NUMERICAL STUDY OF MELTING IN AN ANNULAR ENCLOSURE FILLED WITH NANOENHANCED PHASE CHANGE MATERIAL

by

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Heat transfer enhancement during melting in a 2-D cylindrical annulus through dispersion of nanoparticle is investigated numerically. Paraffin-based nanofluid containing various volume fractions of Cu is applied. The governing equations are solved on a non-uniform O type mesh using a pressure-based finite volume method with an enthalpy porosity technique to trace the solid and liquid interface. The effects of nanoparticle dispersion into pure fluid as well as the influences of some significant parameters, namely, nanoparticle volume fraction and natural convection on the fluid flow and heat transfer features are studied. The results are presented in terms of streamlines, isotherms, temperatures and velocity profiles and dimensionless heat flux. It is found that the suspended nanoparticles give rise to the higher thermal conductivity as compared to the pure fluid and consequently the heat transfer is enhanced. In addition, the heat transfer rate and the melting time increases and decreases, respectively, as the volume fraction of nanoparticle increases.

Key words: *nanofluid, phase change material, melting, thermal energy storage, heat transfer enhancement*

Introduction

Continuously increasing energy consumption has led inevitably to the dramatic depletion and scarcity of fossil fuel resources as well as a rise in global warming in recent decades. These effects are significant forces attracting more attention to renewable energy sources, such as wind power and solar energy, as alternatives to conventional sources. New energy storage systems have been developed in recent years as a promising option for providing reliable energy. There are three types of thermal energy storage: sensible heat, latent heat, and thermo-chemical heat. Latent heat storage systems have received considerable attention recently because of their ability to provide high-energy storage density and isothermal behavior of the storage. They have found many applications in areas such as food storage, heating and cooling in building (domestic applications) and refrigeration.

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By choosing suitable storage materials known as phase change materials (PCM), large amounts of heat can be absorbed or released during melting or solidification processes, respectively. Fundamentally, there have been numerous investigations on phase change processes. Alternate melting-freezing heat transfer in composite slabs of single and composite PCM was studied by Gong and Mujumdar [1]. The numerical results indicated that using multiple composite PCM greatly enhances heat flux. Domanski *et al.* [2] experimentally investigated the cooker performance using PCM. It was found that the thermo-physical properties of the PCM strongly affect cooker performance. A comprehensive study on thermal energy storage using solid-liquid phase change was conducted by Zalba *et al*. [3]. The study was structured in three parts: materials, heat transfer and applications. Thermal performance evaluation of hybrid cooling system based on the effect of orientation for PCMs-based heat sinks was done by Wang *et al.* [4]. It was shown that the orientation has limited effect on the thermal performance. PCM as effective technique of storing solar energy were studied by Kenisarin and Mahkamov [5]. They also studied the properties of various PCM and methods of heat transfer enhancement. Sharma *et al*. [6] investigated the available thermal energy storage technology as well as a wide range of PCM applications. The constrained melting of PCM in a spherical capsule was investigated numerically and experimentally by Tan *et al*. [7]. Various characteristics of two mechanisms of heat transfer namely conduction and convection in different regions of a sphere were studied at different times. Ismail and Moraes [8] investigated the effects of different PCM as well as variation of the surface temperature, material and diameter of spherical shells on the time of complete solidification of PCM in the sphere and cylindrical shells. Cabeza *et al*. [9] presented a review of recent publications on the use of PCM in buildings. The review deals with classification of materials, materials available, and different solutions to the problems of using such materials in the past. Thermal energy storage with four tubes inside a cylindrical tank filled with PCM was studied by Tay *et al.* [10] using a CFD model. The numerical results obtained were compared to experimental results indicating the high ability of the CFD model to predict accurately the behavior of the thermal energy storage system during charging and discharging.

Also, several researches were done on melting of phase change material in annulus. Ng *et al*. [11] simulated the convection-dominated melting of a PCM in a horizontal annulus heated isothermally from the inside wall using the finite element method. They found that a multiple cellular pattern formed at high Rayleigh numbers increased the heat transfer rate. Melting of a pure PCM in a horizontal annulus of arbitrary cross-section was studied numerically by Khillarkar *et al.* [12]. They found that the effect of heating both walls was the same as heating the inside wall and the outside wall separately until there was interaction between the two melt zones. A numerical study of melting inside concentric and eccentric horizontal annuli was conducted by Darzi *et al.* [13]. They investigated the conduction and convection mechanism through the melting process.

