

NUMERICAL STUDY OF HEAT AND MASS TRANSFER DURING EVAPORATION OF A TURBULENT BINARY LIQUID FILM

by

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This paper deals with a computational study for analysing heat and mass exchanges in the evaporation of a turbulent binary liquid film (water-ethanol and water-methanol) along a vertical tube. The film is in co-current with the dry air and the tube wall is subjected to a uniform heat flux. The effect of gas-liquid phase coupling, variable thermophysical properties and film vaporization are considered in the analysis. The numerical method applied solves the coupled governing equations together with the boundary and interfacial conditions. The algebraic systems of equations obtained are solved using the Thomas algorithm. The results concern the effects of the inlet liquid Reynolds number and inlet film composition on the intensity of heat and mass transfer. In this study, results obtained show that heat transferred through the latent mode is more pronounced when the concentration of volatile components is higher in the liquid mixture. The comparisons of wall temperature and accumulated mass evaporation rate with the literature results are in good agreement.

Key words: *evaporation, mixed convection, binary liquid film, heat and mass transfer*

Introduction

Film evaporation in ducts is of importance in many engineering application such as: distillation, boiling, film cooling, and combustion premixing. Since the pioneering analytical work of Nusselt [1], there have been numerous studies of internal and external evaporation of pure components. Studies concerning the treatment of flows with multicomponent evaporation have not received much attention.

A large number of works have been made for pure substances. Yan [2] studied the effect of the liquid film thickness and concluded that it can be neglected when the liquid mass flow rate is small. Feddaoui *et al.* [3, 4] and Feddaoui and Mir [5] investigate numerically the co-current turbulent mixed convection heat and mass transfer in falling film of water inside a vertical heated tube. They applied a low Reynolds number $k-\epsilon$ turbulence model in the gas stream. Senhaji *et al.* [6] studied numerically the evaporation in mixed convection of a pure alcohol liquid film ethanol and methanol. The numerical calculations indicate that the metha-

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nol evaporates in more intense way in comparison to ethanol. A small proportion of this fluid absorbs a significant amount of energy supplied from the wall in comparison to ethanol and water using the same liquid Reynolds number. The mechanism of the evaporation of a binary or a ternary liquid film has not been studied in detail, in spite of its important role in many industrial systems. Baumann and Thiele [7] considered the evaporation of a binary liquid film flowing inside a cylindrical duct. For liquid benzene-methanol mixtures into a hot air stream, they show that small portions of a second component in the liquid film can create significant changes in the temperature levels as well as in the heat and mass transfer. Ali Cherif and Daif [8] considered the evaporation of a thin binary liquid film by mixed convection in a vertical channel. They showed the importance of the film thickness and composition in the mass and heat transfers. Palen *et al.* [9] in their work devoted to evaporation of propylene glycol-water solutions, found a weak dependence of heat transfer coefficient on heat flux and liquid flow-rate and strong dependence on the solution composition. El Armouzi *et al.* [10] investigated numerically the evaporation by mixed convection of a binary liquid film flowing down of two coaxial cylinders. They showed that the volatilities of the mixture influenced the heat transferred through the latent mode, which is more pronounced for mixture composed of volatile components. The most recent works are those of Nasr *et al.* [11-13] which examined a numerical analysis of evaporation of water-ethylene glycol into air by forced convection in a channel. They showed that the inversion temperature phenomenon for the evaporation of binary liquid mixture is observed for high liquid concentration of ethylene glycol.

The objective of the present study is to analyze the mixed convection heat and mass transfer processes in moving turbulent binary liquid film. 2-D incompressible boundary layer model is employed for liquid film and gas flows. The effects of inlet liquid conditions on the performance of falling film evaporation are examined.

