.* [2] studied the flow and temperature fields in the boundary layer on a continuous moving surface, both analytically and experimentally and verified the results obtained in [1]. Crane [3] extended this concept to a stretching plate in a quiescent flu-

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id with a stretching velocity that varies with the distance from a fixed point and presented an exact analytic solution. The case of stretching plate developed to stretching cylinder by Ishak and Nazar [4] and extended by Bachok and Ishak. [5] to study the mixed convection boundary layer flow over a vertical cylinder with a prescribed surface heat flux. The effect of heat transfer in a conical cylinder with a porous medium was studied by Ahmed *et al.* [6]. It is well known that Choi [7] was the first to introduce the term of "nanofluid" that represents the fluid in which nano-scale particles are suspended in the base fluid with low thermal conductivity such as water, ethylene glycol, oils, *etc.* [8]. In the recent years, the concept of nanofluid has been proposed as a route for surpassing the performance of heat transfer rate in liquids currently available. An excellent collection of articles on this topic can be founded in [9-22], and in the book by Das *et al.* [23].

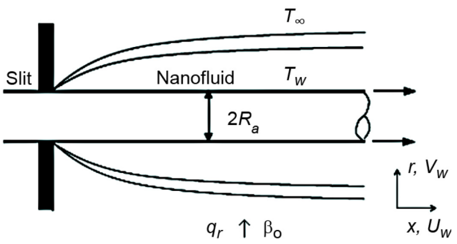

Figure 1. Physical model and co-ordinate system

Formulation of the problem

Consider an unsteady, laminar, and incompressible nanofluid on a continuous moving cylinder. The fluid is a water based nanofluid containing three types of nanoparticles, either copper (Cu), or silver (Ag), or aluminum oxide (Al_2O_3). The nanoparticles are assumed to have a uniform shape and size. Moreover, it is assumed that both the fluid phase and nanoparticles are in thermal equilibrium state. As shown in fig. (1) the x-axis runs along the cylinder, and the r-axis is perpendicular to it.

The conservation equations for the unsteady boundary layer are:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\upsilon_{\mathrm{nf}}}{r}\frac{\partial}{\partial r}r\frac{\partial u}{\partial r} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha_{\mathrm{nf}}}{r}\frac{\partial}{\partial r}r\frac{\partial T}{\partial r}+\frac{T-T_\infty}{(\rho C_p)_{\mathrm{nf}}}Q-\frac{1}{(\rho C_p)_{\mathrm{nf}}}\frac{\partial q}{\partial r} \tag{3}$$

Subjected to the boundary conditions:

$$\begin{aligned} &u=U_w, \quad v=V_w, \quad T=T_w, \quad \text{at } r=R_a \\ &u=0, \quad v=0, \quad T=T_\infty, \quad \text{as } r\to\infty \end{aligned} \tag{4}$$

where u and v are velocity components in the x- and r-directions, respectively, R_a is the radius of the cylinder, t – the time, υ_{nf} – the nanofluid kinematic viscosity, ρ_{nf} – the density of the nanofluid, T – the temperature of the nanofluid, α_{nf} – the thermal diffusion of the nanofluid, C_p – the specific heat of the nanofluid, Q – the heat source, and q_r – the radiative heat flux.

The fluid is considered to be gray, absorbing-emitting radiation but non-scattering medium and the Roseland approximation is used to describe the radiative heat flux in the energy, eq. (3). By using Rosseland approximation for radiative heat flux is simplified as:

$$q_r = -\frac{4\sigma_S}{3\alpha^*}\frac{\partial T^4}{\partial r} \tag{5}$$

where σ_S and α^* are the Stefan-Boltzman constant and the mean absorption coefficient, respectively. Assuming that the temperature differences within the flow are such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms we get:

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4 \tag{6}$$

Using eqs. (5) and (6) in the energy eq. (3), we get:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha_{\mathrm{nf}}}{r}\frac{\partial}{\partial r} r \frac{\partial T}{\partial r} + \frac{T - T_\infty}{(\rho\, C_p)_{\mathrm{nf}}} Q + \frac{16\sigma_S T_\infty^3}{3\alpha^*(\rho C_p)_{\mathrm{nf}}}\frac{\partial^2 T}{\partial r^2} \tag{7}$$

