A SOLUTION FOR POTENTIAL FLOW OVER AN ARC FIBER

by

***Wenxi WANG*[a], *Qing HE*[a], *Nian CHEN*[a], *Yuan ZHOU*[a], *and Mingliang XIE*[a,b*]**

[a] The State Key Laboratory of Coal Combustion, Huazhong University of Science and Technology, Wuhan, China
[b] Department of Mechanical Engineering, University of Delaware, Newark, Delaware, USA

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In the paper, the flow field around an arc fiber is studied. Based on the Zhukovsky conversion, the potential and stream functions are derived. The results show that the flow fields depend on geometric properties of the arc fiber and the orientation angle of the incoming flow. The flow field developed in this paper can be used to predict the single-fiber efficiency for the arc fiber.

Key words: *arc fiber, Zhukovsky conversion, potential flow*

Introduction

With the advancement of technology, synthetic fibers can be made in various shapes, including ellipse, square, rectangular flat and arc plate. As non-circular fibers per unit volume can offer higher surface area than circular fibers and improve the filter performance greatly. There are already many researches on the flow field over ellipse, square and rectangular fibers [1-5], but little over arc plates.

The flow regimes over the obstacles include the Stokes flow regime (Re < 1), transition flow regime ($1 \leq \mathrm{Re} \leq 1000$), and potential flow regime ($\mathrm{Re} \geq 1000$). For viscous Stokes flow around a circle, Kuwabara [6] and Happel [7] arrived at an analytical description of the flow field by neglecting inertial terms in the Navier-Stokes equations and specifying boundary conditions on a unit cell around each fiber. Since then, the Kuwabara-Happel model became fundamental to many theoretical studies of fibrous filtration. The viscous flow assumption is reasonable for the majority of filter applications. For a larger Reynolds number, a potential flow is predicted. In the present study, a solution for the flow field around an arc fiber based on Zhukovsky conversion is obtained.

Theory and method

In the present study, the deduction of flow field over an arc is based on the theory of Zhukovsky conversion [8], which is defined as:

* Corresponding author; e-mail: mlxie@mail.hust.edu.cn

$$z = \frac{1}{2}\left(\zeta + \frac{b^2}{\zeta}\right) \tag{1}$$

where b is a parameter in the transformation, and the inverse transformation is:

$$\zeta = z + \sqrt{z^2 - b^2} \tag{2}$$

In order to obtain the transformation from circle to arc with arbitrary orientation of incoming fluid flow (α) as shown in fig. 1, the equation of a circle with radius a in the ζ (ξ, η) plane is set as:

$$\left|\zeta - f_1\right| = a = \sqrt{f_1^2 + b^2} \tag{3}$$

The corresponding curve equation in $z(x, y)$ plane based on the transformation is given as:

$$x^2 + \left[y + \frac{1}{2}\left(\frac{b^2}{f_1} - f_1\right)\right]^2 = b^2 + \frac{1}{4}\left(\frac{b^2}{f_1} - f_1\right)^2 \tag{4}$$

From the derivation procedure, the direction of arc depends on the positive or negative of f_1, and for a given b, the bending degree of the arc depends on the absolute value of f_1.

