

INFLUENCE OF THERMAL RADIATION AND HEAT GENERATION/ /ABSORPTION ON MHD HEAT TRANSFER FLOW OF A MICROPOLAR FLUID PAST A WEDGE WITH HALL AND ION SLIP CURRENTS

by

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In this paper a numerical model is developed to examine the effect of thermal radiation on magnetohydrodynamic heat transfer flow of a micropolar fluid past a non-conducting wedge in presence of heat source/sink. In the model it is assumed that the fluid is viscous, incompressible, and electrically conducting. The Hall and ion slip effects have also been taken into consideration. The model contains highly non-linear coupled partial differential equations which have been converted into ordinary differential equation by using the similarity transformations. These equations are then solved numerically by Shooting technique along with the Runge-Kutta-Fehlberg integration scheme for entire range of parameters with appropriate boundary conditions. The effects of various parameters involved in the problem have been studied with the help of graphs. Numerical values of skin friction coefficients and Nusselt number are presented in tabular form. The results showed that the micropolar fluids are better to reduce local skin drag as compared to Newtonian fluids and the presence of heat sink increases the heat transfer rate.

Key words: thermal radiation, heat source/sink, micropolar fluids, Hall current, ion slip, wedge, similarity solution

Introduction

The radiative flows of an electrically conducting fluid in presence of magnetic field are encountered in electric power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications and other industrial areas. At high operating temperatures, radiation effects can be quite significant. We know that radiation effect is important under many non-isothermal situations. If the entire system involving the polymer extrusion process is placed in thermally controlled environment, then radiation could become important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with a sought characteristic. Radiation effect on Newtonian and non-Newtonian fluids with or without magnetic field has been considered by many authors [1-6]. These applications involve fluids as a working medium and in such applications unclean fluids (*i. e.* clean fluid + interspersed particles) are very common and clean fluid is an exception. Therefore classical Navier-Stokes equation is not suitable to model such type of problems.

Eringen [7, 8] proposed a theory of micropolar fluids taking into account the inertial characteristics of the substructure particles, which are allowed to undergo rotation. The concept of micropolar fluids deals with a class of fluids that exhibit certain microscopic effects arising

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from the local structure and micro motions of fluid elements. These fluids contain dilute suspensions of rigid macro molecules with individual motions that support stress and body moments influenced by spin inertia. The interaction of the macro velocity field and micro rotation field can be described through new material constants in addition to those of classical Newtonian fluids. Eringen's micropolar fluid model includes the classical Navier-Stokes equations for a viscous and incompressible fluid as a special case. The equations governing the flow of a micropolar fluid involve a micro rotation vector and a gyration parameter in addition to the classical vector field. The micropolar fluid theory requires an additional transport equation representing the principle of conservation of local angular momentum. Extensive review of the theory and applications of micropolar fluids can be found in Eringen [9] and Lukaszewicz [10].

Keeping these things in consideration various researchers worked on micropolar fluids. Rahman [11] investigated the convective flow of micropolar fluids from radiate isothermal porous surfaces with viscous dissipation and joule heating. Bakier [12] studied the effect of thermophoresis on natural convection boundary layer flow of a micropolar fluid. An analytical solution of stagnation flow of a micropolar fluid towards a vertical permeable surface has been discussed by Asgharian *et al.* [13]. Due to the vast applications of magnetohydrodynamic (MHD) flow in industries the flows of micropolar fluids have also been investigated in the presence of magnetic fields, by various authors. Norfifah and Ishak [14] investigated the steady MHD stagnation point flow of a micropolar fluid past a vertical surface under different temperature conditions.

Also, MHD boundary layer flows over wedge shaped bodies are very common in many thermal engineering applications such as geothermal systems, crude oil extraction, ground water pollution, thermal insulation, heat exchanger and the storage of nuclear waste, *etc.* Lin and Lin [15] found the similarity solutions for laminar forced convection heat transfer from wedges to fluids of any Prandtl number. Kim [16] and Kim and Kim [17] have considered the steady boundary layer flow of a micropolar fluid past a wedge with constant surface temperature and constant surface heat flux, respectively. In these studies the similarity variables found by Falkner and Skan [18] were employed to reduce the governing partial differential equations to ordinary differential equations. The study of MHD forced convection flow adjacent to a non-isothermal wedge is done by Yih [19]. This work is extended by Chamka *et al.* [20]. They considered the thermal radiation effects on MHD forced convection flow adjacent to a non-isothermal wedge in the presence of heat source or sink. Singh *et al.* [21] considered the time dependent free stream velocity and gave a mathematical model for unsteady mixed convection flow of an incompressible viscous fluid over a vertical wedge with constant suction/ injection and analyzed the behavior of the flow. The steady two-dimensional laminar forced convection flow and heat transfer of a viscous, incompressible, electrically conducting and heat generating fluid past a permeable wedge embedded in non-Darcy high porosity ambient medium with uniform surface heat flux is studied by Rashad and Bakier [22]. Ishak *et al.* [23] investigated the MHD boundary-layer flow of a micropolar fluid past a wedge with constant wall heat flux.

In a MHD device using weakly ionized gases hall and ion-slip effects appear as the ratio of magnetic field strength to the gas density. A boundary layer analysis is used to study the Hall effect on Couette flow with heat transfer of a dusty conducting fluid by assuming uniform suction/injection and temperature dependent physical properties by Hazem [24, 25]. Elgazery [26] also studied the effects of hall and ion slip currents and chemical reaction, on MHD flow with temperature dependent viscosity and thermal diffusivity.

