DYNAMIC STABILITY OF NON-DILUTE FIBER SHEAR SUSPENSIONS

by

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Temporal stability analysis of fiber suspended shear flow is performed. After introducing the second order structure tensor to determine the Folgar-Tucker inter-fiber interactions based on the Langevin's equation, a system governing the flow stability is derived in conjunction with the fiber orientation closure. Effect of the inter-fiber interactions on the dynamic stability is studied by solving the general eigenvalue problem. Results show that fiber interaction has significant stabilizing effects on the flow. The most unstable wave number changes with the interaction coefficient. For given interaction coefficient, wave number and other relevant parameters, there is a Re number which corresponds to the critical flow. This Re number is related to the wave number.

Key words: *interaction, stability, shear suspension, non-spherical particulate*

Introduction

Suspension of particles can significantly change the flow field. It has been known that the suspensions of particles can significantly change the characteristics of the flow field [1], and the dispersed particles can either increase or decrease the flow instability under different circumstances [2]. This paper addresses the hydrodynamic load on the solid fiber particles. Their constitutive behaviors are described using transversely isotropic fluid models originally derived by Ericksen [3], Hand [4] *et al*. and further developed by Batchelor [5]. The properties of non--spherical fiber particle suspensions depend not only on the suspending fluid and flow process, but also on the aspect ratio of fibers and the particle concentration [6, 7]. In non-dilute systems, the free rotations of fibers are hindered due to the frequent interactions between fibers, and the fiber-fiber interactions cannot be neglected. In some literatures of stability analysis, *e. g*. [8-10], the effects of the fiber-fiber interactions are not included. Therefore, a better understanding of the effect of interaction on the stability is desired. Recently, a stability analysis of the planar mixing layer shows that the hydrodynamic interaction acts as a dissipative term and has stabiliz-

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ing effects on the suspension [11]. The anisotropic diffusion models [12] propose new forms of fiber collision modeling with higher accuracies. On the theoretical side, one modification to the rotary diffusion term is proposed by Rahnama *et al*. [13]. The present study extends previous analysis on the stability of fiber suspension between two coaxes rotating cylinders [14] and aims at how inter-fiber interaction affects the dynamical stability.

Theoretical model

The derivation of stability analysis equations with the moment tensor of fibers is briefly presented here. A fiber with finite length in suspensions undergoes translational motion and rotates in a shear flow along a Jeffery orbit, which satisfies:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{n} = \boldsymbol{\omega}\cdot\mathbf{n} + \lambda\ \dot{\boldsymbol{\gamma}}\cdot\mathbf{n} - \dot{\boldsymbol{\gamma}}:\mathbf{nnn} \tag{1}$$

where $\boldsymbol{\omega}$ and $\dot{\boldsymbol{\gamma}}$ are the vorticity and the rate of strain tensor, respectively. The parameter λ is related to the aspect ratio r_e as $\lambda = (1 - r_e^2)/(1 + r_e^2)$. It attains values between 0 and 1. Hence, a decrease of r_e from infinity to a finite value reduces the relative strength of strain rate with respect to the vorticity in determining the rotation of the fiber particles. Gupta *et al*. [15] investigated the large aspect ratio of $O(10^3)$. In the present study, the range of aspect ratio r_e is given as O(10). For fiber-fiber interaction, the extra part of the contribution from hydrodynamic forces f_b should be added to eq. (1). The modified equation to incorporate the rotary diffusion is referred to as the generalized Fokker-Planck or Smoluchowski equation [16]:

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \frac{\partial}{\partial\mathbf{n}}\left\{ \mathbf{I} - \mathbf{nn}\ \mathbf{D}_r \frac{\partial\psi}{\partial\mathbf{n}} - \left[\boldsymbol{\omega}\cdot\mathbf{n} + \lambda\ \dot{\boldsymbol{\gamma}}\cdot\mathbf{n} - \dot{\boldsymbol{\gamma}}:\mathbf{nnn}\ \right]\psi \right\} \tag{2}$$

