THE FRACTAL HARMONIC LAW AND ITS APPLICATION TO SWIMMING SUIT

by

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Short paper
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Decreasing friction force between a swimming suit and water is the key factor to design swimming suits. Water continuum mechanics forbids discontinuous fluids, but in angstrom scale water is indeed discontinuous. Swimming suit is smooth on large scale, but it is discontinuous when the scale becomes smaller. If fractal dimensions of swimming suit and water are the same, a minimum of friction force is predicted, which means fractal harmonization. In the paper, fractal harmonic law is introduced to design a swimsuit whose surface fractal dimensions on a macroscopic scale should be equal to or closed to the water's fractal dimensions on an Angstrom scale. Various possible microstructures of fabric are analyzed and a method to obtain perfect fractal structure of fabric is proposed by spraying nanofibers to its surface. The fractal harmonic law can be used to design a moving surface with a minimal friction.

Key words: *fractal, fractal harmonic law, friction, swimsuit*

Introduction

Atoms are roughly Angstroms in size (a hydrogen atom is about 1 Å in diameter, a carbon atom is about 2 Å in diameter, and the diameter of an oxygen atom is ca. 1.75 Å). One Angstrom (Å) is one ten-billionth of a meter or one-tenth of a nanometer.

$$1 \text{ nm} = 10 \text{ Å} \tag{1}$$

Water continuum mechanics forbids discontinuous fluids, but water in Angstrom scale is indeed discontinuous. On the Angstrom scale, water, for example, becomes discontinuous and the classical mechanics become invalid, the world of water must look like Universe which is full of empty space, see fig. 1 [1-4].

Fractal dimensions can be calculated by the mathematic expression:

$$D = \log M/\log N \tag{2}$$

where M is the number of new units within the original unit with a new dimension, and N is the ratio of the original dimension to the new dimension. In macro-scale, pure water is definitely continuous with 3 dimensions of space, but on Angstrom-scale, the fractal dimensions read [4]:

$$D_{\text{water}} = 1.92 \tag{3}$$

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That means the pure water is full of empty space in the three dimensional space. Angstrom technology (A-technology) has to be used in many modern applications.

Fractal harmonic law

A fractal harmonic law was proposed in 2010 in [4] to design swimming vest and a moving surface with minimum friction. Fractal harmonic law admits minimal friction between pure water and a swimming vest when the fractal dimensions of water in Angstrom-scale are equal to the fractal dimensions of the vest in micro scale, that is:

$$D_{\mathrm{water}} = D_{\mathrm{vest}} \tag{4}$$

Generally friction is a minimum when the factional dimensions of a solution (*e. g.* air, water, oil) in Angstrom-scale, D_{solution}, are equal to the fractional dimensions of the surface of the moving body in a larger scale (*e. g*. nano-scale, micro-scale) in the solution:

$$D_{\mathrm{solution}} = D_{\mathrm{surface}} \tag{5}$$

Now we image a small ball (circle in fig. 2) with diameter of 1 Å or less in water. The average distance between two water molecules is about 3 Å, so the ball can stand among the water molecules with almost zero pressure. If we could exactly replace some water molecules (A in fig. 2) by a fish, then there would be a minimal pressure on the surface of the fish. The fish's skin is, however, made of macromolecules, so it is impossible to exactly match discontinues water molecules.

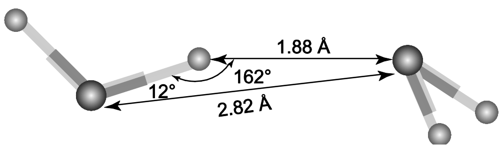

Figure 1. Two water molecule in Angstrom scale

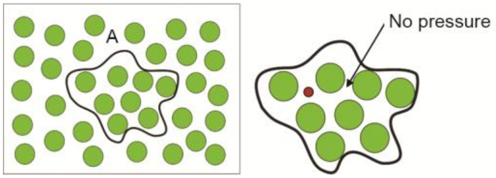

Figure 2. An object A among the water molecules undergoes no pressure

Considering an object constituted of several water molecules (shown in fig. 2), fractal dimensions of A and water are same, so A will stand no pressure in water whatever the shape of A.

Application of fractal harmonic law

Fractal harmonic law can be used to design swimming vest and a moving surface with minimum friction. Swimming vest is smooth on larger scale, but it is discontinuous when the scale tends to smaller as illustrated in fig. 3.

The fractional dimensions of the fabric on a micro scale [5-6] are:

$$D_{\mathrm{fabric}} = \frac{\ln M}{\ln N} = \frac{\ln\left[\pi r_1 a + \pi r_2 b - \pi r_1 \times \pi r_2 \;\; / \, r_1^2\right]}{\ln\left(\dfrac{\sqrt{ab}}{r_1}\right)} \tag{6}$$

where the symbols are the length given in fig. 3. If the swimming vest has the minimal friction which means $D_{\text{vest}} = D_{\text{water}} = 1.92$, then the value of a, b, r_1 and r_2 will be countless.