The study of heat generation or absorption in moving fluid is important in problems dealing with dissociating fluids. Possible heat generation effect may alter the temperature distri-

bution; consequently, the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. Rahman *et al.* [27] studied thermo-micropolar fluid flow along a vertical permeable plate with uniform surface heat flux in the presence of heat generation. Ibrahim [28] found the analytic solution of MHD mixed convection heat and mass transfer over an isothermal, inclined permeable stretching plate immersed in a uniform porous medium in the presence of chemical reaction, internal heating, Dufour effect and Hall effects. Pal and Chatterjee [29] studied heat and mass transfer in MHD non-Darcian flow of a micropolar fluid over a stretching sheet embedded in a porous media with non-uniform heat source and thermal radiation.

From the literature survey it is found that various scientists worked on magneto-micropolar fluid flows about wedge, but they ignored, the radiation effect and heat generation/absorption along with hall and ion slip currents, which cannot be ignored in the view of industrial applications of micropolar fluid flow about wedge shaped bodies. Therefore the objective of the present paper is to study the MHD heat transfer flow of a micropolar fluid past a wedge with heat generation or absorption by considering the radiation effect along with viscous dissipation, joule heating, and Hall and ion-slip effects.

Formulation

A steady viscous, incompressible, electrically conducting and micropolar fluid flowing past a non-conducting wedge is considered. The physical model of the problem is shown in fig 1.

We used rectangular Cartesian co-ordinates (x, y, z) , in which x is the distance measured along the front edge of wedge surface, which makes an angle $\pi\beta$ with the opposite surface, y and z are the distances normal to this surface and along the leading edge of the wedge, respectively. A strong magnetic field $B(x)$ is applied along y -axis and the fluid is considered electrically conducting therefore, Hall and ion-slip currents affect the flow. This effect give rise to force in the direction perpendicular to magnetic field, which induces a cross flow in that direction, *i. e.* z -direction. The generalized Ohm's law including Hall and ion slip is given by, see Sutton and Sherman [30]:

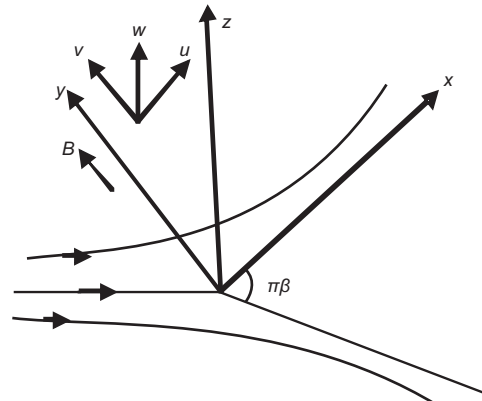


Figure 1. Physical model and co-ordinate system

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}) - \frac{\omega_e \tau_e}{B} (\vec{J} \times \vec{B}) + \frac{\omega_e \tau_e \omega_i \tau_i}{B^2} (\vec{J} \times \vec{B}) \times \vec{B}$$

where, \vec{J} is the electric current density vector, \vec{E} – the electric field intensity vector, \vec{B} – the magnetic induction vector, \vec{V} – the translational velocity vector, ω_e and ω_i are electron and ion cyclotron frequency, and τ_e and τ_i are electron and ion mean free times, respectively.

To simplify the problem we assume that there is no variation of flow and heat transfer quantities in the z -direction as discussed by Rosenhead [31]. The equation of conservation of electric charge $\Delta J = 0$ gives $J_y = \text{constant}$. Since the wedge is non-conducting, thus, $J_y = 0$, everywhere in the flow. Also, $E_x = 0$ and $E_z = 0$ everywhere in the fluid. We assumed that in-

duced magnetic field can be neglected in comparison to applied magnetic field, but the viscous dissipation and Joule heating effects in the fluid are taken into account.

For a steady, incompressible, MHD flow of micropolar fluid with generalized Ohm's law, including Hall and ion-slip under the assumption that the fluid is non-magnetic, neglecting the thermoelectric effect the equations for the flow are:

– conservation of mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\frac{\mu + \kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} + \frac{1}{\rho} \left\{ \frac{\sigma B_y^2}{\alpha_e^2 + \beta_e^2} [\alpha_e (U - u) - \beta_e w] \right\} + U \frac{dU}{dx} \quad (2)$$

– conservation of translational momentum

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \left(\frac{\mu + \kappa}{\rho} \right) \frac{\partial^2 w}{\partial y^2} - \frac{1}{\rho} \frac{\sigma B_y^2}{(\alpha_e^2 + \beta_e^2)} (\alpha_e w - \beta_e u) \quad (3)$$

– conservation of angular momentum (micro-rotation)

$$\rho j \left(u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \frac{\partial}{\partial y} \left(\gamma \frac{\partial N}{\partial y} \right) - \kappa \left(2N + \frac{\partial u}{\partial y} \right) \quad (4)$$

– conservation of energy

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + (\mu + \kappa) \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] + \frac{\sigma B_y^2}{\alpha_e^2 + \beta_e^2} (u^2 + w^2) - \frac{\partial}{\partial y} (q_r) + Q(T - T_\infty) \quad (5)$$

where $\alpha_e = 1 + \beta_i \beta_e$ and u , v , and w are the velocity components along x -, y -, and z -directions, β_e , and β_i – the hall effect parameter and ion slip parameter, μ , κ , γ , j , N , k , C_p , ρ , T , and Q – the dynamic viscosity, vortex viscosity, spin gradient viscosity, micro-inertia, component of microrotation vector normal to xy -plane, thermal conductivity of the fluid, specific heat at constant pressure, fluid density, temperature of the fluid and heat generation parameter, respectively.

