NUSSELT NUMBER EVALUATION FOR COMBINED RADIATIVE AND CONVECTIVE HEAT TRANSFER IN FLOW OF GASEOUS PRODUCTS FROM COMBUSTION

by

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Combined convection and radiation in simultaneously developing laminar flow and heat transfer is numerically considered with a discrete-direction method. Coupled heat transfer in absorbing emitting but not scattering gases is presented in some cases of practical situations such as combustion of natural gas, propane, and heavy fuel. Numerical calculations are performed to evaluate the thermal radiation effects on heat transfer through combustion products flowing inside circular ducts. The radiative properties of the flowing gases are modeled by using the absorption distribution function model. The fluid is a mixture of carbon dioxide, water vapor, and nitrogen. The flow and energy balance equations are solved simultaneously with temperature dependent fluid properties. The bulk mean temperature variations and Nusselt numbers are shown for a uniform inlet temperature. Total, radiative and convective mean Nusselt numbers and their axial evolution for different gas mixtures produced by combustion with oxygen are explored.

Key words: *gas mixture, non-gray medium, convection, thermal radiation, heat transfer, carbon dioxide, water vapor, combustion, numerical simulation*

Introduction

For systems at high temperature such as recuperating energy system from combustion products, regenerating high-temperature heat exchangers, cooling processes in nuclear reactors, industrial furnaces, combustion chambers and re-entry applications, accurate and computationally efficient radiative transfer model is the key factor. In fact, the radiative transfer has sufficiently large effect on thermal and flow fields to require accurate calculation on simultaneously fluid flow and heat transfer.

The assessment of radiation effects on thermal and flow fields usually involves an iterative procedure. To solve the geometrical part of radiative transfer, comparison of various mathematical approaches with different radiation models (discrete ordinate model, Hottel zone model, discrete transfer method, spherical harmonics model, Monte Carlo model) have been done by Crnomarkovic *et al.* [1]. The popular discrete transfer radiation method (often called discrete transfer method or discrete directions method) is used for general radiation predictions.

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This method is easily embedded into computational fluid dynamics (CFD) codes. Baburić *et al.* [2] implemented this technique into a non-structured finite volume CFD software together with the weighted sum of gray gases (WSGG) for the radiative gas.

The complex absorption and emission spectra of combustion products such as H_2O, CO_2, CO, introduce important difficulties in the simulation of the radiative heat transfer at high temperature. On one hand, the complexity of the problem modeling is due to the non-gray radiative behavior of gas mixtures and on the other hand it is due to the multi-dimensional and directional nature of radiative transfer.

The presence of radiating species in flow of combustion products affects the total heat flux directly via the temperature distribution in the fluid and indirectly through the temperature-dependent thermo-physical properties due to the involved large temperature variations.

For radiatively participating media, the interaction of thermal radiation and convection, when considering a radiation field with its spectral nature and with or without taking into account the 3-D radiative flux, has been studied by many investigators with various degrees of approximations.

The models for thermal radiation transfer in view of their potential applications to combustion systems are presented as part of a computational fluid dynamics analysis by Hiu *et al.* [3], Saario *et al.*[4], Dombrovsky *et al.* [5], and Filkoski *et al.* [6] among others. Recently, Caliot *et al.* [7] studied the effects of non-gray thermal radiation on the heating of a methane laminar flow at high temperature. Authors implemented a global model for radiative properties of methane in a widely used computational fluid dynamics code where radiative transfer problem is solved by using the discrete ordinate method.

A study of wide band radiation models applied to non-homogeneous combustion systems is presented by Cumber *et al*. [8]. Cumber and Fairweather [9] predict the radiation fluxes from jet fires and flame tubes by application of a number of flame emission models in conjunction with the discrete ordinate method. Thermal radiation in combustion system has been also investigated by Pessoa-Filho [10] where the non-gray behavior effects are treated by using a narrow band model. Dembele and Wen [11] developed a spectral formulation for radiative heat transfer in one dimensional fires and combustion systems. They present a comparison between results given by the correlated-k band model coupled with the discrete ordinate method and some theoretical and measured radiative intensities for a natural gas flame. Kontogeorgos *et al.* [12] suggest a simplified thermal radiation models which can be easily applied in engineering calculations.

From a radiative behavior viewpoint, different models of thermal radiation useful for engineers have been discussed. The suggested approaches varied from gray to non gray models. The previous studies reveal that the rigorous analysis of emissivities and absorptivities of realistic non-grey combustion gases would require rather complicated and time-consuming computations taking into account the effects of different emitting and absorbing bands in molecular spectra. In fact, the gas radiative properties vary generally so strongly and rapidly across the spectrum and the rigorous and accurate line-by-line radiative property model is generally not practicable in 2-D or 3-D geometries [13-16]. Fortunately, for most engineering problems, these computations can be avoided, since the radiative properties of most practically important combustion gases can be approximated by a global model such as weighted sum of gray gases (WSGG) or absorption distribution function (ADF). The ADF model seems to have clear advantages over the others when a compromise between accuracy and computational efficiency is required [13-15].