It is assumed that the velocity of the surface U_w and the surface temperature T_w are of the form:

$$U_w = \frac{ax}{1-\gamma t} \qquad T_w = T_\infty + \frac{bx}{1-\gamma t} \tag{8}$$

where *a, b*, and γ are constants. The properties of nanofluid are defined [16]:

$$\mu_{\mathrm{nf}} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \qquad \rho_{\mathrm{nf}} = (1-\varphi)\rho_{\mathrm{f}} + \varphi\rho_s, \qquad \alpha_{\mathrm{nf}} = \frac{k_{\mathrm{nf}}}{(\rho C_p)_{\mathrm{nf}}}$$

$$(\rho C_p)_{\mathrm{nf}} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s, \qquad \frac{k_{\mathrm{nf}}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)}$$

where φ is the nanoparticle volume fraction; it is worth mentioning that the study reduces to those of a viscous or regular fluid when $\varphi = 0$.

We now introduce the following dimensionless functions f and θ and the similarity variable η:

$$\left.\begin{aligned} \eta = \frac{r^2 - R_a{}^2}{2R_a}\sqrt{\frac{a}{\upsilon_f(1-\gamma t)}}, \quad \psi = f_{(\eta)}\sqrt{\frac{a\,\upsilon_f}{1-\gamma t}}\, xR_a\,, \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \end{aligned}\right\} \tag{9}$$

where υ_f is the kinematic viscosity of the base (water), and ψ – the stream function, which defined as $u = \partial\psi/r\partial r$ and $v = -\partial\psi/r\partial x$ which satisfies eq. (1). Substituting (9) into eqs. (2) and (7) we obtain:

$$\frac{1+2\eta\rho}{B} f''' + \frac{2\rho}{B} f'' + (ff'' - f'^2) - A\left(f' + \frac{\eta}{2} f''\right) = 0 \tag{10}$$

$$\frac{1+2\eta\rho}{\mathrm{Pr}}\left(L + \frac{4R}{3N}\right)\theta'' + \frac{2L\rho}{\mathrm{Pr}}\theta' + \frac{\delta}{N}\theta + f\theta' - f'\theta - A\left(\theta + \frac{\eta}{2}\theta'\right) = 0 \tag{11}$$

The boundary condition (4) becomes:

$$f(0)=f_w, \qquad f'(0)=1, \qquad \theta(0)=1, \qquad \text{and} \qquad f'(\infty)=0, \qquad \theta(\infty)=0 \tag{12}$$

where $A = \gamma/a$ is the unsteadiness parameter, $\rho = \{[\upsilon_f(1-\gamma t)]/aR_a^2\}^{1/2}$ – the curvature parameter, $R = 4\sigma_S T_\infty^3/\alpha^* k_f$ – the radiation parameter, $\delta = Qx/U_w(\rho C_p)_f$ – the heat source parameter, and $\Pr = (\upsilon\rho C_p/k)_f$ is the Prandtl number.

$$B=(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right), \quad N=\left[1-\varphi+\varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right], \quad \text{and} \quad L=\frac{\dfrac{k_{\text{nf}}}{k_f}}{1-\varphi+\varphi\dfrac{(\rho C_p)_s}{(\rho C_p)_f}}$$

Numerical solution

We first convert the eqs. (10) and (11) to a system of differential equations of first order, by using $S_1 = f$, $S_2 = f'$, $S_3 = f''$, $S_4 = \theta$, and $S_5 = \theta'$.

$$\begin{aligned} S_1' &= S_2 \\ S_2' &= S_3 \\ S_3' &= \frac{B}{1+2\eta\rho}\left[S_2{}^2 - S_1S_3 - \frac{2\rho}{B}S_3 + A\left(S_2 + \frac{\eta S_3}{2}\right)\right] \\ S_4' &= S_5 \\ S_5' &= \frac{3\Pr N}{(4R+3NL)(1+2\eta\rho)}\left[S_2S_4 - S_1S_5 - \frac{\delta}{N}S_4 - \frac{2\rho L}{\Pr}S_5 + A\left(S_4 + \frac{\eta S_5}{2}\right)\right] \end{aligned} \tag{13}$$

Subjected to the initial conditions:

$$S_1(0)=f_w, \qquad S_2(0)=1, \qquad S_3(0)=m, \qquad S_4(0)=1, \qquad S_5(0)=n \tag{14}$$

where m and n are unknown to be determined as a part of the numerical solution.