The boundary conditions for the flow are:

$$u = v = w = 0, \quad N = -n \frac{\partial u}{\partial y}, \quad q_r = -k \frac{\partial T}{\partial y} \quad \text{at } y = 0 \quad (6)$$

$$u \rightarrow U, \quad w \rightarrow 0, \quad N \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \quad (7)$$

The radiative heat flux q_r under Rosseland approximation by Brewster [32] has the form:

$$q_r = -\frac{4\sigma_1}{3k_1} \frac{\partial T^4}{\partial y}$$

where σ_1 is the Stefan-Boltzmann constant, and k_1 – the mean absorption coefficient.

We assume that the temperature differences within the flow are so small that T^4 can be expressed as a linear function of T_∞ . This is obtained by expanding T^4 in a Taylor series about T_∞ and neglecting the higher order terms. Thus we get:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4$$

Following Falkner and Skan [18], we assume that the free stream velocity is $U = ax^m$, where $m = \beta/(2 - \beta)$, and β is the Hartree pressure gradient parameter which corresponds to $\beta = \Omega/\pi$ for an angle Ω of the wedge, and a is a positive constant. As discussed by Ishak et al. [23], we assume that:

$$B_y = B_0 x^{\left(\frac{m-1}{2}\right)}$$

where $0 \leq m \leq 1$ with $m = 0$ for boundary layer flow over a stationary flat plate and $m = 1$ for the flow near the stagnation point on an infinite wall. In the boundary conditions n is a constant such that $0 \leq n \leq 1$. The case when $n = 0$ is called strong concentration, which indicates $N = 0$ near the wedge that represents concentrated particle flows where the micro-elements close to the wedge surface are unable to rotate, see Jena and Mathur [33]. The case when $n = 1/2$ indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentrations, whereas $n = 1$ is used for the modeling of turbulent boundary layer flows. Here we are taking the case of $n = 1/2$. It is known that $\gamma = (\mu + \kappa/2)j = \mu(1 + K/2)j$, where $K = \kappa/\mu$ is dimensionless viscosity ratio and is called as material parameter. We also assumed that $Q = Q(x) = Q_0/x$.

We introduce following similarity variables:

$$\psi = f(\eta) \sqrt{\frac{2\nu x U}{m+1}}, \quad N = Uh(\eta) \sqrt{\frac{(m+1)U}{2\nu x}}, \quad w = Ug(\eta), \quad \phi = \frac{2xQ(x)}{\rho C_p U}$$

$$\eta = \sqrt{\frac{(m+1)U}{2\nu x}} y, \quad \theta(\eta) = \frac{k(T - T_\infty)}{q_w} \sqrt{\frac{(m+1)U}{2\nu x}}, \quad M = \frac{2\sigma B_0^2}{a\rho(m+1)}$$

Equation (1)-(5) along with boundary conditions (6) and (7) reduce to:

$$(1 + K)f''' + ff'' + \frac{2m}{m+1}(1 - f'^2) + Kh' + \frac{M}{\alpha_e^2 + \beta_e^2}[\alpha_e(1 - f') - \beta_e g] = 0 \quad (8)$$

$$(1 + K)g'' + g'f' - \frac{2m}{m+1}gf' - \frac{M}{\alpha_e^2 + \beta_e^2}(\alpha_e g - \beta_e f') = 0 \quad (9)$$

$$\left(1 + \frac{K}{2}\right)h'' - \left(\frac{3m-1}{m+1}hf' - fh'\right) - \frac{2KI}{m+1}(2h + f'') = 0 \quad (10)$$

$$\begin{aligned} \frac{4+3N}{3NPr} \theta'' + \left[(m-1)f' + (1+K)Ec(f'^2 + g'^2) + \frac{EcM}{(\alpha_e^2 + \beta_e^2)}(f'^2 + g^2) + \phi \right] \theta = \\ = -(m+1)f\theta' \end{aligned} \quad (11)$$

where (') represents the derivative with respect to η , ψ is the stream function. *i. e.* $u = \partial\psi/\partial y$ and $v = -(\partial\psi/\partial x)$, $I = (v^2 \text{Re})/(jU^2)$ (inertial parameter) and $\text{Ec} = U^2/[C_p(T - T_\infty)]$ (Eckert number).

Boundary conditions in non-dimensional form are:

$$f(0) = 0, \quad f'(0) = 0, \quad g(0) = 0, \quad h(0) = -\frac{1}{2}f''(0), \quad \theta'(0) = -1 \quad (12)$$

$$f'(\infty) \rightarrow 1, \quad g(\infty) \rightarrow 0, \quad h(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0 \quad (13)$$

The quantities of physical interest are local skin friction coefficients (C_{fx} and C_{fz}) and local Nusselt number, which are defined as:

$$C_{fx} = \frac{\tau_w}{\frac{1}{2}\rho U^2}, \quad C_{fz} = \frac{\tau_z}{\frac{1}{2}\rho U^2}, \quad \text{and} \quad \text{Nu} = \frac{xq_w}{k(T - T_\infty)}, \text{ respectively,}$$

where

$$\tau_w = \left[(\mu + \kappa) \frac{\partial u}{\partial y} + \kappa N \right]_{y=0}, \quad \tau_z = \left[(\mu + \kappa) \frac{\partial w}{\partial y} \right]_{y=0},$$

$$q_w = -k \frac{\partial T}{\partial y}, \quad \text{and} \quad \text{Re} = \frac{Ux}{\nu} \text{ (local Reynolds number).}$$