Equation (2) consists of partial differential equations in 5-spacial dimensions. The second order orientation tensor is defined as:

$$\mathbf{a}_2 = \oint \mathbf{nn}\psi\,\mathrm{d}\mathbf{n} \tag{3}$$

Similar expressions for other higher moment tensors can also be obtained. Taking the material time derivative of eq. (3) and incorporating eqs. (1) and (2), we have:

$$\mathbf{a}_{2\,1} = \frac{\chi - 1}{2}\ \dot{\boldsymbol{\gamma}}\cdot\mathbf{a}_2 + \mathbf{a}_2\cdot\dot{\boldsymbol{\gamma}}\ - \chi\ \dot{\boldsymbol{\gamma}}:\mathbf{a}_4\ + 2\mathbf{D}_r\left[\boldsymbol{\delta} - \ 2 + D_z\ \ \mathbf{a}_2\right] \tag{4}$$

where $\boldsymbol{a}_{2(1)}$ is the upper convected derivative of the second order orientation tensor, and D_z – a thickness moment coefficient with $0 \leq D_z \leq 1$. A smaller D_z implies less out-of-plane orientation. For three dimension, set $D_z = 3$. The orientation tensor approach requires knowledge of the next higher-order moment tensor, thus some form of closure is required. Here we adopt the closure approximation of Hinch *et al.* [17]. The stability analysis for Taylor-Couette shear flow is: assume the base flow is disturbed. The velocity, pressure and the orientation tensor are represented by the base profile plus an infinitesimally small perturbation. Substitution of the base profile plus perturbation into the system of governing equation. After linearization, we obtain the stability equations. The Chebyshev pseudo-spectral technique is applied to solve the equations.

Results and discussion

Note that the terms in eqs. (2) and (4) contain the $\boldsymbol{D}_r$, which accounts for the inter-fiber interactions. So far, there have been several models proposed to deal with this diffusive contribution. Generally, the diffusivity should be dependent on the orientation state, being higher when the fibers are randomly oriented and lower when the fibers are aligned. The Folgar-Tucker model of diffusion for fiber interactions within a suspension has been used for several decades to compute fiber orientation and has been implemented in many industrial and scientific computer simulations. However, this model is phenomenological: the interaction coefficient is inversely proportional to the average inter-fiber spacing; the rotary diffusion is isotropic; the model gives the steady-state distributions of fiber orientation, and it reaches an upper limit in volume fraction and aspect ratio when the average inter-fiber spacing approaches zero. The model define the diffusive contribution as:

$$\mathbf{D}_r = C_I |\dot{\gamma}|, \quad |\dot{\gamma}| = \sqrt{\text{tr} \;\; \dot{\gamma} \cdot \dot{\gamma} \;\; /2}$$

Yamane *et al*. [18] simulated the fiber motion incorporated the effects of short-range interactions by lubrication forces between neighboring fibers. They extracted the values of C_I at the range 10^{-4} to 10^{-8}. Folgar *et al*. [19] measured it as about 10^{-2} to 10^{-4}.