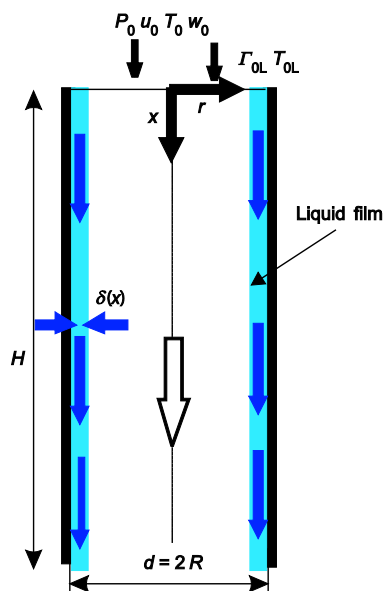


Figure 1. Schematic diagram of the physical system

Analysis

Physical model and assumption

The physical model concerned in this study is a vertical tube of height H and a radius R (fig. 1). The tube wall is subjected to a uniform heat flux. The binary liquid film is fed with an inlet liquid temperature T_{0L} , and inlet liquid mass flow rate Γ_{0L} .

For the mathematical formulation of the problem, the following simplifying assumptions are taking into consideration:

- the flow is considered to be incompressible and axis-symmetric,
- the solubility of air in the liquid film is negligible,
- the vapour and liquid phases are in thermodynamic equilibrium at the interface, and
- radiation heat transfer, viscous dissipation and other secondary effects are negligible.

Governing equations

With the mentioned assumptions, the 2-D boundary layer flow steady turbulent momentum and heat transfer equations for the liquid film can be expressed as following.

Basic equations for the liquid film

– continuity equation

$$\frac{\partial}{\partial x} (\rho_L u_L) + \frac{1}{r} \frac{\partial}{\partial r} (\rho_L r v_L) = 0 \quad (1)$$

– axial momentum equation

$$\rho_L \left(u_L \frac{\partial u_L}{\partial x} + v_L \frac{\partial u_L}{\partial r} \right) = - \frac{dP}{dx} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \mu_{Leff} \frac{\partial u_L}{\partial r} \right) + \rho_L g \quad (2)$$

– energy equation

$$\rho_L c_{pL} \left(u_L \frac{\partial T_L}{\partial x} + v_L \frac{\partial T_L}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda_{Leff} \frac{\partial T_L}{\partial r} \right) \quad (3)$$

Effective viscosity and conductivity in liquid film are defined as $\mu_{Leff} = \mu_{Lt} + \mu_L$ and $\lambda_{Leff} = \lambda_{Lt} + \lambda_L$ (μ_{Leff} and λ_{Leff} are turbulent viscosity and conductivity).

Basic equations for the gas flow

The laminar mixed convection heat and mass transfer in the gas flow is governed by the following conservation equations.

– continuity equation

$$\frac{\partial}{\partial x} (\rho_G u_G) + \frac{1}{r} \frac{\partial}{\partial r} (\rho_G r v_G) = 0 \quad (4)$$

– axial momentum equation

$$\rho_G \left(u_G \frac{\partial u_G}{\partial x} + v_G \frac{\partial u_G}{\partial r} \right) = - \frac{dP}{dx} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \mu_G \frac{\partial u_G}{\partial r} \right) - (\rho_0 - \rho_G) g \quad (5)$$

– energy equation

$$\begin{aligned} \rho_G c_{pG} \left(u_G \frac{\partial T_G}{\partial x} + v_G \frac{\partial T_G}{\partial r} \right) &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda_G \frac{\partial T_G}{\partial r} \right) + \\ &+ \rho_G \left\{ \sum_{i=1}^2 \left[(D_{G,im} c_{pi} - D_{G,am} c_{pa}) \frac{\partial w_i}{\partial r} \right] \right\} \frac{\partial T_G}{\partial r} \end{aligned} \quad (6)$$

– species concentration equations

$$\rho_G \left(u_G \frac{\partial w_i}{\partial x} + v_G \frac{\partial w_i}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left(r D_{G,im} \rho_G \frac{\partial w_i}{\partial r} \right), \quad (i = 1, 2) \quad (7)$$

Two additional equations are required to complete the mathematical model. At every axial location, the overall mass balance in the gas flow and liquid film should be satisfied:

$$\Gamma_{0L} = 2\pi \int_{R-\delta_x}^R r \rho_L u_L dr - 2\pi \int_0^x (R-\delta_x) \rho_G v_S dx \quad (8)$$

$$\frac{1}{2} (R-\delta_0)^2 u_0 \rho_0 = \int_0^{R-\delta_x} r \rho_G u_G dr + \int_0^x (R-\delta_x) \rho_G v_S dx \quad (9)$$