Nanotechnology has great potential for enhancing thermal conductivity and consequently higher thermal performance as a result of the presence of nanoparticle in the fluids. The pioneering theoretical work on a new class of fluid namely high-thermal-conductivity nanofluids was carried out by Choi [14]. He showed that a dramatic reduction in heat exchange pumping power is one of the advantageous of nanofluids. Natural convection heat transfer enhancement in cylindrical annulus using a wide range of volume fractions of nanoparticle at different Rayleigh numbers was investigated by Abu-Nada *et al.* [15]. Abu-Nada [16] also evaluated the effects of different viscosity and thermal conductivity of Al_2O_3-water nanofluid on heat transfer enhancement of natural convection in horizontal annuli. High thermal conductivity is one of the major factors which determines the ability of PCM to store or release latent heat in a

given volume of the storage material in a short time. Since a significant deficiency of PCM is their low thermal conductivity, many researchers have attempted to overcome this weakness through various methods such as using composite PCM [17-19], inserting fins [20] and nanoparticle-enhance phase change materials (NEPCM). The first report of using nanoparticles in PCM known as NEPCM was presented by Khodadadi and Hosseinizadeh [21]. The NEPCM show enhanced thermal conductivity compared to conventional PCM. This indicated the great potential of NEPCM for thermal energy storage applications. A numerical study of heat transfer enhancement during solidification in horizontal annuli filled with NEPCM using an enthalpy porosity technique with finite volume approach was conducted by Sebti *et al.* [22]. The results revealed that as the nanoparticle volume fraction increased, the rate of heat transfer increased. Ranjbar *et al.* [23] reported the results of solidification in a 3-D rectangular enclosure based on the various parameters affecting the solidification. Kashani *et al.* [24] performed a numerical study of solidification of NEPCM in an enclouser with two wavy walls. The significant factors that affect the solidification of a Cu-water nanofluid such as surface waviness and nanoparticle dispersion were investigated.

To the best of our knowledge, no attempts have been made as yet to study the effects of nanoparticle dispersion into pure fluid on heat transfer characteristics to explore the conditions for heat transfer enhancement during the melting process in a horizontal cylindrical annulus. The present study aims to numerically investigate the melting of Cu-paraffin nanofluid as NEPCM in a two-dimensional horizontal cylindrical annulus using the enthalpy porosity technique. Particular attention is paid to examine the effects of volume fractions of Cu ranging from 0 to 0.075 on the melting performance and natural convection.

Mathematical formulation

The schematic of the cavity configuration as well as the co-ordinate system employed in the present study are shown in fig. 1. The physical model under consideration is a two-dimensional concentric cylindrical annulus with inner and outer radius of 2.5 mm and 7.5 mm, respectively, filled with Cu-paraffin nanofluid as phase change material. The nanofluid including nanoparticles and base fluid as continuous media is Newtonian, incompressible and assumed to be in thermal equilibrium as well as no-slip condition is imposed between them. As shown in tab. 1. thermo-physical properties are considered to be constant except the density which is modeled by Boussinesq approximation to account for the thermal buoyancy. The inner cylinder is maintained at the constant temperature above the melting temperature of mixture whereas the outer cylinder is assumed to be thermally insulated. The temperature difference

Figure 1. Physical configuration and co-ordinate system

between the inner cylinder and melting temperature of mixture is 10 °C. With respect to the above mentioned assumptions the governing equations can be written as:

Table 1. Thermo-physical properties of nanofluid

	Cu nanoparticle	Base fluid $\phi = 0.000$	Nanofluid $\phi = 0.025$	Nanofluid $\phi = 0.050$	Nanofluid $\phi = 0.075$
ρ [kgm^{-3}]	8954	780	984.35	1188.7	1393.05
μ [Pa·s]		0.00287	0.003057	0.00326	0.003487
c_p [Jkg^{-1}K^{-1}]	383	2310	1871.78	1584	1381
k [Wm^{-1}K^{-1}]	400	0.1505	0.1620	0.1742	0.1870
α [m^{2}s^{-1}]	1.166e–4	8.35e–8	8.79e–8	9.25e–8	9.72e–8
β [K^{-1}]		9.1e–4	0.000706	0.00057	0.00047
L [Jkg^{-1}]		228900	176846	142689	118553
Pr		44.06	35.31	29.66	25.74
Fusion point [°C]		18.3	18.3	18.3	18.3