Using Mathematica, a function (F) has been defined such that F [m_, n_]:=NDSolve [system of eqs. (10), (11)], The values of m and n are determined upon solving the equations, $S_2(\eta_{\max}) = 0$, and $S_4(\eta_{\max}) = 0$ to get the solution, NDSolve first searches for initial conditions that satisfy the equations, using a combination of solve and a procedure much like Find Root. Once m and n are determined the system of eqs. (10) and (11) is closed, it can be solved numerically using the NDSolve function.

To validate the numerical method used in this study the results for $-\theta'(0)$ and $-1/\theta'(0)$ are compared with the numerical solution which reported in Ishak and Nazar [4], and Bachok and Ishak [5], respectively.

Table 1. Values of $-\theta'(0)$ for a various values of Pr at $\rho = 0$, $A = 0$, $f_w = 0$, $R = 0$, $\delta = 0$, and $\varphi = 0$

Pr	Ishak and Nazar [4]	Present results
1	1.0000	1.0000
10	3.7207	3.7207

Table 2. Values of $-1/\theta'(0)$ for a various values of Pr at $\rho = 1$, $A = 0$, $f_w = 0$, $R = 0$, $\delta = 0$, and $\varphi = 0$

Pr	Bachok and Ishak [5]	Present results
1	0.7439	0.7439
6.7	0.2966	0.2965
10	0.2442	0.2441

From the engineering point of view, the most important characteristics of the flow are the skin friction coefficient, and Nusselt number which are indicate physically to surface shear stress, and rate of heat transfer, respectively. This characteristics affect directly on the mechanical properties of the surface during heat treatment process, such that increasing the rate of heat transfer (heat flux) from the surface accelerates the cooling of the surface which improve the hardness and shear strength of the surface but on the other hand decrease the ductility of the surface and increase surface cracking.

Surface shear stress

$$\tau_w = \mu_{\mathrm{nf}} \left(\frac{\partial u}{\partial r} \right)_{r=R_a} = \frac{\mu_f U_w}{(1-\phi)^{2.5}} \sqrt{\frac{a}{\upsilon_f \left(1-\gamma t\right)}} f''(0)$$

Since the skin friction coefficient is given by:

$$C_f = \frac{2\tau_w}{\rho U_w^2}, \qquad i.\,e. \qquad \frac{2f''(0)}{(1-\varphi)^{2.5}} = C_f \sqrt{\mathrm{Re}}$$

Surface heat flux

$$q_w = -k_{\mathrm{nf}} \left(\frac{\partial T}{\partial r} \right)_{r=R_a} = -k_{\mathrm{nf}} (T_w - T_\infty) \sqrt{\frac{a}{\upsilon_f (1-\gamma t)}} \quad \theta'(0)$$

Since the Nusselt number is given by:

$$\mathrm{Nu} = \frac{x q_w}{k_f (T_w - T_\infty)}, \qquad i.\,e. \qquad \frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = -\frac{k_{\mathrm{nf}}}{k_f} \theta'(0)$$

Discussion

In the study authors present a mathematical model of a moving continuous cylinder embedded into a nanofluid. The influence of unsteadiness parameter A, thermal radiation R, heat source δ, suction/injection f_w, nanoparticles type, and nanoparticle volume fraction φ on the velocity and temperature within the boundary layer are shown in figs. (2)-(9). Three different types of nanoparticles are considered, Cu, Ag, and Al_2O_3 with water as the base fluid. Table 3 shows the thermo physical properties of water and the elements Cu, Ag, and Al_2O_3. The Prandtl number of the base fluid (water) is kept constant at 6.2.