Turbulence model

In this work, a modified Van Driest eddy viscosity model for the turbulent liquid film proposed by Yih and Liu [14] was used. The turbulent eddy viscosity is given by:

$$\frac{\mu_{Lt}}{\mu_L} = -0.5 + 0.5 \sqrt{1 + 0.64 y^{+2} \frac{\tau}{\tau_w} \left[1 - \exp \left(-y^+ \frac{\sqrt{\frac{\tau}{\tau_w}}}{A^+} \right) \right]^2} f^2 \quad (10)$$

for $(R-r)/\delta_x < 0.6$, where:

$$A^+ = 25.1, \quad f = \exp \left[-1.66 \left(1 - \frac{\tau}{\tau_w} \right) \right], \quad y^+ = \frac{(R-r)u^*}{\nu} \quad (11)$$

For $0.6 < (R-r)/\delta_x < 1$ the turbulent eddy viscosity for the liquid film was taken as constant and equal to its value at $(R-r)/\delta_x = 0.6$ which may be readily obtained from eq. (10).

The turbulent conductivity λ_{Lt} , can then be obtained by introducing the turbulent Prandtl number, Pr_t :

$$\lambda_{Lt} = \mu_{Lt} \frac{c_{pL}}{Pr_t} \quad (12)$$

where the turbulent Prandtl number can be evaluated from Cebeci and Smith [15]:

$$Pr_t = \frac{1 - \exp \left(-y^+ \frac{\sqrt{\frac{\tau}{\tau_w}}}{A^+} \right)}{1 - \exp \left(-y^+ \frac{\sqrt{\frac{\tau}{\tau_w}}}{B^+} \right)} \quad (13)$$

In eq. (13), B^+ is given by Habiba and Na [16].

Boundary and interfacial conditions

The boundary conditions are:

– at the entry ($x = 0$)

$$u_G = u_0, \quad T_G = T_0, \quad w_i = w_{0i}, \quad P = P_0, \quad T_L = T_{0L} \quad (14)$$

– at the tube wall ($r = R$)

$$u_L = v_L = 0, \quad \lambda_{\text{Leff}} \frac{\partial T_L}{\partial r} = Q_w \quad (15)$$

– at the centre line ($r = 0$)

$$\frac{\partial u_G}{\partial r} = 0, \quad \frac{\partial T_G}{\partial r} = 0, \quad \frac{\partial w_i}{\partial r} = 0, \quad v_G = 0 \quad (16)$$

At the liquid-gas interface ($r = R - \delta_x$), the matching conditions are:

– continuity of velocity and temperature

$$T_S(x) = T_{G,S} = T_{L,S}, \quad u_S(x) = u_{G,S} = u_{L,S} \quad (17)$$

– continuity of shear stress

$$\tau_S = \left(\mu_{\text{eff}} \frac{\partial u}{\partial r} \right)_{L,S} = \left(\mu \frac{\partial u}{\partial r} \right)_{G,S} \quad (18)$$

The transverse velocity component of the air-vapour mixture is deduced by assuming the liquid-gas interface to be semi-permeable [17] (that is, the solubility of air into the liquid is negligibly small and the transverse velocity of air is zero at the interface):

$$v_S = - \frac{\sum_{i=1}^2 D_{im} \frac{\partial w_i}{\partial r}}{1 - \sum_{i=1}^2 w_{S,i}} \quad (19)$$

The mass fraction at the interface of species i vapour can be evaluated by [13]:

$$w_{S,1} = \frac{P_{VS,1}^*}{P_{VS,1}^* + \left(P_{VS,2}^* \frac{M_2}{M_1} \right) + \left[(P - P_{VS,1}^* - P_{VS,2}^*) \frac{M_A}{M_1} \right]} \quad (20)$$

A similar relationship holds for the second component.

The P and $P_{VS,i}^*$ are the total pressure and the partial pressure of species i at the interface, respectively. M_A and M_i are the molecular weights of air and of species i ($i = 1, 2$).

The heat balance at the interface implies:

$$\left(\lambda_{\text{eff}} \frac{\partial T}{\partial r} \right)_{L,S} = \left(\lambda \frac{\partial T}{\partial r} \right)_{G,S} + \dot{m}_S h_{fg} \quad (21)$$

where h_{dg} is the enthalpy of evaporation and $\dot{m}_S$ the vapour generation rate ($= \rho_G v_S$).