– *continuity*

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

– *momentum equations*

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}}\left(-\frac{\partial P}{\partial x} + \mu_{nf}\nabla^2 u\right) + S_x \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho_{nf}}\left[-\frac{\partial P}{\partial y} + \mu_{nf}\nabla^2 v + (\rho\beta)_{nf}g(T - T_{ref})\right] + S_y \tag{3}$$

– *energy equation*

$$\frac{\partial(C_{p,nf}T)}{\partial t} + u\frac{\partial(C_{p,nf}T)}{\partial x} + v\frac{\partial(C_{p,nf}T)}{\partial y} = \frac{\partial}{\partial x}\left[\frac{k_{eff}}{(\rho C_p)_{nf}}\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{k_{eff}}{(\rho C_p)_{nf}}\frac{\partial T}{\partial x}\right] - S_h \tag{4}$$

The density of nanofluid is written as:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{5}$$

Whereas the heat capacities of the nanofluid and part of the Boussinesq term can be expressed as:

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s \tag{6}$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s \tag{7}$$

The effective dynamic viscosity of the nanofluid given by Brinkman [25], is:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{8}$$

The effective thermal conductivity of the nanofluid for spherical nanoparticles, according to Maxwell [26], is:

$$\frac{k_{eff}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \tag{9}$$

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Numerical procedure

A home-generated FORTRAN code based on the finite volume method on collocated body-fitted grids using the SIMPLE algorithm has been developed to solve the governing equations. The discre- tized equations have been solved using a strongly implicit procedure (SIP). The enthalpy--porosity [27, 28] method is applied to simulate the NEPCM melting in annulus. In this method, the fraction of each cell volume which is in liquid phase is set equal to porosity in that cell. The liquid fraction is computed at each iteration, based on the enthalpy balance. Due to the steep gradient of the flow properties near the cavity walls, finer grid has to be used in these regions to obtain accurate numerical solutions. Therefore, the grid becomes finer towards the inner and outer walls of the cavity. To test the mesh dependency of the solution, five different grids systems 64×20, 82×50, 82×60, 102×70, and 122×80 are chosen. Figure 2 shows the effects of grid size on the variation of dimensionless heat flux at $t = 500$ s during the melting in annulus. From fig. 2, the numerical results are computed on the grid of 82×60, since the gird independency results do not show much variation between the grid size of 82×60 and finer grid sizes.

Validation of the numerical procedure

To verify the validity and accuracy of the results of present study, the obtained results are compared with other available numerical and experimental results. In order to evaluate the ability of the enthalpy-porosity method in modeling of phase change process the experimental and numerical results of melting gallium in a rectangular cavity obtained by Gau *et al.* [29] and Brent *et al.* [30], respectively, are compared with the present results. The top and bottom walls are assumed to be thermally insulated and the right and left walls are kept in constant low (cooling) and high (heating) temperatures, respectively. Figure 3(a) displays the results of melting front progression with time in present work and those obtained by Gau and Brent. As it is clear from fig. 3(a), there is an acceptable agreement between the results. The next validation is per-
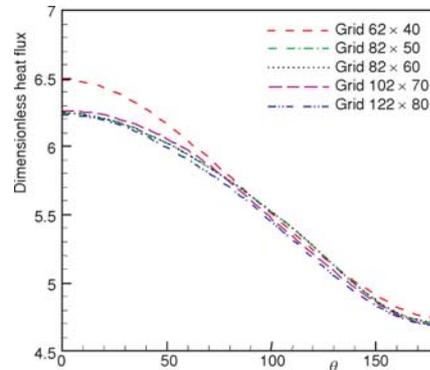
Figure 2. Effects of grid size on the variation of dimensionless heat flux at *t* = 500 s during the melting in annulus
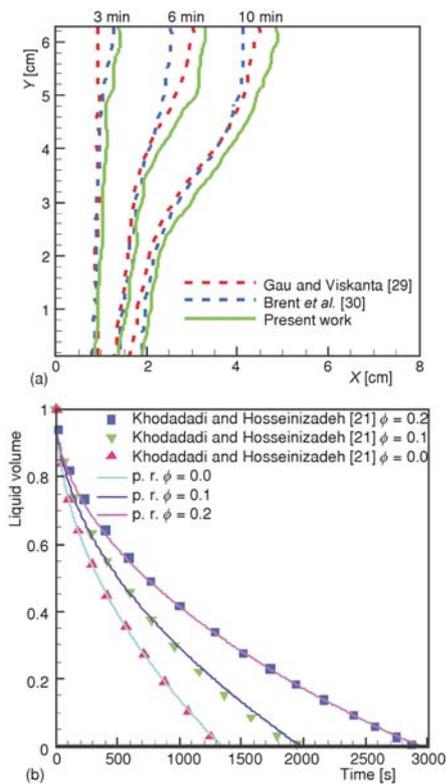
Figure 3. (a) Melting phase front progression with time: comparison of the present results with those of Gau and Viskanta [29] and Brent *et al.* [30], (b) change in the liquid volume fraction with time: comparison of the present results (p. r.) with those of Khodadadi and Hosseinzadeh [21]