In this paper, special attention is devoted to the treatment of the spectral nature of radiation in real H_2O-CO_2-N_2 mixtures produced, at high temperature, by combustion with air or with oxygen of propane, natural gas and heavy fuel.

To solve the geometrical part of radiative transfer, a discrete-direction method is simultaneously applied with the parabolic boundary-layer equations of the flow. The radiative properties of the flowing gases are modeled by using the absorption distribution function (ADF) model.

To solve the mass, momentum and energy equations, an implicit finite difference technique is used. The flow equations and energy balance equation are solved simultaneously with temperature-dependent fluid properties. The resolution of the problem is undertaken in dimensional form. As the velocity and temperature fields obtained by numerical solution cannot be easily used for engineering purposes, total Nusselt number with combined convection/radiation and Nusselt number with pure convection, for different gaseous combustion products for cooled flows inside circular ducts is presented.

Although the dimensionless treatment has a limited utility in the case of realistic gas mixtures band radiation, the main aim of this study is to give a first-order estimation calculation of Nusselt number which can be useful for engineering.

In the following section, we recall briefly the thermal radiation modeling for the combustion products. In the next section, we present the basic formulation and the computational procedure. The main results of the paper are summarized in the last section.

Thermal radiation modelling for combustion products

The modelling of thermal radiation in conjunction with solutions of fluid dynamics equations for combustion products at high temperature is a difficult task. The fundamental difficulty introduced by thermal radiation in real gases at high temperature is related to the treatment of its spectral nature. The spectral intensity, as the solution of the radiative transfer equation, is the fundamental quantity of interest. It is a function of location, orientation, wave number and it depends in a complex way on the temperature. To determine the spectral radiation fields, the radiation flux vector and its divergence, which need to solve the thermal energy equation, solution of combined heat transfer problems requires both the knowledge of the spectral radiative properties and efficient methods for solving the radiative transfer equation (RTE).

The high-resolution structure of emission and absorption spectra of combustion products like H_2O, CO_2, CO generates spectral correlation effects between emission, transmission and absorption by consecutive elementary columns. These effects strongly modify the local intensity field and consequently the radiative fluxes and the volumetric radiative source term at each point of the medium.

To solve the radiative transfer problem, we use in this study the discrete-direction method, developed initially for radiative transfer in axisymmetric systems using statistical narrow-band models [17, 18]. This method is used to solve the geometric part of radiative transfer equation. At first, a directional integration quadrature is used for a regular integration, with prescribed angular increments and solid angles. After that, intensity interpolations are used over control volumes. A detailed description of this method applied for cylindrical enclosure with the correlated-k band model or with the ADF model, is given in refs. [17, 19].

The absorption distribution function (ADF) infrared radiative property model [7, 14, 15] used in this work takes into account the spectral correlation phenomena. Parameters of ADF models are deduced from line-by-line calculations founded on the same high temperature has to be written (Energetics and Combustion Laboratory – E.M2.C) spectroscopic data bases [20, 21].

Some details related to the model principles, parameters and the practical way to use this approach in radiative transfer calculations can be found in the references given in [11-16].

The ADF model is a global model based on a cumulative distribution function of the absorption coefficient depending on two temperatures. This approach is able to treat the spectral correlations between the spectra at different points of a non-homogenous or non-isothermal medium. It takes into account the variations of the gas absorption coefficient like in the narrow band model but it is extended to the whole spectrum. The absorption coefficient values are discretized into N ranges for a reference thermo-physical condition (molar fraction of the absorbing species, local pressure and temperature).

Table 1. Different molar fractions and partial pressures for combustion products

Hydrocarbon	Combustion with oxygen	Combustion with air
Natural gas	$X_{H_2O} = 0.66$, $X_{H_2O}/X_{CO_2} = 1.9$	$X_{H_2O} = 0.19$, $X_{H_2O}/X_{CO_2} = 1.9$
Propane	$X_{H_2O} = 0.57$, $X_{H_2O}/X_{CO_2} = 1.33$	$X_{H_2O} = 0.155$, $X_{H_2O}/X_{CO_2} = 1.33$
Heavy fuel	$X_{H_2O} = 0.45$, $X_{H_2O}/X_{CO_2} = 0.8$	$X_{H_2O} = 0.114$, $X_{H_2O}/X_{CO_2} = 0.8$

If k_j designates a mean value of absorption coefficient inside the range $[k_j^-, k_j^+]$ at the reference thermo-physical condition (X_{ref}, p_{ref}, T_{ref}) the value of k_j in any other thermo-physical condition X, p, T is given by an implicit relation [14-16] which constitutes the correlation approximation. The ADF parameters used in this study are those developed in [14-16].