At the interface, the heat is transferred from the liquid film (eq. 21) into the gas flow by two modes. The first is the sensible heat due to the gas temperature gradient Q_S , whereas the second is via the latent heat associated with the liquid film vaporization Q_L .

In order to evaluate the importance of the different processes of energy transfer a non-dimensional accumulated mass evaporation rate is introduced:

$$Mr = \frac{2\pi \int_0^x \dot{m}_s dx}{\Gamma_{0L}} \quad (22)$$

The pure component data are approximated by polynomials in term of temperature and mass fraction. For further details, the thermophysical properties are available in Reid and Sherwood [18], and Bird *et al.* [19].

Numerical method

The governing equations for continuity, momentum, energy, and species concentration with appropriate boundary conditions are solved by the finite-difference numerical method. A fully implicit numerical scheme in which the axial convection terms are approximated by the backward difference and the radial convection and diffusion terms by the central difference is employed to transform the governing equations into finites differences equations. Each system of the finites differences equations forms a tri-diagonal set which can be efficiently solved by the Thomas algorithm [20]. The correction of the pressure gradient and axial velocity profile at each axial station in order to satisfy the global mass flow constraint is achieved using a method proposed by Raithby and Schneider [21].

Marching procedure

After specifying the flow and thermal conditions, the numerical solution is advanced forward and step by step:

- (1) For any axial location x , guess the values of dP_g/dx and δ_x .
- (2) Solve the finite-difference forms of eqs. (2) and (5) simultaneously for velocities u_L and u_G .
- (3) Integrate numerically the continuity equations of liquid film and gas flow to find v_L and v_G :

$$v = -\frac{1}{\rho} \frac{1}{r} \frac{\partial}{\partial x} \int_0^r \rho u r dr \quad (23)$$

- (4) Solve the finite-difference forms of eqs. (3) and (6) for temperatures T_L and T_G .
- (5) Solve the finite-difference forms of eq. (7) for mass fraction of species w_i ($i = 1, 2$).
- (6) Check the satisfaction of the overall conservation of mass in both gas flow and liquid film. If the following criteria:

$$\left| \frac{\rho_0 u_0 \frac{(R - \delta_0)^2}{2} - \left[\int_0^{R - \delta_x} r \rho_G u_G dr + \int_0^x (R - \delta_x) \rho_G v_I dx \right]}{\rho_0 u_0 \frac{(R - \delta_0)^2}{2}} \right| < 10^{-4} \quad (24a)$$

and

$$\left| \frac{\Gamma_{0L} - \left[2\pi \int_{R - \delta_x}^R (r \rho u dr)_L - 2\pi \int_0^x (R - \delta_x) \rho_G v_I dx \right]}{\Gamma_{0L}} \right| < 10^{-4} \quad (24b)$$

are met, then test the convergence of the velocity, temperature and the mass fraction of species.

If the maximum relative errors between two consecutive iterations satisfy the criterion:

$$\frac{\left| \psi_{i,j}^n - \psi_{i,j}^{n-1} \right|_{\max}}{\left| \psi_{i,j}^n \right|_{\max}} < 10^{-4} \quad (25)$$

where ψ represents the variables u , T and w_i ($i = 1, 2$).

The solution for the current axial location is complete. Now if eq. (25) is not simultaneously met, repeatedly solve the finite-difference equations for u , v , T , and w_i ($i = 1, 2$) in the gas flow and liquid film until the condition specified in eq. (25) is fulfilled. If eqs. (24) are not satisfied, adjust dP_d/dx and δ_x and repeat procedures (1)-(6) for the current axial location.

A better approximation of the liquid film thickness δ_x is obtained in function of the error in the mass flow E_m using the secant method [22]. Thus:

$$\delta_x^{n+1} = \delta_x^n - \frac{\delta_x^n - \delta_x^{n-1}}{E_m^n - E_m^{n-1}} E_m^n \quad (26)$$

where E_m^n is the error in the mass flow at iteration n , calculated from mass balance in the liquid film.

The convergence criteria used is $E_m < 10^{-5}$. Usually four to five iterations are sufficient to obtain converged solution.