Therefore

$$C_{fx} = \sqrt{\frac{2(m+1)}{\text{Re}}} \left(1 + \frac{K}{2} \right) f''(0), \quad C_{fz} = \sqrt{\frac{2(m+1)}{\text{Re}}} (1 + K) g'(0),$$

$$\text{and} \quad \text{Nu} = \sqrt{\frac{(m+1)\text{Re}}{2}} \frac{1}{\theta(0)}, \text{ respectively.}$$

Method of solution

The non-linear ordinary differential eqs. (8)-(11) subjected to boundary conditions (12) and (13) have been solved by using Runge-Kutta-Fehlberg method along with shooting method. This method is based on the discretization of the problem domain and the calculation of unknown boundary conditions.

The domain of the problem is discretized and the boundary conditions for $\eta = \infty$ are replaced by $f'(\eta_{\max}) = 1$, $g(\eta_{\max}) = 1$, $h(\eta_{\max}) = 1$, and $\theta(\eta_{\max}) = 0$, where η_{\max} is a sufficiently large value of η (corresponding to step size) at which the boundary conditions (13) for $f(\eta)$ is satisfied. In the present work we have set $\eta_{\max} = 3.0$ taking into account the consistency and stability criteria. To solve the problem we first converted non-linear eqs. (8)-(11) into nine first order linear ordinary differential equations. There are five conditions at the boundary $\eta = 0$ (values of the function) and four conditions at the boundary $\eta = \infty$. To get the solution of the problem we need four more conditions at $\eta = 0$, these conditions have been found by Shooting technique. We ran the computer program for different values of step size $\Delta\eta$, to check the stability and consistency of the method, and found that method is stable and consistent for step size $\Delta\eta = 0.001$. Finally the problem has been solved by Runge-Kutta-Fehlberg method along with calculated boundary conditions.

Results and discussions

In order to analyze the effects of various physical parameters on the flow, a numerical computation has been performed. To validate our results, we have compared the results with published work [15, 19, 20, 23]. These comparisons are given in tabs. 1 and 2. These results show that our results are in very good agreement with the previously published work. The effect of various physical parameters on horizontal velocity, transverse velocity, angular velocity profiles and temperature distribution have been discussed and shown in figs. 2-15. The effect of all these parameters on the local skin friction coefficients (due to horizontal velocity as well as transverse velocity) and local Nusselt number have been given in tabs. 1-6.

Table 1. Values of $(1/2)C_{f_x}Re^{1/2}$ for various values of K , M , and m at $I = 0.5$, $\beta_i = 0.4$, $\beta_e = 0$, and $Ec = 0$

K	M	m	Yih [19]	Chamka et al. [20]	Ishak et al. [23]	Present result
0	0	0	0.332057	0.332206	0.3321	0.3466
0	0	1/3	0.757448	0.757586	0.7575	0.7586
0	0	1	1.232588	1.232710	1.2326	1.2328

Table 2. Values of $NuRe^{-1/2}$ for various values of Pr at $I = 0.5$, $\beta_i = 0.4$, $\beta_e = 0$, and $Ec = 0$

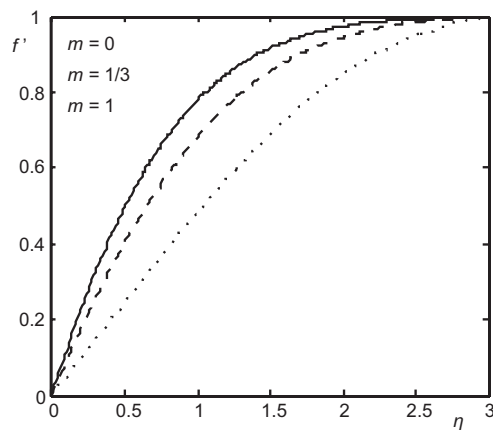
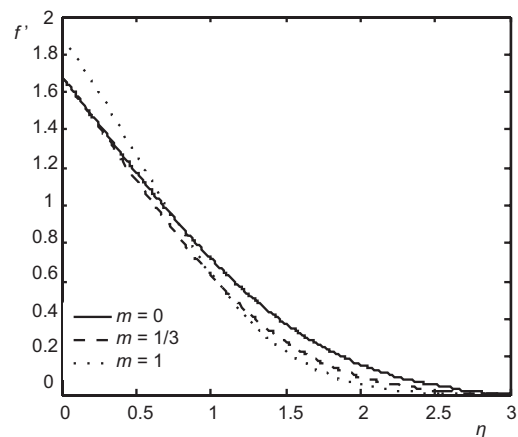
K	M	m	Ec	Pr	Lin and Lin [15]	Ishak et al. [23]	Present result
0	0	0	0	1	0.45897	0.4590	0.460439
0	0	0	0	10	0.99789	0.9980	1.00012
0	0	0	0	100	2.15197	2.1520	2.163009
0	0	0	0	1000	4.63674	4.6367	4.647032
0	0	0	0	10000	9.98965	9.9897	10.00013
0	0	0	0.1	1	–	–	0.458130
0	0	0	0.5	1	–	–	0.419986