This model, called ADF8 (N = 8), uses 7 non-zero absorption coefficients and one (equal to zero) corresponding to the transparency regions of the gas. More details on model parameters are given in [16, 22]. Authors conclude that no significant differences appear between ADF8 and ADF50 results.

Different implementations of the ADF model in the case of absorbing gas mixtures are discussed by Taine and Soufiani [22]. Comparisons between the results from ADF model and from the more elaborated correlated-k band model, applied to the study of combined forced convection and radiation, have shown a reasonable accuracy of the ADF model [14].

For the treatment of gas mixtures with global models, Pierrot *et al.* [15] considered three approaches. In the first approach, they assumed that the ratio of the absorbing gas molar fractions is uniform throughout the medium. This restrictive hypothesis is roughly valid in some combustion applications. In the second and the third approaches, no restriction is imposed on the absorbing gas molar fractions. They take into account the correlations of the spectra of the two absorbing gases by using the joint distribution function of the two absorption coefficients. Authors conclude that when X_{H_2O}/X_{CO_2} is constant throughout the medium, the gas mixture can be considered as a single absorbing gas, and the models keep the same accuracy without increasing the number of their coefficients. But when X_{H_2O}/X_{CO_2} is not constant throughout the medium, joint distribution functions of absorption coefficients of H_2O and CO_2 are needed to rigorously deal with the correlations of their spectra.

In the present investigation, we assume that the ratio of the absorbing gas molar fractions is uniform. The mixtures are treated as single gas with the spectral absorption coefficient:

$$\kappa_\nu = \kappa_{\nu,H_2O} + \kappa_{\nu,CO_2} \tag{1}$$

The chosen reference conditions for ADF model were T_{ref} = 1100 K and X_{H_2O} or X_{CO_2} equal to 0.1.

These parameters have been calculated at atmospheric pressure and different temperatures with $N = 8$ (one gas corresponding to the transparency regions and 7 active gases).

The total radiative intensity $I(\vec{u}, s)$ at curvilinear abscissa s and for the direction $\vec{u}$ is written as the sum of the partial intensities:

$$I(\vec{u}, s) = \sum_{j=1}^{N} I_j(\vec{u}, s) \tag{2}$$

Each partial intensity $I_j(\vec{u}, s)$ satisfies the radiative transfer equation:

$$\frac{\partial I_j(\vec{u}, s)}{\partial s} = k_j(s) X_{H_2O} p(s) \left[a_j(s) \frac{\sigma T^4}{\pi} - I_j(\vec{u}, s) \right] \tag{3}$$

The divergence of the radiative flux or the radiative source term, appearing in the energy equation, is given by:

$$\mathrm{div}\vec{q}^{\,rad} = \sum_{j=1}^{N=8} k_j(s) X_{H_2O} p(s) \int_{4\pi} \left[a_j(s) \frac{\sigma T^4}{\pi} - I_j(\vec{u}, s) \right] \mathrm{d}\Omega \tag{4}$$

Accuracy of several global and narrow-band gas infrared radiative property models applied to radiative transfer in a planar geometry with different types of temperature profiles are tested by Pierrot *et al.* [15]. They conclude that the use of global models offers a good compromise between accuracy and CPU time.

Basic formulation and solution method

The situation under study corresponds, for instance, to the case of regenerative exchangers used for energy recuperation from combustion products, where infrared active hot gas mixtures flow inside circular duct. Although the above applications are in general in an intermediate regime between constant wall temperature and constant wall heat flux, we assume here a constant wall temperature to gain insight on the effects of radiative transfer on global heat transfer.

The investigated situations correspond to a laminar steady cooled flow of non-scattering (H_2O-CO_2-N_2) gas mixtures (see fig. 1).

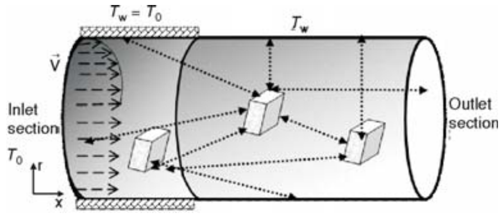

Figure 1. Geometrical and physical conditions

In the formulation of the heat transfer phenomena without radiation all the usual boundary layer approximations were made. Furthermore, both viscous dissipation and body forces are neglected. We restrict our considerations to axially symmetric flows. The mass, momentum and energy balance equations are given, respectively, by:

$$\nabla(\rho \vec{V}) = 0 \tag{5}$$

$$\nabla(\rho \vec{V}) = -\nabla p + \nabla\nabla(\mu \vec{V}) \tag{6}$$

$$\nabla(\rho h \vec{V}) = -\nabla \vec{q}^{\,cd} - \nabla \vec{q}^{\,rad} \tag{7}$$

where $\vec{V}$ is the velocity vector at a given point $M(x, r)$ h, ρ, and μ designate the enthalpy per unit mass, the temperature-dependent density and the viscosity, respectively. $\vec{q}^{\,rad}$ and $\vec{q}^{\,cd}$ are respectively, the radiative flux vector and the conductive flux vector.