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formed through the comparison of the present results of NEPCM solidification with the results of Khodadadi *et al.* [21]. A good agreement between the results can be observed in fig. 3 (b).

Results and discussion

Figure 4 shows the results of streamlines and isotherms of the nanofluid at various times for different volume fractions of nanoparticles in the melting process. The right and left halves-sections of the cavity indicate the streamlines and temperature contours, respectively, as shown in fig. 4. Note that the red and blue colors (see journal web site) are indicatives of liquid and solid phases, respectively. It should be mentioned that the patterns of isotherms and streamlines change with the increase in the volume fraction of the nanoparticles and time.

Since the temperature of the hot wall is maintained over the melting temperature of the nanofluid, the nanofluid starts melting on the inner cylinder. As it is obvious from fig. 4, the melting occurs faster in upper region of the cavity compared to the lower region due to the higher temperature in the upper section. It is worth noting that at the beginning of the melting process, isotherms are parallel lines near the inner cylinder implying that the domination of the conduction mechanism and convection mode cannot play a significant role in heat transfer due to its limited range of motion as a result of the resistance imposed by viscose forces. This situation is thermally unstable, therefore as time goes by, the phase front moves from the inner cylinder toward the outer cylinder and the liquid phase occupies a major part of the cavity expediting the melting process and advection in the liquid phase. So near the inner cylinder, the natural convection flow occurs and the flow moves upwards in close proximity to the hot wall which has a significant effect on the thermal field and consequently the speed of phase front travelling. At $t = 500$ s the liquid-solid interface is more flat compared to that at $t = 1000$ s when it is more distorted. The advancement of the liquid-solid interface where the hot fluid moves upwards is more pronounced than near the outer cylinder where the cold nanofluid descends. After a long enough time, thermal stratification and departure of the isotherms from being parallel or distortion of isotherms occurs, the convection mode becomes more dominant and a thermal boundary layer develops near the inner cylinder. Far enough from the inner cylinder (*i. e.*, in the central region of the cavity), the flow is not much affected by thermal boundary layers and hence the tempera-
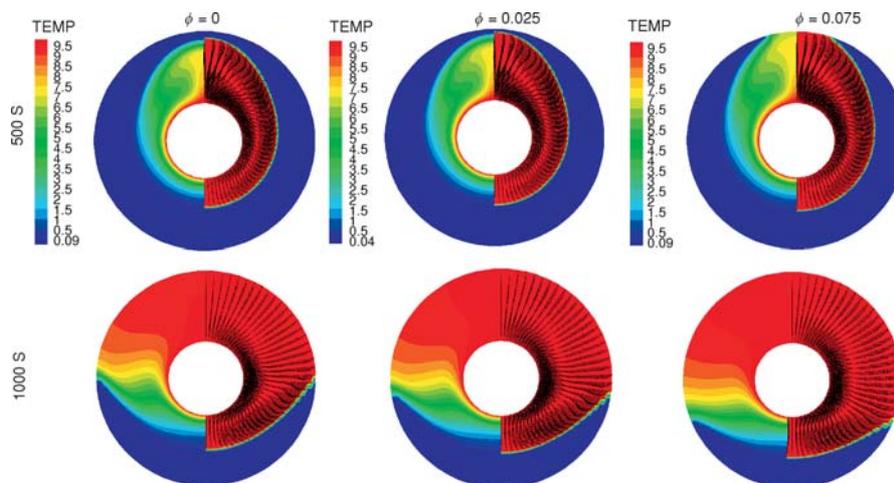
Figure 4. Streamlines (right) and isotherms (left) in the melting zone using various nanoparticle volume fractions of Cu at different times (for color image see journal web site)