To validate our results, we first considered the case of Newtonian fluid ($K = 0$) in the absence of magnetic field ($M = 0$). We analyzed the effect of m or Ω (the angle of wedge) on velocity profile and temperature distribution as shown in figs. 2 and 3. We also calculated the values of skin friction coefficient and Nusselt number as given in tabs. 2 and 3. It is depicted from fig. 2, that the horizontal velocity $f'(\eta)$ increases with m and the momentum boundary layer thickness decreases. This in turn increase the velocity gradient at the surface ($\eta = 0$) and hence increases the skin friction coefficient. The skin friction coefficient for $m = 1$ is largest. Thus, the skin friction coefficient near the stagnation point of a plate is largest in comparison of flow past a horizontal plate ($m = 0$) and flow past a wedge ($0 < m < 1$). This results in the increase in local Nusselt number as given in tab. 3.

Figure 3 depicts the effect of m on temperature distribution. The value of temperature at the surface increases slightly with the increase in the value of m , but at $m = 1$ it increases rapidly. But as we move away from the surface the effect is just reversed. *i. e.* temperature decreases with the increase in m from 0 to 1. From the graph it is also clear that, the thermal boundary layer thickness decreases with the increase in m .

Table 3. Values of $(1/2)C_{fx}Re^{1/2}$, $(1/2)C_{fz}Re^{1/2}$, and $NuRe^{-1/2}$ for various values of m and M

$I = 0.5, \beta_i = 0.4, \beta_e = 0, M = 0, K = 0,$ $Pr = 1, \text{ and } Ec = 0.5$				$M = 1, m = 1/3, \beta_i = 0.4, \beta_e = 2, I = 0.5,$ $Pr = 1, \text{ and } Ec = 0.5$			
m	$(1/2)C_{fx}Re^{1/2}$	$NuRe^{-1/2}$	$(1/2)C_{fz}Re^{1/2}$	K	$(1/2)C_{fx}Re^{1/2}$	$NuRe^{-1/2}$	$(1/2)C_{fz}Re^{1/2}$
0	0.3466	0.4200	–	0	0.8425	0.4726	0.1225
1/3	0.7586	0.4845	–	0.5	0.8386	0.4244	0.1295
1	1.2328	0.5297	–	1	0.4672	0.3999	0.1150

**Figure 2. Horizontal velocity profile vs. m for $K = 0, M = 0, \beta_e = 0, \beta_i = 0.4, I = Ec = 0.5,$ and $Pr = 1$** **Figure 3. Temperature distribution vs. m for $K = 0, M = 0, \beta_e = 0, \beta_i = 0.4, I = Ec = 0.5,$ and $Pr = 1$**

Figures 4-9 depict the effect of β_e and β_i on velocities and temperature distribution. Figure 4 depicts that with the increase in β_e horizontal velocity $f'(\eta)$ increases slightly near the surface of the wedge, but after a certain distance it decreases. This increases the velocity gradient near the wall and results the increase in local skin friction coefficient $(C_{fx}Re^{1/2})/2$. The values of skin friction coefficients for various values of β_e and β_i are given in tab. 4.

Table 4. Values of $(1/2)C_{fx}Re^{1/2}$, $(1/2)C_{fz}Re^{1/2}$ and $NuRe^{-1/2}$ for various values of β_e and β_i

$M = 1, m = 1/3, \beta_i = 0.4, K = 1, I = 0.5,$ $Pr = 1, \text{ and } Ec = 0.5$				$M = 1, m = 1/3, \beta_e = 5, I = 0.5, K = 1,$ $Pr = 1, \text{ and } Ec = 0.5$			
β_e	$(1/2)C_{fx}Re^{1/2}$	$NuRe^{-1/2}$	$(1/2)C_{fz}Re^{1/2}$	β_i	$(1/2)C_{fx}Re^{1/2}$	$NuRe^{-1/2}$	$(1/2)C_{fz}Re^{1/2}$
2	0.4672	0.3999	0.1150	0	0.5330	0.4055	0.0882
4	0.5090	0.4050	0.0799	0.2	0.5233	0.4056	0.0781
5	0.5162	0.4058	0.0660	0.4	0.5162	0.4058	0.0660

The effect of Hall current and ion slip on transverse component of velocity is shown in figs. 5 and 6. These figs. show that $g(\eta)$ decrease with the increase in β_e and β_i both. This causes the decrease in transverse velocity gradient, thus decreases the local skin friction coefficient $(C_{fz}Re^{1/2})/2$. This result is also verified by the calculated values given in tab 4.

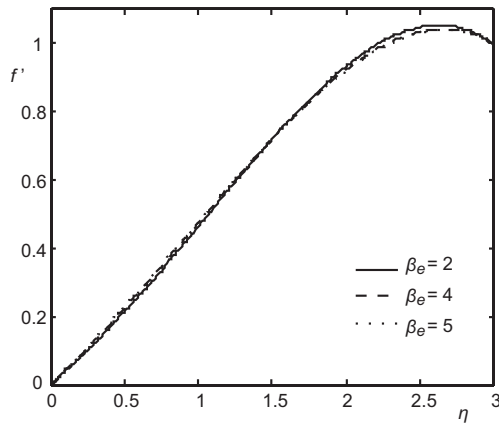


Figure 4. Horizontal velocity profile vs. β_e for $K = 1, M = 1, m = 1/3, \beta_i = 0.4, I = 0.5, Ec = 0.5,$ and $Pr = 1$