Grid pattern

To obtain enhanced accuracy in the numerical computations, grids are chosen to be non-uniform in both axial and radial directions. The grids are transversely clustered near the gas-liquid interface, and the grid density is also higher in the region near the inlet. Several different grid distributions have been tested to ensure that the calculated results are grid independent (tab. 1). It is noted that the difference in the local Nusselt number, Nu_x , from computations using either $101 \times (61 + 31)$ or $101 \times (101 + 41)$ grids are always less than 4%. In light of those results all further calculations were performed with the $101 \times (61 + 31)$ grid.

Table 1. Comparison of local Nusselt number Nu_x for various grids

x/d	$51 \times (81 + 21)$	$101 \times (61 + 31)$	$101 \times (101 + 41)$	$201 \times (121 + 81)$
10.25	347.45	351.25	355.35	363.76
15.38	172.98	174.17	175.52	178.80
20.44	118.64	120.05	120.79	121.80
30.48	79.46	79.56	79.92	81.08
50.1	58.04	58.22	58.32	58.67
79.4	52.99	52.89	52.88	52.84
100	55.18	54.98	54.86	54.72

Parameters used in test case are: $Re_G = 2000$, $Q_w = 5000 \text{ W/m}^2$, $P_0 = 1 \text{ atm}$, $R = 0.01 \text{ m}$, $H = 2 \text{ m}$, $Re_L = 500$, $T_0 = T_{OL} = 20 \text{ }^\circ\text{C}$, 50% water-ethanol mixture

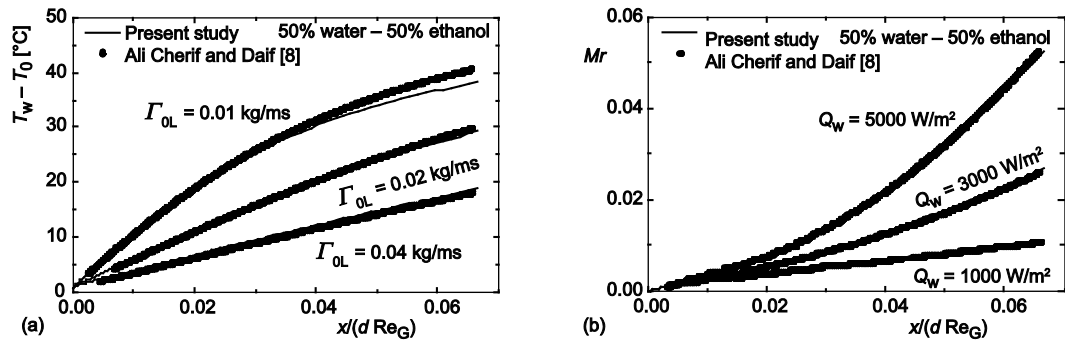


Figure 2. Validation of calculated (a) wall temperature, (b) total accumulated evaporation rate

In figs. 2(a) and (b) the present predictions of wall temperature, and total accumulated evaporation rate are compared with those of Ali Cherif and Daif [8]. It is clear that generally the agreement between our prediction and study of Ali Cherif and Daif [8]. Through this program test, the proposed numerical algorithm is considered to be suitable for the practical purpose.

Results and discussion

In the numerical study, computations were performed by considering a liquid film composed of water-ethanol or water-methanol mixtures. The length of the tube is equal to 2 m and radius is equal to 0.01 m. The wall of the tube is subjected to a uniform heat flux density ranging between 1000 W/m^2 and 5000 W/m^2 . Inlet liquid Reynolds number is chosen to be from 500 to 2000, and inlet gas stream Reynolds number Re_G is 2000. Inlet air temperature is taken equal to 20°C while the inlet liquid temperature at 20°C .

Figure 3 illustrates the effect of inlet liquid Reynolds number on the interfacial and wall temperatures along the tube for two mixtures (water-ethanol and water-methanol). The results indicate that T_s and T_w increase monotonically in the flow direction for both mixtures. This feature is due to the fact that the liquid mixture absorbs sensible heat from gas flow as it falls along the tube. The reason for the decrease of wall and interfacial temperatures with higher liquid flow rate can be justified by the fact that the energy required for the evaporation is sufficient in the case of lower inlet liquid film.