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ture gradient is not severe. As the volume fraction of nanoparticle increases, the isotherms in this region become almost horizontal which restricts the vertical motion of the flow. It should be noted that by adding nanoparticles to the base fluid the heat transfer rate is enhanced because of the increase in thermal conductivity of nanofluid. It is obvious from fig. 4 that an increase in the nanoparticle volume fraction results in higher rate of phase front progression at the same times and consequently heat transfer enhancement.

The temperature distribution and velocity variation of nanofluid with various volume fractions of nanoparticle on radial plane of angle 90 at different times are depicted in figs. 5(a) and (b), respectively. According to these figures, the temperature is found to increases with the increasing volume fraction of the nanoparticles causing an enhanced heat transfer rate especially via a conduction mechanism. The temperature distribution tends to become more uniform as time goes by and this occurs faster for higher volume fractions of nanoparticle. It is worth mentioning that the increase of volume fraction of nanoparticle has two major effects. The first consequence is the enhanced thermal conductivity leading to higher rate of heat transfer and the second one is increasing of nanofluid viscosity resulting in decreasing of velocity field.

A better understanding of the effect of nanoparticle addition to a base fluid on the rate of heat transfer through convection and conduction modes requires an accurate study of dimensionless heat flux ($\partial T^*/\partial r^*$) on the hot wall. Figure 6 shows the variation of dimensionless heat flux along the hot wall for various volume fractions of nanoparticle at different times. This
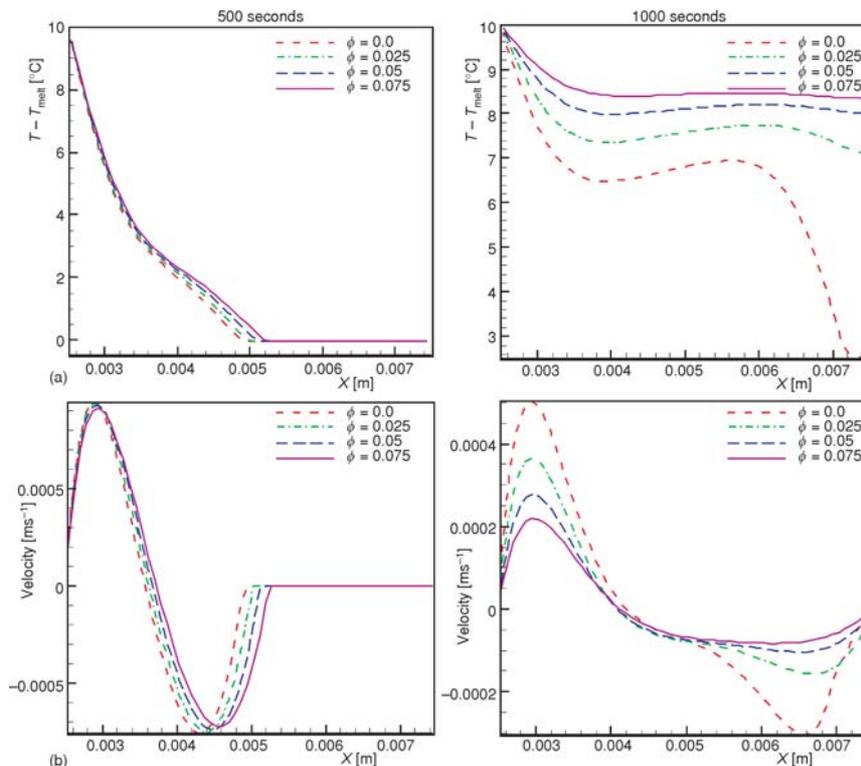
Figure 5. (a) Temperature distribution along the radial plane of angle 90 using various nanoparticle volume fractions at different times, (b) velocity distribution along the radial plane of angle 90 using various nanoparticle volume fractions at different times

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parameter can be considered as a local dimensionless temperature gradient indicating the ratio of conductive resistance to convective resistance.