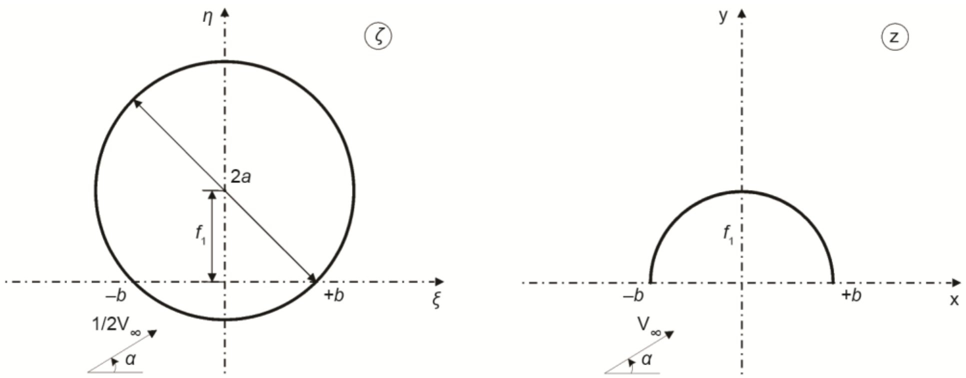

Figure 1. Transformation from circle to arc

The velocity components of incoming flow at the infinity in the ζ (ξ, η) plane is given by:

$$\left(\frac{d\chi}{d\zeta}\right)_{\zeta=\infty} = \left(\frac{d\chi}{dz}\cdot\frac{dz}{d\zeta}\right)_{\zeta=\infty} = \frac{1}{2}V_\infty e^{-i\alpha} \tag{5}$$

Then, the complex potential function in $z(x, y)$ plane can be gained using the inverse transformation equation:

$$\chi(\zeta)=\frac{1}{2}V_\infty\left[\mathrm{e}^{-i\alpha}\left(z+\sqrt{z^2-b^2}-f_1\right)+\frac{a^2\mathrm{e}^{i\alpha}}{z+\sqrt{z^2-b^2}-f_1}\right] \tag{6}$$

According to the definition of complex potential function, the potential function (real part) and stream function (imaginary part) in $z(x, y)$ plane can be written, respectively:

$$\begin{aligned}\varphi&=\frac{1}{2}V_\infty\left[\xi\cos\alpha+(\eta-f_1)\sin\alpha\right]\left[1+\frac{a^2}{\xi^2+(\eta-f_1)^2}\right]\\ \psi&=\frac{1}{2}V_\infty\left[(\eta-f_1)\cos\alpha-\xi\sin\alpha\right]\left[1-\frac{a^2}{\xi^2+(\eta-f_1)^2}\right]\end{aligned} \tag{7}$$

where ξ and η are given as:

$$\begin{aligned}\xi&=x\pm\sqrt{\frac{\sqrt{\left(x^2-y^2-b^2\right)^2+4x^2y^2}+\left(x^2-y^2-b^2\right)}{2}}\\ \eta&=y\pm\sqrt{\frac{\sqrt{\left(x^2-y^2-b^2\right)^2+4x^2y^2}-\left(x^2-y^2-b^2\right)}{2}}\end{aligned} \tag{8}$$

Results and discussions

The solution of the flow field around an arc includes four parameters, *i. e.*, the radius of circle, a, the transformation parameter, b, the eccentric distance of circle, f_1, and the orientation of incoming fluid flow, α. To simplify the analysis of potential and stream functions, the four parameters can be combined as two variables based on the Gougu theorem (Pythagorean theorem), *i. e.*, f_1/b and α. Figure 2 shows streamlines and equipotent lines for $f_1/b = 0$, 1/2, 1, 3/2, and $\alpha = \pi/4$. The streamlines are divided into two parts that pass around different sides of the fiber. When $f_1/b = 0$, the arc becomes a flat plate. The upstream and downstream branches of this streamline are consequently given by eq. 7 for different quadrant, and it can clearly be seen that the flow field is asymmetric. Figure 3 shows streamlines for α set to 0, $\pi/6$, $\pi/3$ and $\pi/2$, and $f_1/b = 2/3$. For the limited cases ($\alpha = \pi/2$), the streamlines are symmetric relative y-axis. But for any other incoming flow angle α, the streamlines are asymmetric.

Conclusions

The developed theory can predict the velocity field around an arc filter fiber, and the derived equations depend on the ratio of arc fiber (f_1/b), and the orientation of the incoming flow (α). The velocity field developed in this paper can be directly used for prediction of the single-fiber efficiency for an arc fiber.