Table 3. Thermo-physical properties of water and the elements Cu, Ag, and Al_2O_3

Properties	Fluid (water)	Cu	Ag	Al_2O_3
C_p [$Jkg^{-1}K^{-1}$]	4179	385	235	765
ρ [kgm^{-3}]	997.1	8933	10500	3970
K [$WM^{-1}k^{-1}$]	0.613	400	429	40
$\alpha \cdot 10$ [m^2s^{-1}]	1.47	1163.1	1738.6	31.7

The effects of unsteadiness parameter A on the velocity and temperature within the boundary layer of (Cu-nanofluid) are shown in figs. 2 and 3, respectively. It is observed that the increases in this parameter decrease both the velocity and temperature within the boundary layer; also it is observed that the boundary layer thickness decreases with increase of A.

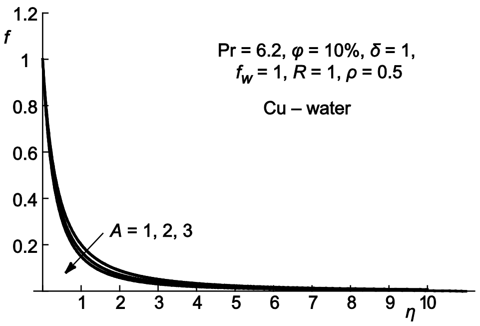

Figure 2. The velocity profiles with increasing of unsteadiness parameter (*A*)

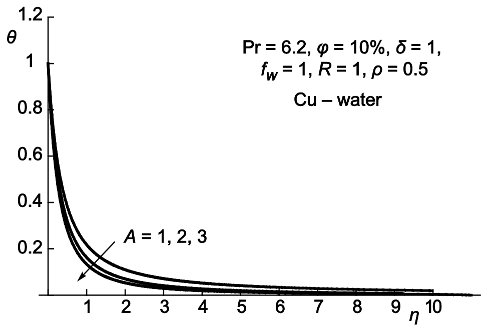

Figure 3. The temperature profiles with increasing of unsteadiness parameter (*A*)

Figures 4 and 5 show the effect of suction/injection parameter on the velocity and temperature within the boundary layer of Cu-nanofluid respectively. It is observed that the increases of suction/injection parameter decrease both the velocity and temperature within the boundary layer.

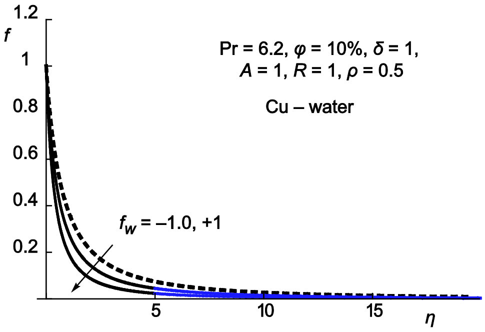

Figure 4. The velocity profiles with increasing of suction parameter (f_w)

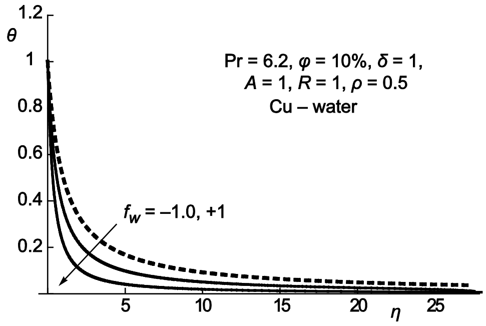

Figure 5. The temperature profiles with increasing of suction parameter (f_w)

The effects of nanoparticles concentration (volume fraction) φ on the velocity and temperature within the boundary layer of Cu-nanofluid are shown in figs. 6 and 7. It is observed that increase the nanoparticle volume fraction decrease the velocity but increase the temperature within the boundary layer.