As the volume fraction of nanoparticle increases, the dimensionless heat flux decreases in spite of the increasing heat transfer rate. With respect to the reduction of local dimensionless temperature gradient on the inner cylinder on the one hand and increase in thermal conductivity of nanofluid resulting in decreased conductive thermal resistance on the other hand, it can be noted that increasing volume fraction of nanoparticle can lead to decrease in convective thermal resistance. But the rate of decrease in conductive thermal resistance is much higher than that in convective thermal
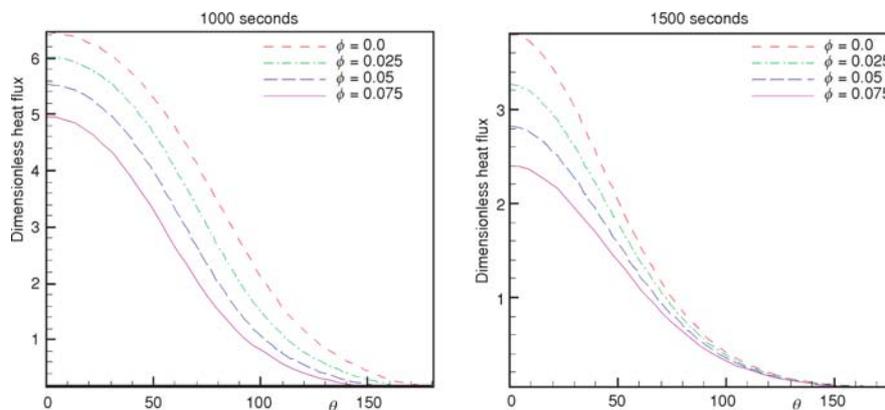
Figure 6. Dimensionless heat flux distribution along the inner cylinder using various nanoparticle volume fractions at different times
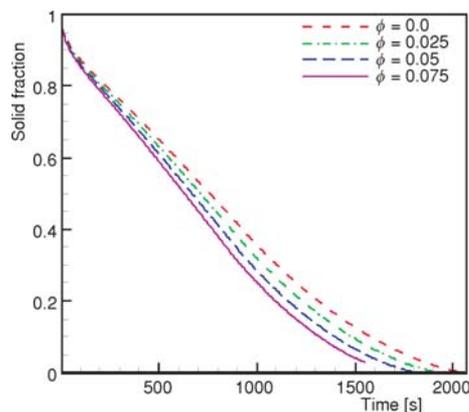
Figure 7. Solid fraction variation with time using various nanoparticle volume fractions

To study the effect of nanoparticle addition on time of melting process which is an important factor in the design of latent thermal energy storage systems, the variation in the solid fraction over time for various volume fractions of nanoparticle is investigated in fig. 7. As mentioned before, since the nanoparticle presence in base fluid leads to higher thermal conductivity as well as lower latent heat, it has an effective role in reducing the time of melting process. It can be concluded from fig. 7 that adding nanoparticle volume fraction of 7.5%, the total melting time decreases by approximately 15% compared to pure fluid

Conclusions

In this paper, a numerical investigation of melting in a two-dimensional cylindrical annulus filled with nanoenhanced phase change material (NEPCM) was carried out. The effects of nanoparticle dispersion during melting with various volume fractions of Cu is studied numerically using a pressure based finite volume method with an enthalpy porosity technique. The fluid flow and the shape of phase front depend on the liquid layer thickness during the progress of melting. At the beginning of the melting process, conductive heat transfer dominates and iso-

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therms are parallel near the hot wall. Buoyancy-driven convection is strengthened and melting occurs faster in the top region of the cavity. The solid fraction decreases approximately linearly with time but will be late at the end of the melting process. It is found that the dispersion nanoparticle in a phase change material results in increase of thermal conductivity and enhanced heat transfer rate and consequently, decreases the melting time. Furthermore, the rate of heat transfer increased by increasing nanoparticle volume fraction. On the other hand, the volume occupied by pure fluid is diminished by increasing volume fraction of nanoparticle which in turn decreases the amount of energy stored by nanofluid. Finally, it can be concluded that using proper suspension of nanoparticle into conventional phase change materials has the great potential of improving the traditional energy storage system.