The boundary conditions for the considered problem are the no-slip condition, the impermeability of the walls, and the imposed wall temperature T_w. The gas mixtures are assumed to obey the perfect gas law and the thermo-physical properties are approximated by temperature dependent functions generated from the data given in [23].

Since the system of equations formulated above is assumed parabolic in x-direction, no flow boundary conditions are required at the outlet section of the computational domain.

We also assumed that the flow rate is fixed by a measuring device and the velocity profiles are fully developed at the inlet conditions for $x = 0$. The isothermal region corresponding to $x < L_0$, allows to account for realistic radiative inlet conditions with the possibility of pre-cooling of the gas mixtures before the section $x = L_0$. In addition, the inlet section ($x = 0$) is considered for radiative transfer as a black wall at temperature T_0 while the outlet section ($x = L_0 + L$) is a black wall at T_w. L is chosen sufficiently large so that this last condition has no influence on the results near $x = L_0$.

Mass, momentum and energy equations are solved simultaneously in order to account for variable fluid properties due to significant temperature difference imposed between the walls and the gas.

The walls considered gray with a constant emissivity are also assumed to emit and reflect thermal radiation isotropically.

Engineering parameters

Several calculations have been carried out. Velocity, temperature, and radiative power fields are computed at each point of the medium. The velocity, temperature and volumetric radiative power profiles obtained cannot be presented here in full, nor they easy to use for engineering purposes. Of interest to engineers are: bulk temperature, Nusselt number, and heat transfer coefficients.

In the study, the mean bulk temperature is defined by:

$$T_b(x) = \frac{\int \rho u(x,r)T(x,r)2\pi r \mathrm{d}r}{\int \rho u(x,r)2\pi r \mathrm{d}r} \tag{5}$$

where C_p and $u(r, x)$ designate specific heat at constant pressure and axial velocity, respectively.

The energy transport from gas flow to duct in the presence of thermal radiation depends on two related factors: the fluid temperature gradient at the wall $q^{\mathrm{cd}}(x)$, and the rate of radiative heat exchange, $\phi^{\mathrm{rad}}(\mathrm{x})$.

So, the overall heat flux at the wall is a sum of the convective heat flux and the radiative heat flux:

$$\phi^{\mathrm{tt}}(x) = \phi^{\mathrm{conv}}(x) + \phi^{\mathrm{rad}}(x) \tag{6}$$

where $$\phi^{conv}(x) = q^{cd}(x) = -\lambda \left.\frac{\partial T(x,r)}{\partial r}\right|_{r=R} \quad \text{and} \quad \phi^{rad}(x) = \frac{\int_S \mathrm{div}\vec{q}^{\,rad}\mathrm{d}S}{2\pi r} \tag{7}$$

The total heat transfer coefficient is defined as the contribution of convective and radiative energy transport:

$$H^{conv}(x) = \frac{\phi^{conv}(x)}{T_\mathrm{w} - T_\mathrm{b}(x)}, \quad H^{rad}(x) = \frac{\phi^{rad}(x)}{T_\mathrm{w} - T_\mathrm{b}(x)}, \quad H^{tt}(x) = \frac{\phi^{tt}(x)}{T_\mathrm{w} - T_\mathrm{b}(x)} \tag{8}$$

and $$H^{tt}(x) = H^{conv}(x) + H^{rad}(x) \tag{9}$$

The overall heat flux at the wall is a sum of the convective heat flux and the radiative one. So, the local Nusselt number is defined at the distance x from the entrance, as the sum of convective and radiative contributions:

$$\mathrm{Nu}^{\mathrm{tt}}(x) = \mathrm{Nu}^{\mathrm{conv}}(x) + \mathrm{Nu}^{\mathrm{rad}}(x) \tag{10}$$

$$\mathrm{Nu}^{\mathrm{conv}} = \frac{D\phi^{conv}(x)}{\lambda[T_{\mathrm{w}} - T_{\mathrm{b}}(x)]} \quad and \quad \mathrm{Nu}^{\mathrm{rad}} = \frac{D\phi^{rad}(x)}{\lambda[T_{\mathrm{w}} - T_{\mathrm{b}}(x)]} \tag{11}$$

where D is the diameter of the duct, λ – the thermal conductivity, T_{w} and T_{b} are the wall and the bulk temperature, respectively.