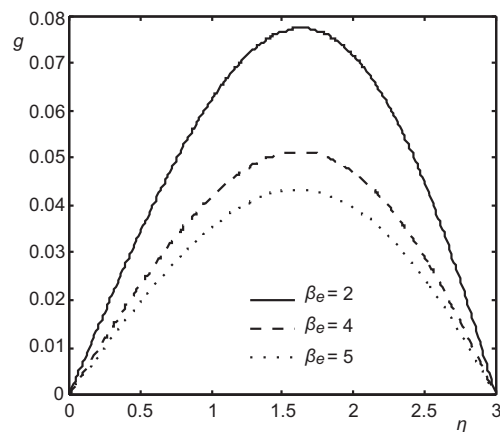


Figure 5. Horizontal velocity profile vs. β_e for $K = 1, M = 1, m = 1/3, \beta_i = 0.4, I = 0.5, Ec = 0.5,$ and $Pr = 1$

From fig. 7 it is observed that the magnitude of angular velocity decreases with the increase in β_e , this causes the decrease in angular velocity gradient near the wall, and results in the decreased wall couple stress. Figure 8 shows that magnitude of angular velocity increases with the increase in β_i and therefore increases the wall couple stress. It is clear from fig. 9 that the temperature decreases slightly with the increase in β_e . This concludes that the local Nusselt number *i. e.* heat transfer rate increases slightly with the increase in β_e , but there is no effect of β_i on the Nusselt number, as given in tab. 4.

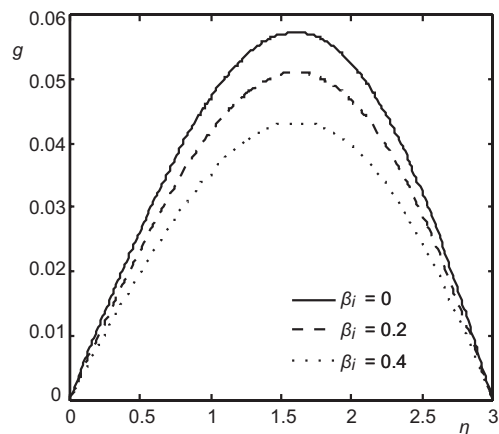


Figure 6. Transverse velocity profile vs. β_i for $K = M = 1, m = 1/3, \beta_e = 5, I = 0.5, Ec = 0.5,$ and $Pr = 1$

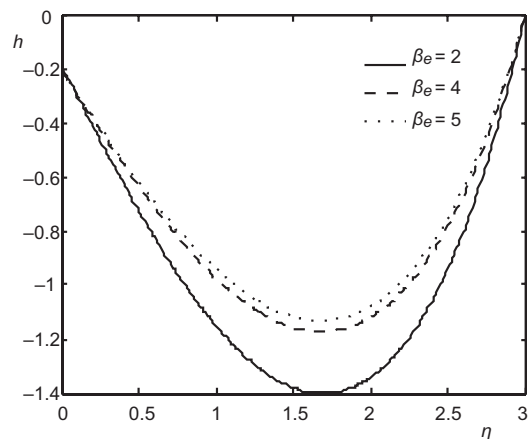


Figure 7. Angular velocity profile vs. β_e for $K = 1, M = 1, m = 1/3, \beta_i = 0.4, I = 0.5, Ec = 0.5,$ and $Pr = 1$

The effect of radiation on temperature is seen from fig. 10. It is clear from the figure that as we move away from the surface of the wedge, the temperature decreases for all the values of radiation parameter N , but as N increases the temperature decreases faster. Thus for higher value of radiation parameter the fluid cools rapidly. Also the values of local Nusselt

number have been calculated for various values of N and presented in tab. 5. From the table it is found that the value of Nusselt number increase with the increase in radiation parameter, but after a certain value of N , it becomes constant. Figure 11 depicts the effect of heat source/sink parameter on temperature. In the graph the positive values of Φ represents the presence of heat source and the negative values corresponds to heat sink. It is noted that as Φ increases from negative to positive values, the temperature as well as thermal boundary layer thickness increases, which results in the decrease in local Nusselt number as given in tab. 5.

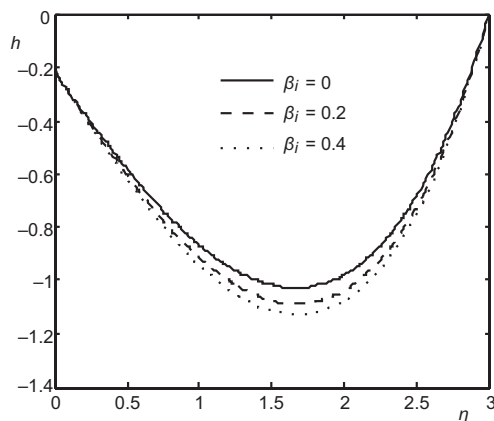


Figure 8. Angular velocity profile vs. β_i for $K = 1$, $M = 1$, $m = 1/3$, $\beta_e = 5$, $I = 0.5$, $Ec = 0.5$, and $Pr = 1$

