MONTE CARLO SIMULATION OF A TWO-PHASE FLOW IN AN UNSATURATED POROUS MEDIA

by

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Relative permeability is a significant transport property which describes the simultaneous flow of immiscible fluids in porous media. A pore-scale physical model is developed for the two-phase immiscible flow in an unsaturated porous media according to the statistically fractal scaling laws of natural porous media, and a predictive calculation of two-phase relative permeability is presented by Monte Carlo simulation. The tortuosity is introduced to characterize the highly irregular and convoluted property of capillary pathways for fluid flow through a porous medium. The computed relative permeabilities are compared with empirical formulas and experimental measurements to validate the current model. The effect of fractal dimensions and saturation on the relative permeabilities is also discussed.

Key words: *porous media, relative permeability, saturation, fractal, Monte Carlo*

Introduction

The seepage flow of gas and liquid (two-phase fluids) in partially saturated porous media is of great interest to enhanced oil and gas recovery, coal gasification, and exploitation of geothermal energy and potential safe permanent storage facilities for geological disposal of high-level nuclear wastes, *etc.* [1]. The relative permeability defined as the ratio of the effective permeability of a particular fluid to the specific (absolute) permeability of the material is one of important properties governing multiphase flow through porous media, thus, evaluation of the relative permeability in the unsaturated porous media has received considerable attention from hydro-geologists and petroleum engineers [2, 3]. A number of measurement techniques for determination of relative permeability of porous media have been developed. Generally, two kinds of laboratory measurement techniques, steady and unsteady state methods, are employed to determine relative permeability [4]. Because the measurement of relative permeability is difficult, empirical formulas and numerical simulations have been pro-

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posed by many researchers [5-7]. Multiphase fluid flow processes are governed by geometrical pore-space characteristics such as pore size distribution, tortuosity, connectivity, *etc*. However, most current models for multiphase fluid flow do not treat pore-scale geometrical characteristics as a major determining factor other than by the empirical fitting of model parameters. This can be attributed to inherent complexity and heterogeneity of microstructures of real porous media. Since Katz *et al.* [8] presented the experimental evidence indicating that the pore size spaces of a set of sandstone samples are fractals and self-similar over 3-4 orders of magnitude in length extending from 10 Å to 100 μm, fractal geometry has been widely applied to derive physically based expressions for hydraulic properties. For example, Xu *et al.* [9] studied the soil-water characteristics and hydraulic conductivity of unsaturated soils with fractal approach; Yu *et al.* [10] proposed the analytical permeability expressions in unsaturated bi-dispersed porous media based on fractal capillary bundle model; Cihan *et al.* [11] predicted relative permeability from water retention based on fractal geometry. Motivated by the above and aware of possible limitations, we develop a statistical permeability model for the two phase fluid flow (water and air) in the unsaturated porous media in which a fractal probability law is applied to characterize the pore size distribution by Monte Carlo technique. The predictions from the present Monte Carlo simulations are then validated by comparison with empirical formulas and experimental measurements, and the effect of fractal dimensions and saturation on the relative permeability is also discussed.

Fractal model

It has been proven that the size distributions of pores in natural porous media follow the fractal scaling laws [12]. The probability density function for pore size distribution in fractal porous media can be given by:

$$f(\lambda) = D_f \lambda_{\min}^{D_f} \lambda^{-(D_f+1)} \tag{1}$$

where λ is the pore size and D_f – the fractal dimension of pore size distribution. A bundle of tortuous capillary is used to represent the flow paths in the porous sample. Based on the probability density function, the i^{th} capillary tube chosen randomly can be generated with Monte Carlo technique by:

$$\lambda_i = \frac{\lambda_{\min}}{(1-R_i)^{1/D_f}} \tag{2}$$

where R is in the range of 0~1, and approximately corresponds to a set of random numbers of 0~1. The subscripts min and max indicate the *minimum* and *maximum* values, and *w* and *nw* denote the *wetting* and *non-wetting* phases, respectively.