Nomenclature

C_p – specific heat at constant pressure, $[\text{Jkg}^{-1}\text{K}^{-1}]$
g – gravitational acceleration, $[\text{ms}^{-2}]$
H – enthalpy, $[\text{Jkg}^{-1}]$
k – thermal conductivity $[\text{Wm}^{-1}\text{K}^{-1}]$
L – latent heat, $[\text{Jkg}^{-1}]$
p – pressure [Pa]
Pr – Prandtl number
r – radial co-ordinate [m]
$r*$ – $(r - r_i)/(r_0 - r_i)$
S – source term
T – temperature [K]
$T*$ – $(T - T_{met})/(T_i - T_{melt})$
t – time [s]
u – velocity component in x-direction, $[\text{ms}^{-1}]$
v – velocity component in y-direction, $[\text{ms}^{-1}]$
x – horizontal co-ordinate, [m]
y – vertical co-ordinate, [m]

Greek symbols

α – thermal diffusivity, $[\text{m}^2\text{s}^{-1}]$
β – thermal expansion coefficient, $[\text{K}^{-1}]$
θ – angle, [rad]
ρ – fluid density, $[\text{kgm}^{-3}]$
μ – dynamic viscosity, $[\text{Nsm}^{-2}]$
ϕ – volume fraction of nanoparticles

Subscripts

i – inner wall
o – outer wall
nf – nanofluid
x – along x-axis
y – along y-axis
h – enthalpy
s – solid
f – base fluid
eff – effective
melt – melting

References

[1] Gong, Z. X., Mujumdar, A. S., Enhancement of Energy Charge-Discharge Rates in Composite Slabs of Different Phase Change Materials, *International Journal of Heat and Mass Transfer, 39* (1995), 4, pp. 725-733
[2] Domanski, R., *et al*., Cooking During Off-Sunshine Hours Using PCMs as Strong Media, *Energy, 20* (1995), 7, pp. 607-616
[3] Zalba, B., *et al*., Review on Thermal Energy Storage with Phase Change: Materials, Heat Transfer Analysis and Applications, *Applied Thermal Engineering, 23* (2003), 3, pp. 251-283
[4] Wang, X. Q., *et al.*, Effect of Orientation for Phase Change Material (PCM)-Based Heat Sinks for Transient Thermal Management of Electric Component, *International Communications in Heat and Mass Transfer, 34* (2007), 7, pp. 801-808
[5] Kenisarin, M., Mahkamov, K., Solar Energy Storage Using Phase Change Materials, *Renewable and Sustainable Energy Reviews, 11* (2007), 9, pp. 1913-1965
[6] Sharma, A., *et al*., Review on Thermal Energy Storage with Phase Change Materials and Applications, *Renewable and Sustainable Energy Reviews, 13* (2009), 12, pp. 318-45
[7] Tan, F. L., *et al*., Experimental and Computational Study of Constrained Melting of Phase Change Materials (PCM) Inside a Spherical Capsule, *International Journal of Heat and Mass Transfer, 52* (2009), 15-16, pp. 3464-3472
[8] Ismail, K. A. R., Moraes, R. I. R., A Numerical and Experimental Investigation of Different Containers and PCM Options for Cold Storage Modular Units for Domestic Applications, *International Journal of Heat and Mass Transfer, 52* (2009), 19-20, pp. 4195-4202

[9]   Cabeza, L. F., *et al.*, Materials Used as PCM in Thermal Energy Storage in Buildings: A Review, *Renewable and Sustainable Energy Reviews, 15* (2011), 3, pp. 1675-1695

[10] Tay, N. H. S., *et al.*, Experimental Validation of a CFD Model for Tubes in a Phase Change Thermal Energy Storage System, *International Journal of Heat and Mass Transfer, 55* (2012), 4, pp 574-585

[11] Ng, K. W., *et al.*, Heat Transfer in Free Convection-Dominated Melting of a Phase Change Material in a Horizontal Annulus, *International Communications in Heat and Mass Transfer, 25* (1998), 5, pp 631-640

[12] Khillarkar, D. B., *et al.*, Melting of a Phase Change Material in Concentric Horizontal Annuli of Arbitrary Cross-Section, *Applied Thermal Engineering, 20* (2000), 10, pp. 893-912