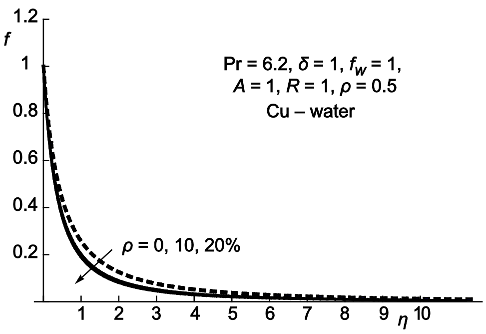

Figure 6. The velocity profiles with increasing of nanoparticle volume fraction (φ)

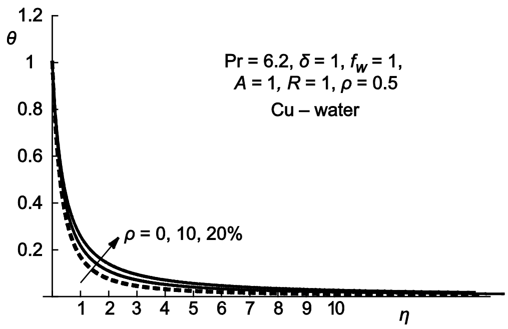

Figure 7. The temperature profiles with increasing of nanoparticle volume fraction (φ)

The effects of curvature parameter ρ on the velocity and temperature within the boundary layer of (Cu-nanofluid) are shown in figs. 8 and 9, respectively. It is observed from fig. 8 that the effect of this parameter on the velocity appears far the surface, such that the increases of ρ increase the velocity within the boundary layer, also one can observe that the terminal velocity of the boundary layer increases with increase of curvature parameter.

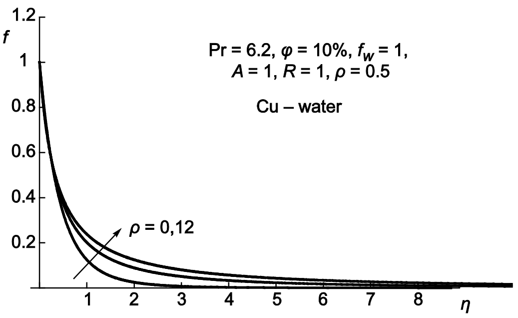

Figure 8. The velocity profiles with increasing of curvature parameter (ρ)

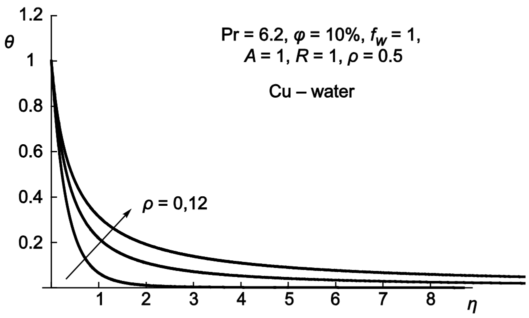

Figure 9. The temperature profiles with increasing of curvature parameter (ρ)

On the other hand it is worth mentioning that the study reduces to flat surface when $\rho \to 0$ which means that the velocity within the boundary layer in the case of cylinder is large than that in the case of flat surface, also means that the increasing in the cylinder diameter leads to decrease in the velocity within the boundary layer.

Figure 9 shows that the temperature within the boundary layer increases with increase the curvature parameter, also one can observe that the temperature within the boundary layer in the case of cylinder is large than that in the case of flat surface.

Tables 4-7 show the values of velocity gradient and temperature gradient at the surface and the corresponding values of skin friction and Nusselt number for different values of embedded parameters at Re = $5 \cdot 10^5$. The effects of nanoparticles type, nanoparticle concentration, steady and unsteady motion, suction and injection, thermal radiation and heat generation on the surface shear stress, surface heat flux, and the mechanical properties (hardness, stiffness, strength, surface cracking, *etc.*) are discussed below.

