A NOVEL FRICTION LAW

by

Hai-Yan KONG and Ji-Huan HE**

National Engineering Laboratory for Modern Silk,
College of Textile and Clothing Engineering,
Soochow University, Suzhou, China

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Frictional force is a vitally important factor in the application of engineering either in macroscopic contacts or in micro/nanoscale contacts. Understanding of the influencing factors about frictional force is essential for the design of miniaturized devices and the use of minimal friction force. In the paper, dimensional analysis is used to analysis factors relative to frictional force. We show that the frictional force scales with $A^{a_1}N^{1-a_1}$*, where A is the contact area and N is the normal contact force. An experiment is carried out to verify the new friction law.*

Key words: *dimension analysis, friction, fractal dimension*

Introduction

Friction force is important, especially the microscale friction force. For any moving surfaces, decrease of friction means energy saving. Macroscopic laws of friction do not generally apply to nanoscale contacts. An understanding of how friction force depends on applied load, contact area and some other factors is essential for the design of miniaturized devices and the use of minimal friction force.

According to the well-known Amontons' law [1] of macroscopic friction, formulated in 1699, the friction force is proportional to the applied load and independent of the macroscopic contact area, which becomes invalid for nanoscale or microscale contacts. For macroscopic contacts, the linear dependence of the friction force on the load is conventionally explained by the theory of Bowden *et al.* [2], which is based on the assumption that the macroscopic contacts are rough and composed of a large number of small asperities.

While continuum mechanics models have been quite successful in describing contact behavior at the microscale, a number of studies [3-6] have suggested that continuum mechanics breaks down when the contact size reaches nanometer dimensions. A model for non--adhesive contact between homogenous, isotropic, linear elastic spheres was first developed by Hertz [7], who showed that the asperity contact area is relative to $L^{2/3}$. Adhesion effects were included in a number of subsequent models, among which Mauhis-Dugdale theory [8] has been frequently used because of its high degree of flexibility. Mo *et al*. [9] showed that simple friction laws do apply at the nanoscale: the friction force depends linearly on the num-

* Corresponding author; e-mail: hejihuan@suda.edu.cn

ber of atoms. As for non-adhesive contact, dependence of friction on the real contact area has been directly verified. Linear dependence of friction on load is due to atomic roughness and small contact pressures. As for adhesive contact, adhesion induces transition from linear to sublinear behavior.

In order to understanding what affects friction force, dimensional analysis has done to illustrate the influencing factors of friction force and a novel friction law is elucidated in the paper.

Dimensional analysis

Dimensional analysis is a conceptual tool often applied in physics, chemistry, and engineering to understand physical situations involving a mix of different kinds of physical quantities.

Two tangent objects, when they will take place or have occurred relative motion, a hindrance force to relative motion will engender on the contact surface. The force is called frictional force. In order to completely analysis friction force, more relative factors should be considered as far as possible. We suppose that friction force F_f is relative to the contact area A, the normal contact force N, the acceleration a, the speed V, the speed gradation $\partial u/\partial x$ and the elastic modulus E. That is:

$$F_f = F_f\left(A, N, a, V, \frac{\partial u}{\partial x}, E\right) \tag{1}$$

According to the dimensional analysis, the friction force can be expressed:

$$[F_f] = [A]^{a_1}[N]^{a_2}[a]^{a_3}[V]^{a_4}\left[\frac{\partial u}{\partial x}\right]^{a_5}[E]^{a_6} \tag{2}$$

where a_1, a_2, a_3, a_4, a_5, and a_6 are constants.

The International System of Units (SI) defines seven basic dimensions. They are length (L), mass (m), time (t), electricity (I), thermodynamic temperature (Q), amount of substance (N), and light intensity (J). Other units can be derived from these basic units according to their definition. According to SI, the dimension of F_f is $[MLT^{-2}]$, the dimension of A is $[L^2]$, the dimension of N is $[MLT^{-2}]$, the dimension of a is $[LT^{-2}]$, the dimension of V is $[LT^{-1}]$, the dimension of $\partial u/\partial x$ is $[T^{-1}]$. The elastic modulus (E) means the ratio between normal stress and strain when material is in the process of elastic deformation, the unit of E is Pa, so the dimension of E is $[M^{-1}LT^{-2}]$.

So the eq. (2) can be expressed as:

$$[MLT^{-2}] = [L^2]^{a_1}[MLT^{-2}]^{a_2}[LT^{-2}]^{a_3}[LT^{-1}]^{a_4}[T^{-1}]^{a_5}[ML^{-1}T^{-2}]^{a_6} \tag{3}$$

from which we obtain:

$$2a_1 + a_2 + a_3 + a_4 - a_6 = 1 \tag{4}$$

$$a_2 + a_6 = 1 \tag{5}$$

$$2a_2 + 2a_3 + a_4 + a_5 + 2a_6 = 2 \tag{6}$$

Solving *c*, *e*, and *f* from eqs. (4)-(6), we have:

$$a_3 = 2 - 2a_1 - 2a_2 - a_4 \tag{7}$$

$$a_5 = 4a_1 + 4a_2 + a_4 + 4 \tag{8}$$

$$a_6 = 1 - a_2 \tag{9}$$

Thus the frictional force F_f can be written as:

$$F_f = kA^{a_1} N^{a_2} a^{2-2a_1-2a_2-a_4} V^{a_4} \left(\frac{\partial u}{\partial x}\right)^{4a_1+4a_2+a_4+4} E^{1-a_2} \tag{10}$$

where *k* is the constant of the equation.