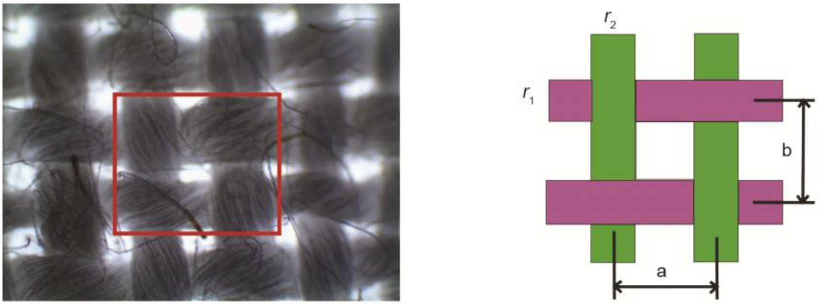

Figure 3. Microstructure of a fabric

Presuming that $r_2 = mr_1$, $b = na$, $a = kr_1$ then:

$$D_{\text{fabric}} = \frac{\ln(\pi k + \pi mnk - \pi^2 m)}{\ln\sqrt{nk}} \tag{7}$$

According to fractal harmonic law, $D_{\text{fabric}} = D_{\text{water}}$:

$$D_{\text{fabric}} = \frac{\ln(\pi k + \pi mnk - \pi^2 m)}{\ln\sqrt{nk}} = 1.92 \tag{8}$$

$$\pi k + \pi mnk - \pi^2 m = \left(\sqrt{nk}\right)^{1.92} \tag{9}$$

Considering that $m = n = 1$, then coefficient k can be calculated as $k \approx 4.78$ and 2.48, the microstructure of the fabric will be illustrated in fig.4. The fractal dimensions of this fabric are equal to those of water. The structure of fabric can be produced.

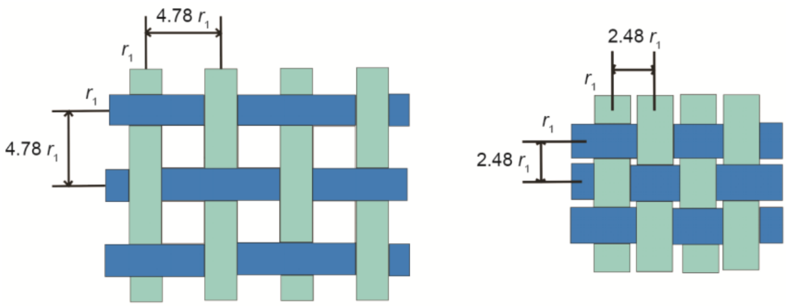

Figure 4. Fabric microstructure with $a = b$, $r_1 = r_2$, $a = 4.78\ r_1$, and $a = 2.48\ r_1$

To get the minimum friction force, the fractal dimensions of swimsuit should be equal to those of water. According to eq. (5), there are countless possibilities. Table 1 lists several possibilities of fabric structure.

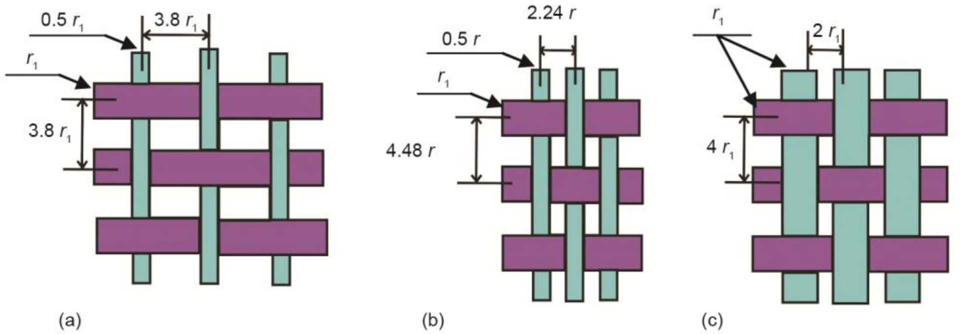

Figure 5. Fabric microstructures with minimum friction

Changing fabric dimensions by electrospinning

There are many holes in warps and wefts interweaving fabric. For a certain fabric, the value of its fractal dimensions is a constant, then how to change the fractal dimensions of the same fabric? A method by spraying nanoparticles or nanofibers on fabric is proposed, fig. 6.

Table 1 Possible fabric structures of minimum friction

Number	m	n	k	Structure
1	0.5	1	3.8	Figure 5a
2	0.5	2	2.24	Figure 5b
3	1	2	4	Figure 5c

Considering a plain weave which is sprayed nanoparticles on it. The fabric has the same warp and weft with radius of 100 μm. Length and width of fabric holes are both 1000 μm. We spray 9 nanoparticles with radius of 500 nm on the hole, then we can calculate the fractal dimensions of the fabric and sprayed fabric, which are 1.723 and 1.778, respectively, fig.6(1). The fabric fractal dimensions increase and are closer to 1.92, the fractional dimensions of water. With this method, we can produce swimsuits whose fractal dimensions are approximately to those of water so that they will have a minimum of friction force in water. This method can be used to change the fractal dimensions of moving surface as well to obtain the minimal friction force.

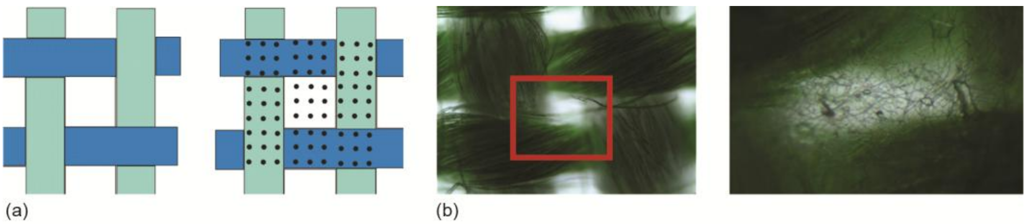

Figure 6. Fabric spraying; (a) nanoparticles spraying, (b) nanofiber spraying

Conclusions

This paper discusses the indispensable importance of fractal harmonic law in the design of swimsuits with minimum friction force. If the fractal dimensions of swimsuits are equal or close to those of water, the friction force between them reaches a minimum. In order to obtain an ideal fractal dimensions in accordance with water, a method to spray nanoparticles or nanofibers on fabric is proposed. The theory can be used to design moving surface so as to get the minimum friction.

Acknowledgments

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