Nanoparticles types

It is clear from tab. 4 that the values of velocity gradient at the surface increased gradually by changing the nanoparticle from Al_2O_3 to Cu to Ag. The same effect occurs on temperature gradient, but in the case of suction only such that the injection process increase the temperature gradient for Cu to be large than that in Al_2O_3. On the other hand the skin friction and surface shear stress are higher in the case of Ag-nanofluid than that in Cu and Al_2O_3-nanofluid, also the Nusselt number and rate of heat transfer from the surface are higher in the case of Cu-nanofluid than that in Al_2O_3 and Ag-nanofluid in the case of suction and Al_2O_3-nanofluid than that in Cu-nanofluid, which means that using Cu-nanofluid as a cooling medium is more useful for the surface hardness and strength for the case of suction and Al_2O_3-nanofluid for the case of injection.

Concentration of nanoparticles within the base fluid

Table 4 shows that the values of velocity gradient at the surface increases gradually by increase the particle volume fraction from 10% to 20% in the case of Cu and

Ag-nanoparticle and decrease for Al_2O_3 nanoparticle, But the temperature gradient decreases by increase of it for all types of nanoparticles. On the other hand the skin friction and rate of heat transfer both increase with increase of the concentration of nanoparticle within the base fluid. So one can say that the nanofluid with 20% nanoparticle is more affect on the mechanical properties than that with 10% nanoparticle.

Table 4. Values of velocity gradient and temperature gradient at the surface at A = 1, R = 1, δ = 0.5, and Pr = 6.2

f_w	ρ	φ	Cu-water nanofluid				Ag-water nanofluid				Al_2O_3-water nanofluid			
			$-f''(0)$	$-\theta'(0)$	C_{fx}	Nu	$-f''(0)$	$-\theta'(0)$	C_{fx}	Nu	$-f''(0)$	$-\theta'(0)$	C_{fx}	Nu
−1	0	0	0.94398	1.01353	0.00267	716.68	0.94398	1.01353	0.00267	716.68	0.94398	1.01353	0.00267	716.68
		0.1	1.04966	0.95531	0.00386	899.53	1.07770	0.93826	0.00397	883.55	0.94319	0.96307	0.00347	896.79
		0.2	1.07385	0.90199	0.00531	1113.42	1.11231	0.86890	0.00550	1072.75	0.91514	0.91205	0.00452	1101.93
	1	0	1.31361	1.04088	0.00372	736.01	1.31361	1.04088	0.00372	736.01	1.31361	1.04088	0.00372	736.01
		0.1	1.41186	1.00757	0.00520	948.74	1.43728	0.98941	0.00529	931.71	1.31286	1.01423	0.00483	944.44
		0.2	1.43381	0.98005	0.00708	1209.78	1.46826	0.94444	0.00725	1166.01	1.36625	0.95262	0.00675	1150.95
1	0	0	1.87640	3.65230	0.00531	2582.57	1.87640	3.65230	0.00531	2582.57	1.87640	3.65230	0.00531	2582.57
		0.1	2.34079	3.15602	0.00862	2971.74	2.48324	3.06627	0.00914	2887.48	1.87330	3.20206	0.00690	2981.71
		0.2	2.46312	2.72922	0.01217	3368.96	2.67234	2.57302	0.01320	3176.66	2.53780	2.71269	0.01254	3277.45
	1	0	2.24980	3.09244	0.00636	2186.68	2.24980	3.09244	0.00636	2186.68	2.24980	3.09244	0.00636	2186.68
		0.1	2.70465	2.72115	0.00996	2562.27	2.84363	2.63347	0.01047	2479.92	2.24675	2.76727	0.00827	2576.84
		0.2	2.82401	2.41885	0.01395	2985.83	3.02785	2.26829	0.01496	2800.45	2.14162	2.47882	0.01058	2994.89

In general using a nanofluid in the cooling process is more active to improve the mechanical properties of the surface, such that using nanofluid increase the rate of heat transfer by 10-40% more than in the case of pure water that leads to accelerate the cooling of the surface which increases the surface hardness and strength.

Shape of the moving surface

Table 4 shows that the values of velocity gradient and skin friction at the surface increases gradually by increase the curvature parameter, But the behavior of the temperature gradient and Nusselt number both depend on the other parameters behind the curvature parameter such that in the presence of injection the temperature gradient and Nusselt number both decrease with increase of curvature parameter and the opposite effect appears in the presence of suction. It is worth mentioning that the increase of ρ from zero to one indicates to transform the shape of the surface from flat surface to cylinder.