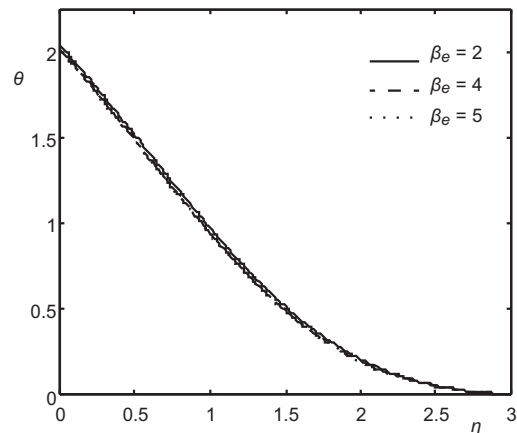


Figure 9. Temperature distribution vs. β_e for $K = M = 1$, $m = 1/3$, $\beta_i = 0.4$, $I = 0.5$, $Ec = 0.5$, and $Pr = 1$

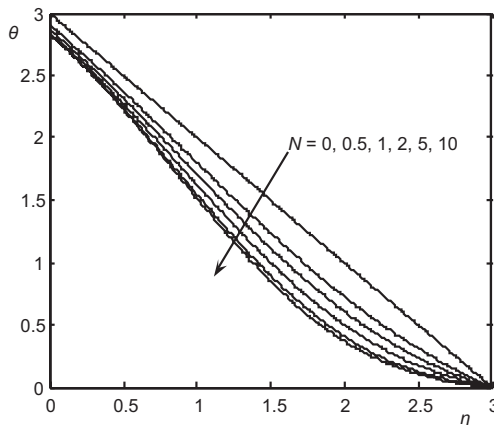


Figure 10. Temperature distribution vs. N for $K = 1$, $M = 1$, $m = 1/3$, $\beta_e = 5$, $\beta_i = 0.4$, $I = 0.5$, $Ec = 0.5$, $\Phi = 0.2$, and $Pr = 1$

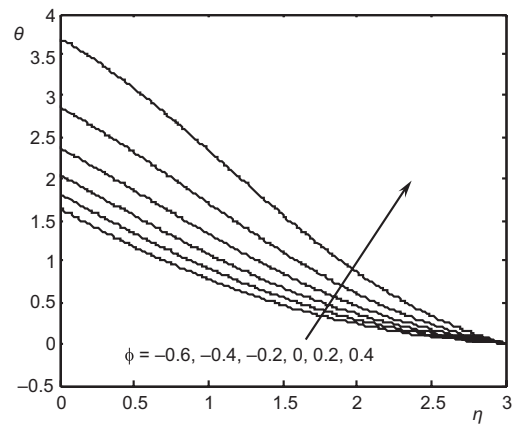


Figure 11. Temperature distribution vs. Φ for $K = 1$, $M = 1$, $m = 1/3$, $\beta_e = 5$, $I = 0.5$, $Ec = 0.5$, $N = 1$, and $Pr = 1$

From fig. 12 it is clear that the temperature as well as thermal boundary layer thickness increase with the increase in value of K . Therefore, the local Nusselt number decreases with the increasing value of material parameter. The variation of Nusselt number is shown in tab. 3. This table also signifies that increasing skin friction increases the heat transfer rate also.

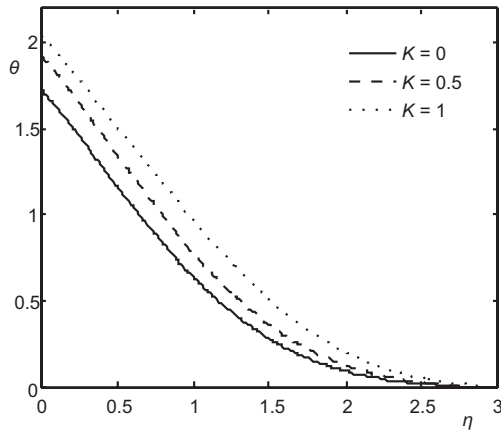


Figure 12. Temperature distribution vs. K for $M = N = 1, m = 1/3, \beta_e = 2, \beta_i = 0.4, I = 0.5, Ec = 0.5,$ and $Pr = 1$

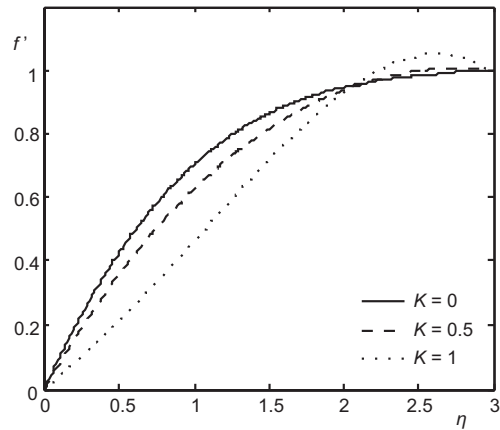


Figure 13. Horizontal velocity profile vs. K for $M = 1, m = 1/3, \beta_e = 2, \beta_i = 0.4, I = 0.5, Ec = 0.5,$ and $Pr = 1$

Table 5. Values of $NuRe^{-1/2}$ for $I = 0.5, \beta_i = 0.4, \beta_e = 5, M = 1, m = 1/3, K = 1, Pr = 1,$ and $Ec = 0.5$

Φ	N	$NuRe^{-1/2}$	N	Φ	$NuRe^{-1/2}$
0.2	0	0.2722	1	-0.6	0.4981
0.2	0.5	0.2812	1	-0.4	0.4499
0.2	1	0.2848	1	-0.2	0.3896
0.2	2	0.2874	1	0	0.3438
0.2	5	0.2886	1	0.2	0.2848
0.2	10	0.2886	1	0.4	0.2209

With the increase in material parameter K , horizontal velocity decreases and after a fixed distance away from the surface it increases, while the transverse velocity shows a uniform decreasing pattern with the increase in the value of K as depicted in figs. 13 and 14. It is also clear from the figs. that the velocity gradients near the surface decrease, and hence produces a decrease in skin friction coefficients. Thus micropolar fluids show a reduction in local skin drag as compared to Newtonian fluids. The numerical values of skin friction coefficients for various values of K are given in tab. 3.