Results obtained with various physical parameters (Reynolds number, diameter of duct, gas mixture composition, wall and gas inlet temperature) cannot be put in general correlation formula since gas thermo-physical properties depend strongly on the temperature, and radiative transfer depends in a highly non-linear manner on length scales and temperature levels. Nusselt number and bulk gas temperature will be presented as first order estimates and can be useful for engineering applications.

Computational procedure

The basic idea to solve simultaneously the convective and radiative problem relies on the use of a coupled radiative calculation code with a flow field computational code.

The mesh structures used to solve numerically the combined convective and radiative problem is illustrated in fig. 2. For the reasons of computational efficiency, a coarser grid is used in the radiative heat transfer solution code than in the CFD code for solving mass and momentum balance equations. The temperature field needed for calculating the radiative source term and the thermo-physical properties is given by the CFD code used in the flow calculation. After calculating the volumetric radiative power ($\nabla\vec{\mathrm{q}}^{\,rad}$), given by eq. 4, in the coarse mesh structure, this source term is interpolated or extrapolated over the fine mesh structure and added to the overall energy balance equations.

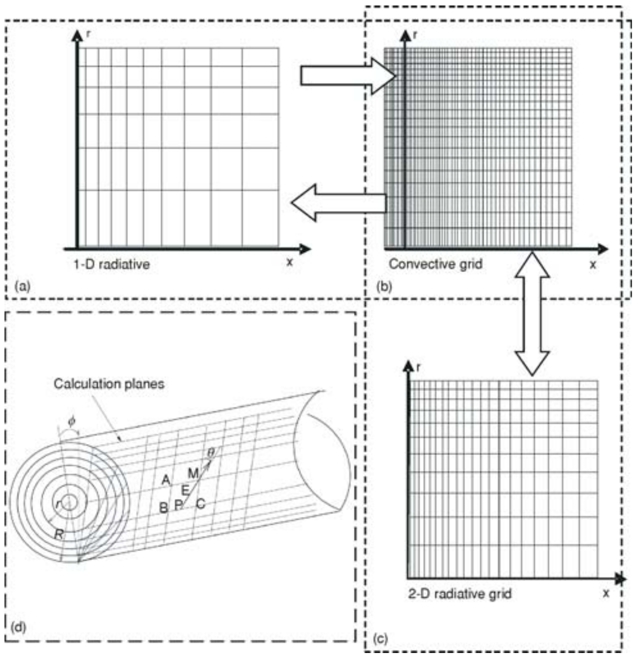

Figure 2. Different mesh structures used in the calculations of the 1-D radiative heat transfer (a) the flow field (b), the 2-D radiative heat transfer (c) and spatial and directional discretizations for radiative transfer (d)

This method starts first with the prediction of the temperature and velocity fields using local 1-D radiative transfer calculations and a marching procedure along the axial direction.

The radiative source term or the volumetric radiative power dissipated in the medium for a given section is calculated from a 1-D temperature field. In practice, for 1-D radiative transfer calculations the same numerical procedure is used with six discretized axial sections and a very large spacing between these sections in comparison with the tube diameter.

In a second step, an iterative procedure is undertaken. 2-D radiative powers and fluxes are calculated from the 2-D temperature field and then the flow and energy equations are solved using the marching procedure and the computed radiative powers as source terms.

The required number of global iterations depends on the optical thickness of the medium. A constant mesh size is used for the radial direction and a variable one in the axial direction. This mesh size increases far from the entrance, which allows a higher accuracy for the temperature and velocity fields in the thermal entrance region where steep axial gradients occur. Typically, we use 160 radial points and 20000 axial ones to compute the flow field. Such refined grid is not required for 2-D radiation calculations; numerical tests show that typically 40 × 100 nodes are sufficient for the above configuration. Numerical linear interpolations and extrapolations are used to convert the results from the convective grid mesh to the radiative one and *vice versa*.

In order to compute the radiative flux and radiative powers, the method uses two discretizations: (1) a spatial discretization which consists in defining the radial and axial grid points where radiative intensities are calculated, (2) a directional discretization of the two angles ϕ and θ as shown in fig. 2(d) The discretization of ϕ is based on the radial discretization. The calculations are carried out in the planes parallel to the system axis and tangential to the coaxial cylinders, defined by the radial discretization.

The radiative intensity at a point M is step by step calculated between upstream grid points by using:

$$I_j(\vec{\mathrm{u}}, M) = I_j(\vec{\mathrm{u}}, P)\mathrm{e}^{[-k_j(\mathrm{E})XpPM]}\{1-\mathrm{e}^{[-k_j(\mathrm{E})XpPM]}\}\,a_j(E)\frac{\sigma T^4 E}{\pi} \tag{12}$$

where the intensity $I_j(\vec{\mathrm{u}}, P)$ at point P is obtained by an interpolation between the intensities: $I_j(\vec{\mathrm{u}}, B)$ and $I_j(\vec{\mathrm{u}}, C)$ or $I_j(\vec{\mathrm{u}}, A)$ at the neighbor grid points. E is the middle point of the segment PM at temperature T_{E}.