Steady and unsteady motion

It is clear from tab. 5 that the unsteady motion of the surface has a direct effect on the mechanical properties such that increase the unsteadiness parameter increase the velocity gradient, skin friction, and surface shear stress also increase the temperature gradient, Nusselt number, and rate of heat transfer. It is worth mentioning that increasing the unsteadiness parameter from 1 to 3 increases Nusselt number and rate of heat transfer from the surface by 30~34% in the case of cylinder and by 20~22% for the case of flat surface.

Table 5. Values of velocity gradient and temperature gradient at the surface at f_w = 1, R = 1, δ = 0.5, and Pr = 6.2

ρ	A	Cu-water nanofluid				Ag-water nanofluid				Al_2O_3-water nanofluid			
		$-f''(0)$	$-\theta'(0)$	C_{fx}	Nu	$-f''(0)$	$-\theta'(0)$	C_{fx}	Nu	$-f''(0)$	$-\theta'(0)$	C_{fx}	Nu
0	1	2.34079	3.15602	0.00862	2971.74	2.48324	3.06627	0.00702	2887.48	1.87330	3.20206	0.00690	2981.71
	2	2.61085	3.50270	0.00961	3298.17	2.76191	3.41949	0.01017	3220.10	2.11099	3.53127	0.00777	3288.26
	3	2.85574	3.82187	0.01051	3598.71	3.01544	3.74065	0.01110	3522.54	2.32412	3.84079	0.00855	3576.49
1	1	2.70465	2.72115	0.00996	2562.27	2.84363	2.63347	0.01047	2479.92	2.24675	2.76727	0.00827	2576.84
	2	3.01491	3.22528	0.01110	3036.95	3.16431	3.14750	0.01165	2963.97	2.51867	3.24709	0.00927	3023.64
	3	3.27960	3.61408	0.01207	3403.05	3.43858	3.53757	0.01266	3331.29	2.74870	3.62666	0.01012	3377.09

Suction and injection process

One can say that the suction/injection process play an important role in the cooling process, such that in the case of suction the velocity gradient, skin friction, surface shear stress, temperature gradient, Nusselt number, and rate of heat transfer all are higher than that in the case of injection. As we know from previous increase the rate of heat transfer from the surface improve the mechanical properties of surface.

Thermal radiation and heat generation

One can observe from tabs. 6 and 7 that increasing the thermal radiation and heat source both decrease the values of Nusselt number and rate of heat transfer that means the hardness and the strength of the surface will be decrease in the presence of both forces.

Table 6. Values of temperature gradient at the surface at f_w = 1, φ = 10% , δ = 0.5, and Pr = 6.2

A	ρ	R	Cu-water nanofluid		Ag-water nanofluid		Al_2O_3-water nanofluid	
			$-\theta'(0)$	Nu	$-\theta'(0)$	Nu	$-\theta'(0)$	Nu
1	0	0.5	4.03387	3798.33	3.92590	3696.98	4.08080	3799.97
		1	3.15602	2971.74	3.06627	2887.48	3.20206	2981.71
		2	2.24073	2109.89	2.17089	2044.31	2.28613	2128.81
	1	0.5	3.76717	3547.21	3.65838	3445.06	3.81749	3554.78
		1	2.72115	2562.27	2.63347	2479.92	2.76727	2576.84
		2	1.72804	1627.14	1.66515	1568.05	1.76650	1644.94

Conclusions

The study presents a mathematical model of a continuous moving cylinder embedded into a nanofluid, the study based on three types of nanoparticle which are the most used types (Cu, Ag, and Al_2O_3). The mechanical properties and heat transfer characteristics of the surface were our goal in this study and the following results are obtained.