Figure 15 depicts that the absolute value of angular velocity near the surface decreases with the increase in the value of material parameter, and as we move away from the wedge this value increases. These changes results an increase in rotation of the particle near the surface and hence increase the wall couple stress. Thus the increase in material parameter is to accelerate the particle rotation near the surface.

The effects of other parameters on local skin friction coefficients and local Nusselt number are given in tab. 6. From this table it is concluded that increasing value of magnetic field M decreases skin friction coefficient due to horizontal motion as well as heat transfer rate, while the skin friction coefficient due to transverse velocity increases with the increase in M . Further, the inertial parameter shows an increase in the skin friction coefficients up to a fixed value of the parameter, but after this value the trend gets reversed. Similarly the local

Table 6. Values of $(1/2)C_{f_x}Re^{1/2}$, $(1/2)C_{f_z}Re^{1/2}$ and $NuRe^{-1/2}$ for various values of M and I

$I = 0.5, \beta_i = 0.4, \beta_e = 5, m = 1/3, K = 1,$ $Pr = 1, \text{ and } Ec = 0.5$				$M = 1, m = 1/3, \beta_i = 0.4, \beta_e = 2, K = 1,$ $Pr = 1, \text{ and } Ec = 0.5$			
M	$(1/2)C_{f_x}Re^{1/2}$	$NuRe^{-1/2}$	$(1/2)C_{f_z}Re^{1/2}$	I	$(1/2)C_{f_x}Re^{1/2}$	$NuRe^{-1/2}$	$(1/2)C_{f_z}Re^{1/2}$
0.5	0.5286	0.4070	0.0336	0.5	0.4672	0.3999	0.1150
2	0.4937	0.4027	0.1266	2	1.0672	0.3636	0.1413
5	0.4473	0.3889	0.2779	4	1.0570	0.3675	0.1395

Nusselt number decreases with the increase in material parameter, but after the same fixed value of the parameter it increases. The oscillations of skin friction coefficients and Nusselt number is due to the combined effect of vortex viscosity parameter (K) and Inertia parameter (I). For a fixed value of K , there is a particular range of inertia effect for which inertia effect is dominant and beyond this value the viscous effects become dominant.

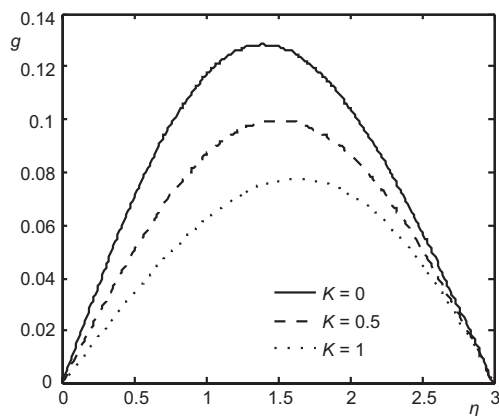


Figure 14. Transverse velocity profile vs. K for $M = 1, m = 1/3, \beta_e = 2, \beta_i = 0.4, I = 0.5, Ec = 0.5,$ and $Pr = 1$

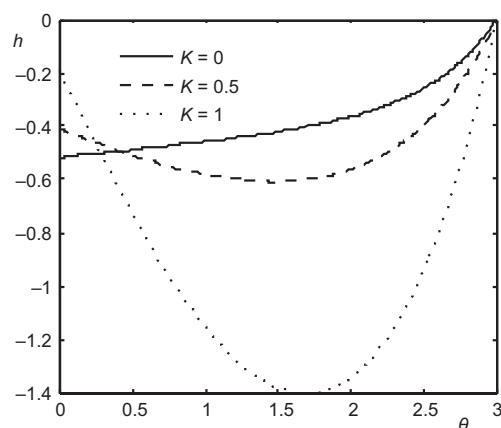


Figure 15. Angular velocity profile vs. K for $M = 1, m = 1/3, \beta_e = 2, \beta_i = 0.4, I = 0.5, Ec = 0.5,$ and $Pr = 1$

Conclusions

We have theoretically studied the problem and obtained the numerical results for skin friction coefficients. From the results it was shown that the findings of the present problem are in very good agreement with the previously published work, for some particular cases of the present problem. The main findings of the present investigation are as follows:

- Local Nusselt number increases with the increase in strength of heat sink as well as radiation parameter.
- Skin friction coefficient and local Nusselt number are small for small wedge angle.
- Skin friction coefficient due to translational motion of the fluid increases with the increase in hall parameter, which causes an increase in heat transfer rate.
- Skin friction coefficient due to rotational motion of the particles of the fluid decreases with the increase in hall parameter.
- Skin friction coefficients due to translational as well as rotational motion of the fluid decreases slightly with the increase in ion slip parameter and therefore, change in the local Nusselt number is negligible.

- Skin friction coefficients and Nusselt number decreases with the increase in material parameter. *i. e.* micropolar fluids reduce the skin friction and heat transfer rate.
- Magnetic field reduces the skin friction due to translational motion as well as heat transfer rate.

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