When radiative transfer problem is solved, the source terms ($\nabla\vec{\mathrm{q}}^{\,rad}$) are known at each point of the medium, then the flow equations become parabolic, an implicit finite marching procedure is used. At each flow cross section, discretization of momentum and energy equations leads to two tridiagonal matrix inversions which are treated by Gaussian elimination.

The following sequences summarize the procedure used to obtain all the variables at each successive cross-sectional plane.

(a) The radiative source term for a given section i is calculated from a local one dimensional temperature field: $T(x_{\mathrm{i}}, r) = T(x_{\mathrm{i-1}}, \mathrm{r})$.
(b) The energy equation is solved with the previous axial velocity field and the previous axial pressure drop. A marching implicit finite volume procedure based on the pressure correction algorithm is used [24].
(c) The temperature profile is used to calculate new thermo-physical and radiative fluid properties.
(d) The velocity field is calculated with the previous pressure drop.
(e) Pressure drop is adjusted in the momentum equation in order to satisfy the integrated continuity equation. Iteration over steps (d) and (e) is then made until the mass flow rate is conserved.

Global iterations between these sequences are performed until convergence. This iterative procedure starts with the 1-D radiation temperature field for the 2-D calculation of the radiative field.

The convection part (non radiative gas) of this model was first validated by comparison with the previous numerical studies of Bankston and McEligot [25] and Worsoe-Schmidt and Leppert [26], where the boundary layer assumption was used. The relative difference observed between our results and those from previous investigations in terms of Nusselt number, is less than 2%.

The geometrical thermal radiation part was validated by comparison with the classical analytic results corresponding to gray medium for finite and infinite cylinders characterized by temperature field and boundary conditions comparison are made with results in [20].

Accuracy of several global and narrow-band gas infrared radiative property models applied to radiative transfer in a planar geometry with different types of temperature profiles are tested by Pierrot *et al.* [15]. They conclude that the use of global models offers a good compromise between accuracy and CPU time and they conclude that no significant differences appear between ADF8 and ADF50 results. Comparisons between the results from ADF model and from the more elaborated correlated-k band model, applied for combined forced convection and radiation, have been done and results have shown a reasonable accuracy of the ADF model. Details of this comparison are presented in [13].

Results and discussions

Several computations have been conducted for various combinations of Reynolds number with fluid properties taken at the inlet temperature T_0, diameter of duct, gas mixture compositions, wall temperature and gas inlet temperature. We limit ourselves, in this presentation, to illustrate the evolution of the Nusselt number and of the bulk mixture temperature flowing inside a cooled duct.

A parabolic profile corresponding to a fully developed isothermal flow is considered from the entrance of the duct until section $x = L_0$. The considered gas mixtures resulting from the combustion of propane, natural gas and heavy fuel are at atmospheric pressure with uniform molar fraction throughout the medium. We have the imposed wall temperature T_w. Mixtures at the inlet section ($x = 0$) are at temperature T_0, and between sections $x = 0$ and $x = L_0$, the wall is assumed at temperature T_0. In this isothermal region a pre-cooling of the gas mixtures occurs before the section $x = L_0$.

Bulk temperature and heat transfer coefficient

Figure 3 illustrates the bulk temperature evolution along the axial duct for gas mixtures obtained from the combustion with oxygen of natural gas, propane and heavy fuel at different partial pressure values. The inlet temperature of gas mixtures is equal to 600 K and the wall temperature of duct is maintained at 400 K. Calculations are made for distances up to the $L = 5$ m and $L_0 = 0.3$ m.

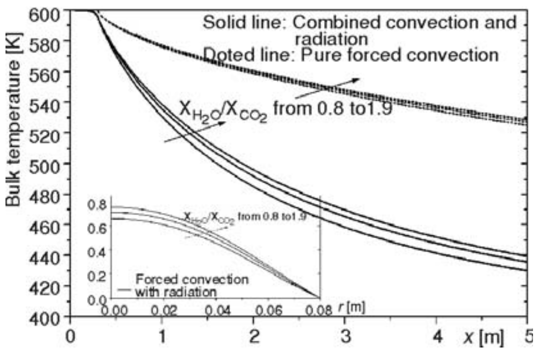

Figure 3. Evolution of the bulk temperature for combustion products at various partial pressure (Re = 1500, T_0 = 600 K, T_w = 400 K, D = 16 cm, combustion with oxygen)

It is shown that the bulk temperature evolves much more rapidly towards wall temperature when accounting for radiation, for instance, at section $x = 4$ m the difference between bulk temperature with radiation and without radiation is about 100 K.