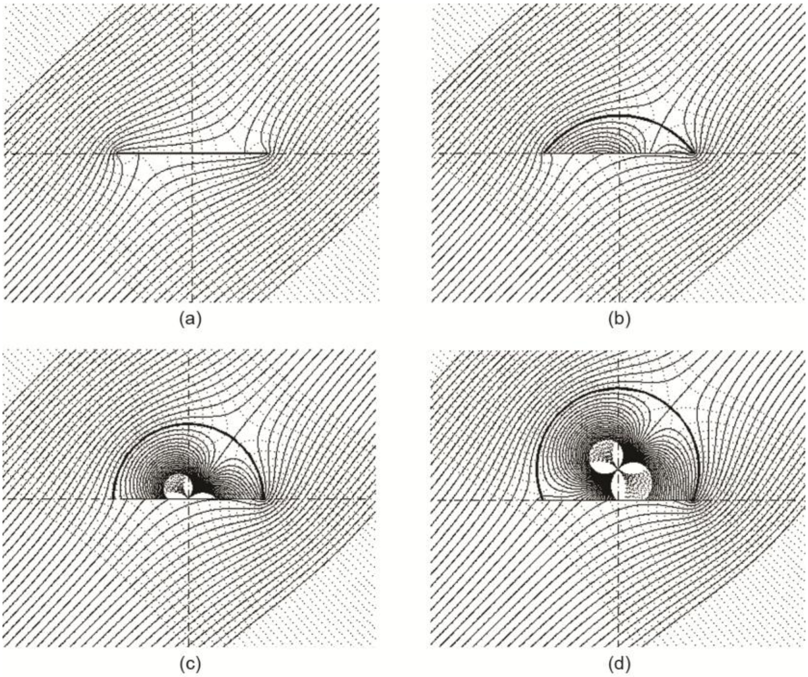

Figure 2. The contours of potential and stream function; (a) $f_1/b = 0$, (b) $f_1/b = ½$, (c) $f_1/b = 1$, (d) $f_1/b = 3/2$

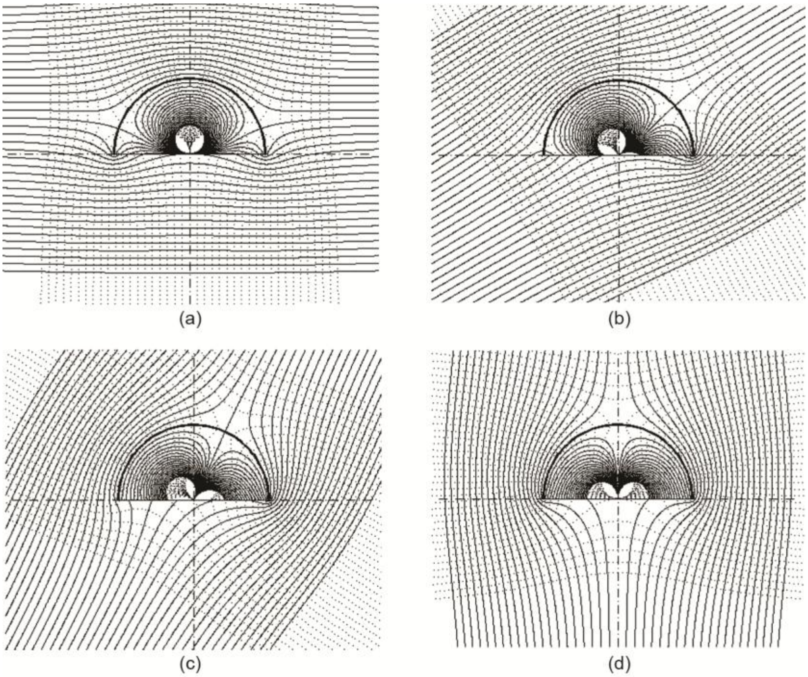

Figure 3. The contours of potential and stream function; (a) $\alpha = 0$, (b) $\alpha = \pi/6$, (c) $\alpha = \pi/3$, (d) $\alpha = \pi/2$

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