Many investigators concluded from experiments that when two immiscible fluids flow simultaneously through a porous medium, each fluid establishes its own tortuous path, which forms every stable channel [1]. At certain value of capillary pressure, the non-wetting fluid occupies all pores larger than a critical value λ_c [1, 13]. The effective pressure at the critical capillary radius λ_c can be expressed as $p = 2\sigma\cos\theta/\lambda_c$, where σ is the surface tension of wetting phase and θ is the contact angle between liquid and solid phase. Note that for $p < p_{\min} = 2\sigma\cos\theta/\lambda_{\max}$, all the capillaries are fully saturated with wetting fluid and non-wetting fluid flow is zero (single-phase flow in the REV). Contrarily, all capillaries are saturated with non-wetting fluid when $p > p_{\max} = 2\sigma\cos\theta/\lambda_{\min}$ and wetting fluid flow is zero. For the sake of simplicity, the steady-state and laminar flow of an incompressible fluid in capillary tubes are assumed. According to Poiseuille's equation and Darcy's law, the relative permeabilities of wetting and non-wetting phases can be gotten as:

$$k_w = S_w \sum_{i=1}^{n_w} \lambda_{iw}^2 / \tau_{iw} / \sum_{i=1}^{n} \lambda_i^2 / \tau_i \quad \text{and} \quad k_{nw} = (1 - S_w) \sum_{i=1}^{n_{nw}} \lambda_{inw}^2 / \tau_{inw} / \sum_{i=1}^{n} \lambda_i^2 / \tau_i \tag{3}$$

where S_w is the wetting fluid saturation, and the value of tortuosity τ_i ranges from 1.25 to 1.78 in natural porous media [1].

Results and discussion

As the pore sizes are randomly chosen in the Monte Carlo simulations, rough oscillations/fluctuations of flow properties can be observed. However, the converged values of flow properties can be obtained as long as an adequate runs of simulations are performed and the convergence criterion is satisfied. The relative permeabilities by the current fractal model are compared with that of empirical formulas firstly. It can be seen from fig. 1 that the predicted relative permeability for wetting phase is very close to Brooks-Corey model [5] and van Genuchten model [6]. Figure 1 indicates that the ratio $\xi = \lambda_{\max}/\lambda_{\max}$ plays a minor effect on the relative permeability of wetting phase under large saturation. The predicted relative permeabilities for wetting and non-wetting phases are compared with experimental data in Berea sandstone at porosity 0.240 [14, 15] in fig. 2. It can be clearly seen that predictions by the present model are close to the experimental data.

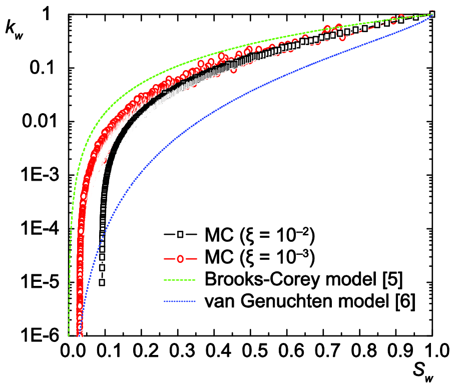

Figure 1. A comparison of the relative permeability for wetting phase between Monte Carlo simulations and empirical formulas

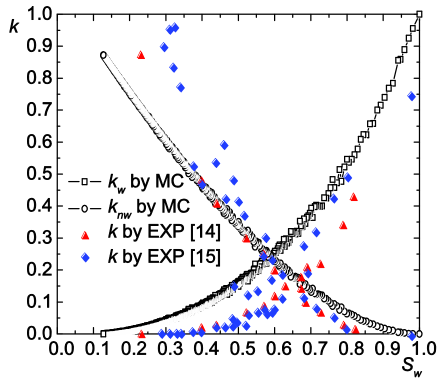

Figure 2. Relative permeabilities by Monte Carlo simulations and experimental data in Berea sandstone at porosity 0.240

Concluding remarks

Based on the fractal scaling laws of pore size distribution in natural porous media, a fractal capillary bundle model is proposed to study the two-phase fluid flow in the unsaturated porous media. The Monte Carlo method has been carried out to calculate the relative permeability and saturation of the unsaturated porous media. The tortuosity is introduced to characterize the highly irregular and convoluted property of capillary pathways for fluid flow through a porous medium. The relative permeabilities for wetting and non-wetting phases as well as effective saturation are numerically calculated. The current numerical model has been validated by comparison with empirical formulas and experimental data, which indicates that the proposed fractal model can capture the complex and multi-scale pore structures and flow path tortuousness and help understanding the transport mechanisms of the two-phase fluid flow in the media. It should be noted that the present model is an idealized conceptual model, more complications such as shape factor of capillary, fluid-structure interaction and trapped non-wetting phase *etc.* can be taken into account to enhance the prediction accuracy.

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