[13] Darzi, A. R., *et al.*, Numerical Study of Melting Inside Concentric and Eccentric Horizontal Annulus, *Applied Mathematical Modelling, 36* (2012), 9, pp. 4080-4086

[14] Choi, U. S., Enhancing Thermal Conductivity of Fluids with Nanoparticles, in: *Developments and Application of Non-Newtonian Flows* (Eds. D. A. Singer, H. P. Wang), *FED-Vol. 231/MD-Vol. 66, ASME, New York, USA, 66* 1995, pp. 99-105

[15] Abu-Nada, E., *et al.*, Natural Convection Heat Transfer Enhancement in Horizontal Concentric Annuli Using Nanofluids, *International Communications in Heat and Mass Transfer, 35* (2008), 5, pp. 657-665

[16] Abu-Nada, E., Effects of Variable Viscosity and Thermal Conductivity of Al_2O_3-Water Nanofluid on Heat Transfer Enhancement in Natural Convection, *International Journal of Heat and Fluid Flow, 30* (2009), 4, pp. 679-690

[17] Bauer, T., *et al.*, PCM-Graphite Composites for High Temperature Thermal Energy Storage. *Proceedings*, ECOSTOCK, 10th International Conference on Thermal Energy Storage, Stockton, USA, 2006

[18] Fang, X., *et al.*, Study on Preparation of Montmorillonite-Based Composite Phase Change Materials and their Applications in Thermal Storage Building Materials, *Energy Conversion and Management, 49* (2008), 4, pp. 718-723

[19] Sari, A., Karaipekli, A., Preparation, Thermal Properties and Thermal Reliability of Palmitic Acid/Expanded Graphite Composite as Form-Stable PCM for Thermal Energy Storage, *Solar Energy Materials and Solar Cells, 93* (2009), 5, pp. 571-576

[20] Shatikian, *et al.*, Numerical Investigation of a PCM-Based Heat Sink with Internal Fins, *International Journal of Heat and Mass Transfer, 48* (2005), 17, pp. 3689-3706

[21] Khodadadi, J. M., Hosseinizadeh, S. F., Nanoparticle-Enhanced Phase Change Materials (NEPCM) with Great Potential for Improved Thermal Energy Storage, *International Communications in Heat and Mass Transfer, 34* (2007), 5, pp. 534-543

[22] Sebti, S. S., *et al.*, A Numerical Investigation of Solidification in Horizontal Concentric Annuli Filled with Nano-Enhanced Phase Change Material (NEPCM), *World Applied Sciences Journal, 13* (2011), 1, pp. 9-15

[23] Ranjbar, A. A., *et al.*, Numerical Heat Transfer Studies of a Latent Heat Storage System Containing Nano-Enhanced Phase Change Material, *Thermal Science, 15* (2011), 1, pp. 169-181

[24] Kashani, S., *et al.*, Solidification of Nano-Enhanced Phase Change Material (NEPCM) in a Wavy Cavity, *Journal of Heat and Mass Transfer, 48* (2012), 7, pp. 1155-1166

[25] Brinkman, H. C., The Viscosity of Concentrated Suspensions and Solution, *Journal of Computational Physics, 20* (1952), pp. 571-581

[26] Maxwell, J., A *Treatise on Electricity and Magnetism*, Oxford University Press, Cambridge, UK, 1904

[27] Voller, V. R., Prakash, C., A Fixed-Grid Numerical Modeling Methodology for Convection-Diffusion Mushy Region Phase-Change Problems, *International Journal of Heat and Mass Transfer, 30* (1987), 8, pp. 1709-1719

[28] Dutta, R., *et al.*, Experimental and Numerical Study of Heat Transfer in Horizontal Concentric Annulus Containing Phase Change Material, *Canadian Journal of Chemical Engineering, 86* (2008), 4, pp. 700-710

[29] Gau, C., Viskanta, R., Melting and Solidification of a Pure Metal on a Vertical Wall, *Journal of Heat Transfer, 108* (1986), 1, pp. 174-181.

[30] Brent, A. D., *et al.*, Enthalpy-Porosity Technique for Modeling Convection- Diffusion Phase Change: Application to the Melting of a Pure Metal, *Numerical Heat Transfer, 13* (1988), 3, pp. 297-318