All results indicate that the decrease of partial pressure values leads to an increase in the heat transfer. In spite of the fact that the water vapour is more efficient on radiative heat transfer than CO_2 due to its spectral structure, results observed in fig. 3 reveal that the heat is more transferred from the fluid to the cooled wall when X_{H_2O} decreases (X_{CO_2} increases). This is a simple consequence of a lower velocity for mixtures with high density. In fact, as it is illustrated by the radial distributions of the axial velocity at section $x = 3$ m, an increase in the density affects indirectly heat transfer and leads to a decrease of the velocity. For example at section $x =$ 3 m the bulk temperatures are about 450K for $X_{H_2O}/X_{CO_2} = 0.8$ and about 475 K for $X_{H_2O}/X_{CO_2} = 1.9$. It is also shown, that when thermal radiation is considered the gas mixture are pre-cooled before the section $x = L_0 = 0.3$ m.

This pre-cooling zone is noticeable but small. This classical result [27] is due to the axial propagation of thermal radiation across the duct.

We can conclude that, although the fact that the water vapour is the most efficient species for radiative heat transfer since its spectrum contains wide absorption bands, the effects of the density on the velocity have an important impact in convective heat transfer. This is only due to thermo-physical properties species difference and particularly their density. In other words, results highlight the indirect radiation effect on the development of the velocity field via temperature dependent thermo-physical properties and particularly for large temperature difference between fluid and wall.

Nusselt number evolutions

Figure 4 shows the evolution of the heat transfer coefficients for H_2O-CO_2-N_2 mixture in cylindrical geometry with diameter $D = 16$ cm and inlet Reynolds number equal to 1500. The total heat transfer coefficient is the sum of convective and radiative heat transfer. The total heat transfer coefficient computed with radiation is of course higher than the one obtained for pure forced convection. Hence, the radiative transfer accelerates the thermal development and the convective transfer coefficient H^{conv} increases when considering radiation. This result is explained by the fact that the absolute difference $|T_w(x) - T_b(x)|$ is lower when the radiation was taken into account.

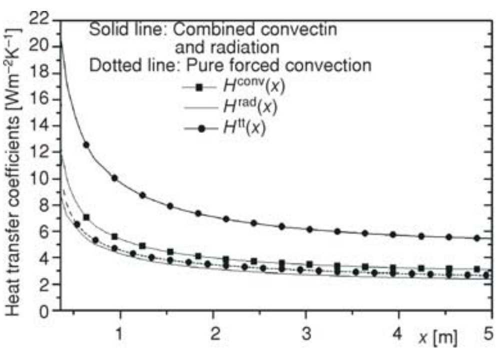

Figure 4. Evolution of heat transfer coefficients for the case $X_{H_2O} = 0.114$ ($X_{H_2O}/X_{CO_2} = 0.8$) produced by the combustion with air of heavy fuel (Re = 1500, $T_0 = 600$ K, $T_w = 400$ K, $D = 16$ cm)

As specified above, the main aim of this study is to give a first order estimates calculations of Nusselt number which can be useful for engineering.

To study the evolution of the total Nusselt number along the axial direction of the duct, we consider laminar flows of different combustion products at Reynolds number equal to 1500. Calculations have been carried out for different gas mixtures produced by the combustion of natural gas ($X_{H_2O}/X_{CO_2} = 1.9$) propane ($X_{H_2O}/X_{CO_2} =$ $= 1.33$) and heavy fuel ($X_{H_2O}/X_{CO_2} = 0.8$), with oxygen and with air separately.

In order to study thermal radiation effects on total Nusselt number evolution, calculus has been done for an optically thick medium (corresponding to diameter D = 16 cm) and for an optically thin medium (corresponding to diameter D = 4 cm). Figure 5 shows the axial evolution of the total Nusselt number for different mixtures produced by the combustion with oxygen. The mixture temperature at the entrance is equal to T_0 = 600 K and the wall is at T_w = 400 K.

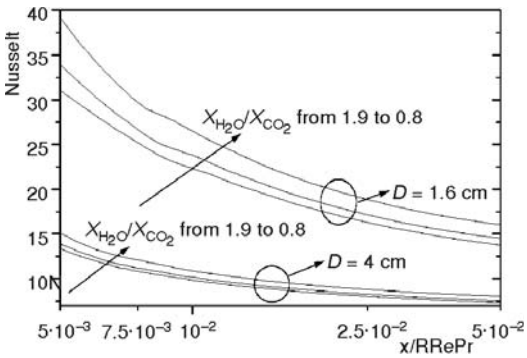

Figure 5. Axial evolution of the total Nusselt number for different mixtures gas produced by the combustion with oxygen of natural gas (X_{H_2O}/X_{CO_2} = 1.9), propane (X_{H_2O}/X_{CO_2} = 1.33), and heavy fuel (X_{H_2O}/X_{CO_2} = 0.8) (Re = 1500, T_0 = 600 K, T_w = 400 K)

The Nusselt number evolves from a high value corresponding to the flat plate (for $x/(R\text{RePr}) \ll 10^{-3}$) and decreases monotonically thereafter. When a variation of partial pressure is introduced a significant variation of the Nusselt number is observed. For both cases (D = 16 cm and 4 cm), the Nusselt number values are high for low partial pressure. As explained above, this is due to the thermal and velocity field dependence on the density of species in medium for constant Reynolds number at the inlet section.