Table 7. Values of temperature gradient at the surface at f_w = 1, φ = 10%, R = 1, and Pr = 6.2

A	ρ	δ	Cu-water nanofluid		Ag-water nanofluid		Al_2O_3-water nanofluid	
			$-\theta'(0)$	Nu	$-\theta'(0)$	Nu	$-\theta'(0)$	Nu
1	0	0.2	3.34573	3150.37	3.26400	3073.68	3.38346	3150.63
		0.5	3.15602	2971.74	3.06627	2887.48	3.20206	2981.71
		0.7	3.00895	2833.25	2.90946	2739.81	3.06503	2854.11
	1	0.2	2.97539	2801.66	2.89862	2729.60	3.01038	2803.22
		0.5	2.72115	2562.27	2.63347	2479.92	2.76727	2576.84
		0.7	2.49759	2351.76	2.39374	2254.16	2.55870	2382.62

- The velocity within the boundary layer increases with increase of curvature parameter and decrease of unsteadiness, suction/injection parameters.
- The temperature within the boundary layer increases with increase of curvature parameter, thermal radiation, and heat source parameters and decrease of unsteadiness, suction/injection parameters.
- In general, the velocity and temperature within the boundary layer over cylinder are larger than that in the case of flat surface.
- Using nanofluid as a cooling medium is useful to improve the mechanical properties (hardness and strength) by 10-40% according to the type and concentration of nanoparticles used.
- We studied three types only of nanoparticles, according to this study the best type used to improve the mechanical properties of the surface (increase the heat flux) is Cu-nanofluid and the best type used to decrease the surface shear stress is Al_2O_3-nanofluid.
- Transform the motion from steady to unsteady leads to increase of skin friction and rate of heat transfer which leads to improve the mechanical properties of the surface by increasing of heat flux.
- Cooling with suction is more useful to increase the heat flux than that with injection.

Nomenclature

A – unsteadiness parameter, (= γ/a), [–]
a, b – constants, [–]
C_f – skin friction, [–]
C_p – fluid specific heat, [$Jkg^{-1}K^{-1}$]
$f(\eta)$ – dimensionless function, [–]
k_f – base fluid thermal conductivity, [$WK^{-1}m^{-1}$]
k_{nf} – nanofluid thermal conductivity, [$WK^{-1}m^{-1}$]
Nu – Nusselt number, [–]
Pr – Prandtl number, $(= \upsilon\rho C_p/k)_f$, [–]
Q – heat source, [K]
q_r – radiative heat flux, [Km^{-2}]
q_w – surface heat flux, [$Ws^{-1}m^{-2}$]
R – radiation parameter, $(= 4\sigma T_\infty^3/\alpha k_f)$, [–]
R_a – cylinder radius, [m]
Re – Reynolds number, (= $U_x x/\upsilon)_f$, [–]
t – time, [s]
T_w – surface temperature, [K]
U_w – surface velocity, [ms^{-1}]
u, v – velocity components along x-, r-directions, [ms^{-1}]

Greek symbols

α^* – mean absorption coefficient, [–]
α_f – thermal diffusion of water, [m^2s^{-1}]
α_{nf} – thermal diffusion of nanofluid, [m^2s^{-1}]
β_0 – magnetic field strength, [T]
γ – constant, [–]
δ – heat source parameter, [= $Qx/U_w(\rho C_p)_f$], [–]
η – similarity variable, [–]
$\theta(\eta)$ – temperature dimensionless function, [–]
μ_f – dynamic viscosity of water, [$kgms^{-1}$]
μ_{nf} – dynamic viscosity of nanofluid, [$kgms^{-1}$]
ρ – curvature parameter, $\rho = \{[\upsilon_f(1-\gamma t)]/aR_a^2\}^{1/2}$, [–]
ρ_f – base fluid (water) density, [kgm^{-3}]

ρ_{nf} – nanofluid density, [kgm^{-3}]
σ – electrical conductivity, [Wm^{-2}K^{-4}]
σ_S – Stefan-Boltzman constant, [–]
τ_w – surface shear stress, [Nm^{-2}]
υ_f – kinematic viscosity of water, [m^{2}s^{-1}]
υ_{nf} – kinematic viscosity of nanofluid, [m^{2}s^{-1}]
φ – nanoparticles volume fraction, [–]
ψ – stream function, [–]

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Paper submitted: October 7, 2012
Paper revised: July 15, 2013
Paper accepted: August 3, 2013