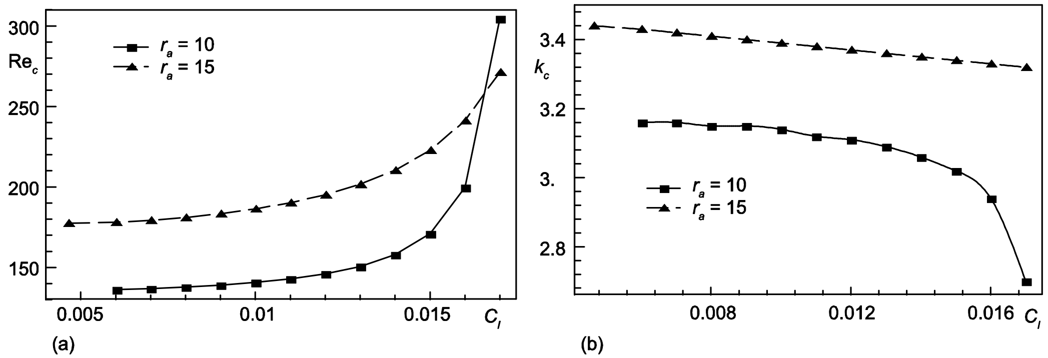

Figure 1. Critical values along C_I for different r_a; (a) Reynolds numbers $\mathrm{Re_c}$, (b) axial numbers k_c

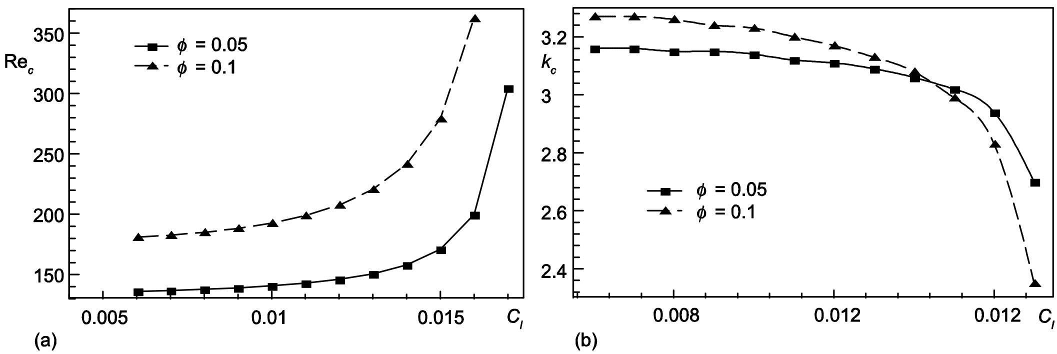

Figure 2. Critical values along C_I for different ϕ; (a) Reynolds numbers $\mathrm{Re_c}$, (b) axial numbers k_c

Figure 1 shows the variation of critical Reynolds number Re_c and critical wave number k_c with C_I for the same volume fraction $\phi = 0.05$ and two aspect ratio $r_a = 10$ and 15, respectively. The similar findings for two volume fraction $\phi = 0.05$, and 0.1, are shown in fig. 2. Here the aspect ratio is 10. According to non-dilute suspension regime, it is expected to find a fiber in a volume dr_e^2. Here the regime is $\phi^* r_a \sim O(1)$. For the low fiber volume fraction, the critical axial wave numbers k_c changes a little with increasing interaction coefficient C_I, fiber volume fraction and fiber aspect ratio. But in the non-dilute concentrated range, the critical axial wave number changes evidently. Especially, for relatively large values of interactions coefficients C_I, the critical axial wave number decreases with C_I. The effects of inter-fiber interaction C_I on Re_c are found to be qualitatively similar with fiber aspect ratio r_a and volume fraction ϕ as in previous study. A recent simulation of fiber diffusion raised from interactions by Fan *et al*. [12] included both long-range hydrodynamic interactions and a lubrication resistance force when the fibers came within a fiber diameter of one another. Slender body theory was used to determine the long-range interaction. They developed a rotary diffusion model in which the scalar interaction coefficient C_I was determined by a fitting formula. We find from eq. (11) and fig. 1 in the cited paper that the value of C_I increases monotonically with the product of aspect ratio and volume fraction, $\phi^* r_a$.

Conclusions

The influence of inter-fiber interaction on the stability of non-dilute fiber shear suspension is analyzed. Results show the strong stabilizing effect of interaction coefficient of $O(10^{-2}\text{-}10^{-3})$ for smaller aspect ratio of O(10) and product of aspect ratio and volume fraction of O(1). Stability property of suspensions is obtained. In addition to the increasing effect of fiber interact on Re, the most unstable wave number has an impact on the evolution of fiber particles.

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