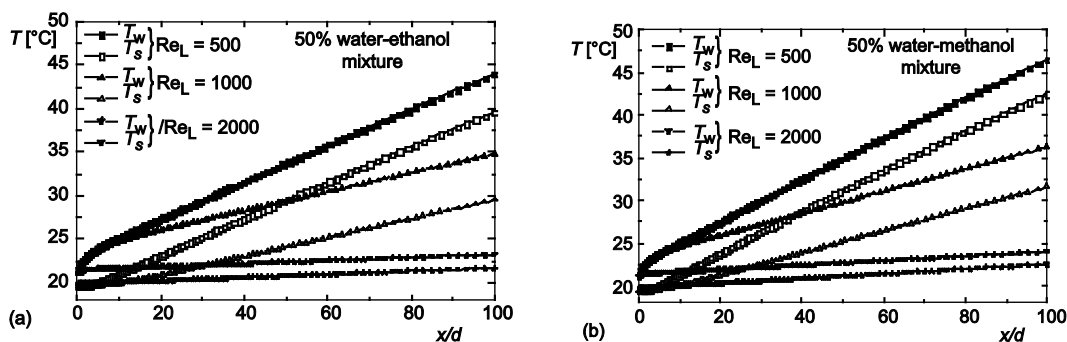


Figure 3. Effect of inlet liquid Reynolds number on the axial evolution of wall and interfacial temperature of (a) water-ethanol film and (b) water-methanol film

The curves also indicate that the wall and interfacial temperatures of water-methanol mixture is higher than the water-ethanol mixture. This results is explained by the fact that methanol component is more volatile than the ethanol component.

The relative importance of the latent heat flux for both mixtures along the tube is illustrated in fig. 4. It is noted that latent heat exchange decrease at the entrance ($x/d < 20$) but as the flow goes downstream ($x/d > 20$) the reverse trend is noticed for lower inlet liquid flow rate $Re_L = 500$ and $Re_L = 1000$. The latent heat flux during water-ethanol and water-methanol evaporation increases with the decrease of inlet liquid Reynolds number. This result can be explained by the fact that an increase of the inlet liquid flow rate induces a decrease of the interfacial temperature and consequently the latent heat flux decreases. This is readily understood by realizing that a reduction in the film flow rate causes greater film evaporation and hence a higher latent heat flux. Thus, it is noted that for the two mixtures considered, the highest latent heat flux is obtained for the liquid film constituted of water-methanol. It is worth to note that at $Re_L = 500$, at the exit tube Q_L/Q_w is always below 12% for water-ethanol mixture, but it is about 25% for water-methanol mixture.

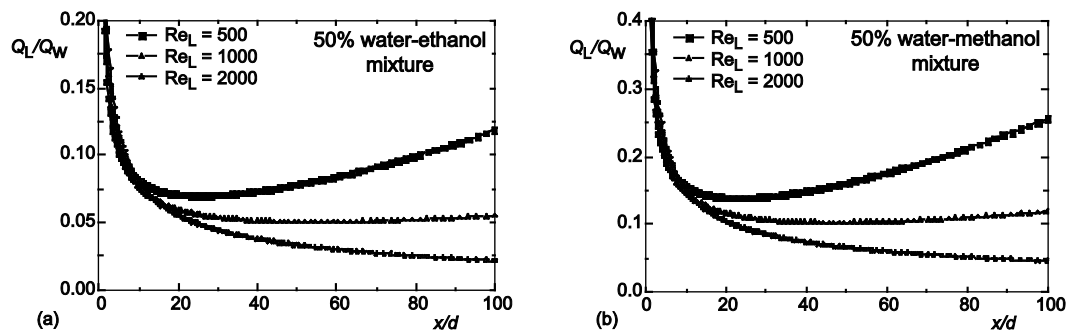


Figure 4. Effect of inlet liquid Reynolds number on the axial evolution of latent heat flux of (a) water-ethanol film and (b) water-methanol film

The total evaporating rate evolution is illustrated in fig. 5. It is observed that this evaporation rate is more important for smaller values of the inlet liquid Reynolds number. This result can be justified by the fact that when we decrease the inlet liquid film flow rate, the total internal energy stored in liquid film evaporates the inlet liquid film flow rate, and