Supposing that the frictional force F_f is independent of the acceleration *a*, *i. e.*, $a_3 = 0$, we can simplify eq. (10) in the form:

$$F_f = k\, A^{a_1} N^{a_2} \left(\frac{V}{\frac{\partial u}{\partial x}}\right)^{2-2a_1-2a_2} E^{1-a_2} \tag{11}$$

Further assuming that the friction is independent of velocity or its gradation, *i. e.* $2 - 2a_1 - 2a_2 = 0$, we have:

$$F_f = k\, A^{a_1} N^{1-a_1} E^{a_1} \tag{12}$$

When $a_1 = 0$, eq. (12) is the well-known Coulomb's law. For a fixed couple of contact, the elastic modulus *E* keeps unchanged, that means $F_f \propto A^{a_1} N^{1-a_1}$. If *N* is a constant, we have $F_f \propto A^{a_1}$.

Experimental

Square metals A, B, and C, with dimensions of 10 cm × 10 cm × 2 cm, were selected as experimental samples. There are, respectively, one, three and five square holes with dimensions of 1 cm × 1 cm × 2 cm in A, B, and C (figs. 1-3). The experimental set-up is illustrated in fig. 4.

Samples A, B, and C are combined together with different orders, so that the contact area differs from each other, while the normal contact force is same.

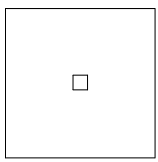

Figure 1. Sample A

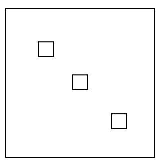

Figure 2. Sample B

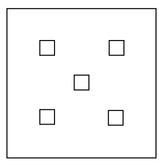

Figure 3. Sample C

Analyses of the experimental data

The combined metal moves down alone the sliding plate with a constant acceleration:

$$L = \frac{1}{2}at^2 \tag{13}$$

where L is the length of the sliding plate, a – the acceleration, and t – the time.

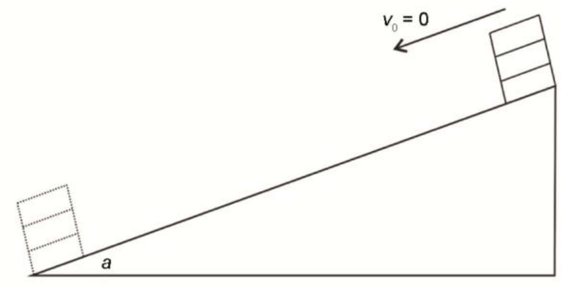

Figure 4. Experimental set-up

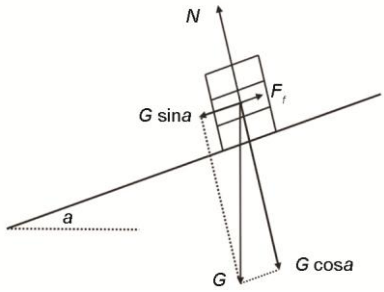

Figure 5. Force analysis

According to Newton's second law, we have (fig. 5):

$$G\cos\alpha - F_f = Ma \tag{14}$$

where G is the gravity, α – the slant angle, F_f – the frictional force, and M – the total mass. From eqs. (12), (13), and (14), we obtain:

$$t = \sqrt{\frac{2ML}{G\cos\alpha - kA^a N^{1-a} E^a}} \tag{15}$$

or

$$t = \sqrt{\frac{2ML}{G\cos\alpha - \beta A^a}} = \sqrt{\frac{1}{m - nA^a}} \tag{16}$$

where β, m, and n are coefficients.

In our experiment, L = 0.6 m, cos α = $(2\cdot 2^{-1/2})/3$, then the coefficient of m can be fixed as:

$$m = \frac{g\cos\alpha}{2L} = 7.7 \tag{17}$$

Table 1. Time record of experiments

Experiment	Time [s]	Area [m^2]
A	2.6	0.0099
B	2.0	0.0097
C	1.8	0.0095

The time of each experiment is listed in tab. 1. Using the data in tab. 1, we obtain:

$$t = \sqrt{\frac{1}{7.7 - 75.2A^{0.498}}} \tag{18}$$

Figure 6 compares the theoretical prediction of eq. (18) with experimental data, showing relatively good agreement.

Conclusions

We obtain a new law of frictional force, a ple experiment shows our suggested law is acceptable for practical applications.

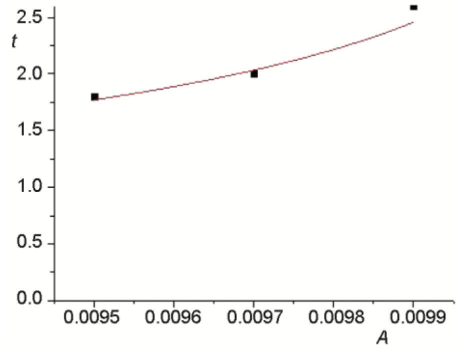

Figure 6. Experimental data and theoretical prediction

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