Figure 5 shows that the total Nusselt number is higher for optically thick medium (corresponding to D = 16 cm) than for optically thin one (corresponding to D = 4 cm) due to the thermal radiation contribution in heat transfer. Effect of this additional mode of heat grows with the optical depth but the conduction mode is predominant for weak diameter D = 4 cm.

In many engineering applications, it is of more interest for the engineer to determine the average Nusselt number which is given by the integration of the local Nusselt number along the duct length.

Table 2 presents the average convective and radiative Nusselt number for the pure forced convection and for combined radiation-convection heat transfer. This table compares total Nusselt numbers for three combustion products resulting from the oxy-combustion of natural gas, propane and heavy fuel. Results show that the Nusselt number computed without radiation is practically independent of molar fractions of combustion products. But, the presence of radiation enhances the heat transfer. For constant Reynolds number at the entrance, the increase of X_{CO_2} induces a flow deceleration and increases total Nusselt number.

Table 2. Different radiative and convective mean Nusselt number for combustion products obtained from combustion with oxygen. D = 16 cm, Re = 500

Hydrocarbon	Radiative molar fractions	Combined heat transfer	Pure forced convection heat transfer
Natural gas	X_{H_2O} = 0.66 X_{CO_2} = 0.347	Nu^{tt} = 14.74	Nu^{conv} = 3.96
Propane	X_{H_2O} 0.57 X_{CO_2} = 0.43	Nu^{tt} = 15.95	Nu^{conv} = 3.94
Heavy fuel	X_{H_2O} = 0.45 X_{CO_2} = 0.55	Nu^{tt} = 17.56	Nu^{conv} = 3.91

Conclusions

The interaction between thermal radiation and forced convection in laminar mixtures gas flows produced by combustion with oxygen has been studied.

The radiative properties of gas mixtures have been taken into account through the Absorption Distribution Function model. Based on the analysis and numerical calculations, the following conclusions can be drawn.

- For gas mixtures produced by combustion with air, the bulk temperature reaches more rapidly the wall temperature than the one produced by combustion with oxygen this is due to the dependence of the velocity on the density.
- The heat transfer process increases with the decrease of the partial pressure value. Such result is due to the low velocity for high density products when Reynolds number is maintained constant at the inlet section.
- The total Nusselt number values increase with the increase of the gas mixtures optical thickness. For optically thick medium, the effect of partial pressure is significant on total Nusselt number resulting from radiative heat process.

Nomenclature

a_j – quadrature weights
C_p – specific heat at constant pressure, [$Jkg^{-1}K^{-1}$]
D – diameter of the duct, [m]
h – enthalpy per unit mass, [Jkg^{-1}]
H^{tt} – total heat transfer coefficient, [$Wm^{-2}K^{-1}$]
H^{conv} – convective heat transfer coefficient, [$Wm^{-2}K^{-1}$]
H^{rad} – radiative heat transfer coefficient, [$Wm^{-2}K^{-1}$]
I – total radiative intensity, [Wm^{-2}]
I_j – partial radiative intensity, [Wm^{-2}]
k_j – reduced absorption coefficient, [$cm^{-1}atm^{-1}$]
L – downstream region length, [m]
L_0 – upstream region length, [m]
Nu – Nusselt number
Pr – Prandtl number
Pe – Peclet number
Nu^{rad} – radiative Nusselt number
Nu^{conv} – convective Nusselt number
P – pressure, [Pa]
q^{cd} – conductive heat flux per unit surface, [Wm^{-2}]
q^{rad} – radiative heat flux per unit surface, [Wm^{-2}]
R – radius of the duct [m]
Re – Reynolds number (= $u_0 D\rho/\mu$)
r, x – radial and axial co-ordinate [m]
s – curvilinear abscissa [m]
T – temperature, [K]
T_b – bulk temperature, [K]
T_w – wall temperature, [K]
u, v – velocity components, [ms^{-1}]
X – molar fraction of the absorbing species

Greek symbols

ε – emissivity of wall
λ – thermal conductivity, [$Wm^{-1}K^{-1}$]
μ – viscosity, [$kgm^{-1}s^{-1}$]
κ_ν – absorption coefficient, [cm^{-1}]
ρ – density, [kgm^{-3}]
σ – Stefan-Boltzmann constant (= $5.67\cdot10^{-8}$), [$Wm^{-2}K^{-4}$]
Ω – solid angle

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