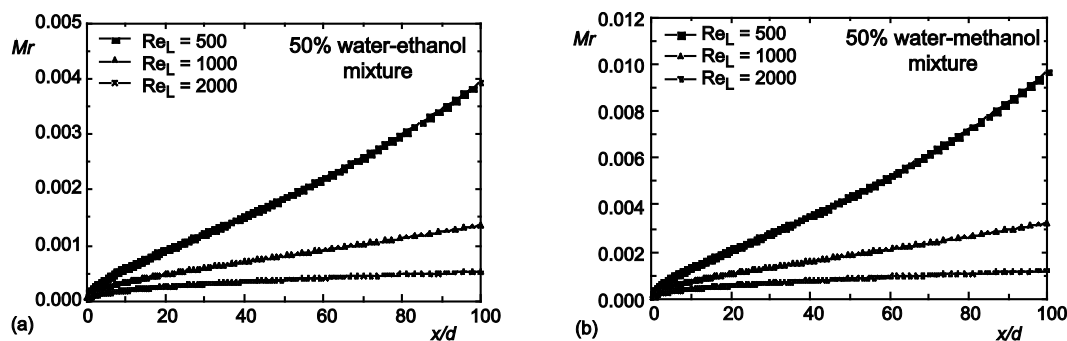


Figure 5. Effect of inlet liquid Reynolds number on the axial evolution of the total accumulated evaporation rate of (a) water-ethanol film and (b) water-methanol film

hence the total accumulated evaporation rate increases. By comparing figs. 5(a) and (b), it is noted that the accumulated evaporation rate of the water-methanol mixture is more important than the water-ethanol mixture. This confirms the results observed previously in fig. 4. The increase of the magnitude of the evaporation is a function of the volatility components in each mixture which is higher for water-methanol mixture than that of the water-ethanol mixture.

Figures 6 and 7 present the axial evolution of the latent heat flux and the total accumulated evaporation rate, respectively, for different conditions. It is observed from figs. 6(a) and (b) that the latent heat flux increases during the evaporation of water-ethanol as the input concentration of the alcohol component in the mixture increase. This result can be justified by the fact that when we increase the inlet liquid concentration of alcohol component, the volatility of the liquid mixture increases (the water is less volatile than the ethanol and methanol), which enhance the evaporation process, and consequently an increase of the heat transferred by latent mode. It becomes apparent, by comparing the magnitude of (Q_L/Q_w) as shown in fig. 6, about 78% of energy supplied from the wall is transported by latent mode using a methanol film, 26% for 50% water-methanol mixture, 21% for ethanol, 12% for 50% water-ethanol mixture and only 7% for water.

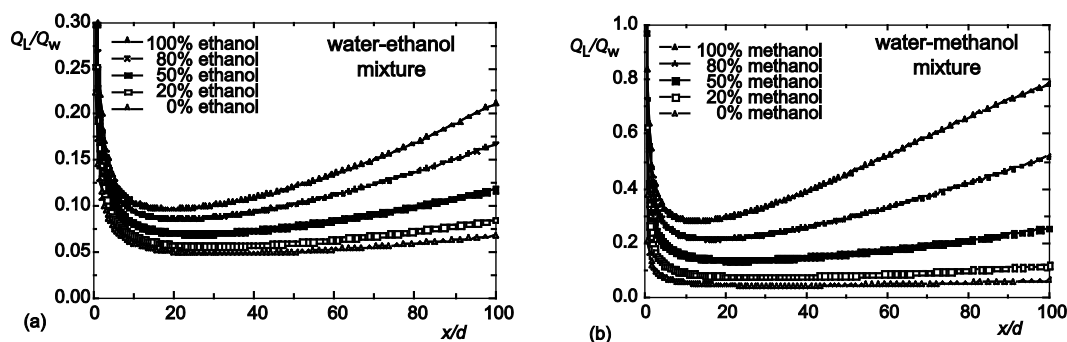


Figure 6. Effect of inlet liquid film composition on the axial evolution of latent heat flux of (a) water-ethanol film and (b) water-methanol film

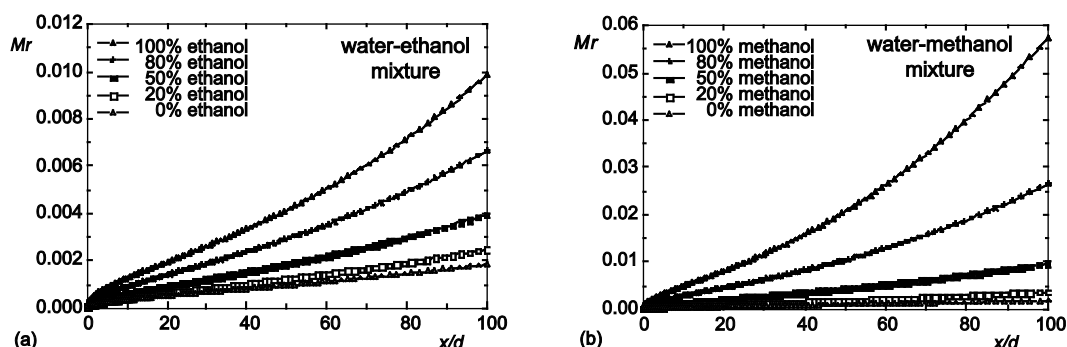


Figure 7. Effect of inlet liquid film composition on the axial evolution of the total accumulated evaporation rate of (a) water-ethanol film and (b) water-methanol film

Figure 7 present the impact of the inlet liquid concentration of alcohol component on the accumulated evaporation rate of both mixtures, the more volatile component (ethanol and methanol) evaporate more dominantly such it's concentration in the liquid mixture in-

crease. In the case of higher evaporation rate at $Re_L = 500$, results reveals that 5.6% for methanol of the inlet liquid mass flow rate evaporates at the end of the tube (1% for 50% water-methanol mixture, 1% for ethanol, 0.4% for 50% water-ethanol mixture and 0.18% for water, respectively).

Conclusions

The evaporation in mixed convection mode of a binary liquid film (water-ethanol and water-methanol mixtures) flows down along a vertical tube under constant heat flux has been numerically studied. The effect of the inlet liquid Reynolds number and the inlet film composition on the heat and mass transfers has been presented and analyzed. The main conclusions from the study are summarized below.

- The heat transferred through the latent mode is more pronounced for mixture composed of more volatile component.
- The mixture evaporating rate Mr increases when the inlet liquid mass flow rate decreases.
- Heat and mass exchanges are more enhanced in mixtures with a higher concentration of volatile component.
- Evaporation in annular flow clearly demonstrates that small portions of alcohol component with water film can contribute significantly to the cooling of heated walls.

Nomenclature

c_p – specific heat at constant pressure, [$Jkg^{-1}K^{-1}$]
 c_{pa} – specific heat for air, [$Jkg^{-1}K^{-1}$]
 c_{pi} – specific heat for species i vapor, [$Jkg^{-1}K^{-1}$]
 D_{im} – mass diffusivity of species i vapor in the gas mixture, [m^2s^{-1}]
 g – gravitational acceleration, [ms^{-1}]
 H – tube length, [m]
 h_{fg} – latent heat of evaporation of mixture, [Jkg^{-1}]
 M_a – molecular weight of air, [$kgmol^{-1}$]
 M_i – molar mass of species i vapor, [$kgmol^{-1}$]
 Mr – total evaporation rate
 Nu_x – local Nusselt number
 P – mixture pressure, [Nm^{-1}]
 P_{vsi} – partial pressure of species saturated i , [Nm^{-1}]
 Pr – Prandtl number
 Q – heat flux, [Wm^{-2}]
 R – tube radius, [m]
 r – radial co-ordinate, [m]
 Re – Reynolds number of gas mixture, ($= u_0d/v_0$)
 Re_L – inlet liquid film Reynolds number, ($= 4\Gamma_{0L}/\pi d\mu_{0L}$)

u – axial velocity, [ms^{-1}]
 v – transversal velocity, [ms^{-1}]
 w – mass fraction
 x – axial co-ordinate, [m]

Greek symbols

Γ_{0L} – inlet liquid flow rate, [$kg s^{-1}$]
 λ – thermal conductivity, [$Wm^{-1}K^{-1}$]
 μ – dynamic viscosity, [$kgm^{-1}s^{-1}$]
 ρ – density, [kgm^{-3}]

Indices

a – dry air
G – vapor phase
i – component indice
L – liquid phase
m – mixture
s – liquid-gas interface
t – turbulent
w – wall condition
0 – inlet condition

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