

## TRANSPORT AND STRAINING OF SUSPENSIONS IN POROUS MEDIA: EXPERIMENTAL AND THEORETICAL STUDY

by

**Kaiser AJI, Zhenjiang YOU\*, and Alexander BADALYAN**

Australian School of Petroleum, The University of Adelaide, Adelaide, Australia

Short paper

DOI: 10.2298/TSCI1205344A

*An analytical model for deep bed filtration of suspension in porous media and straining under size exclusion capture mechanism is developed and validated by laboratory tests on suspension flow in engineered media. The fraction of swept particles is introduced in the inlet boundary condition. The model is successfully matched with the results from column experiments, predicting the suspended particle concentrations at the outlet.*

Key words: *colloidal suspension, porous media, straining*

### Introduction

Colloidal transport is a complex and industrially important process, attracting a great deal of interest for years [1-7]. Flow of colloidal suspensions in natural rocks occurs in many industrial applications ranging from particle filtration to fines migration in oil and gas reservoirs [1]. Through reliable modeling of suspension flow and consequent particle retention in rocks, it is possible to reduce the formation damage in oil and gas wells, to design and plan different oil recovery technologies where the colloidal particles are used to increase sweep efficiency during waterflooding.

Suspended particles can be captured in porous media by electrostatic attraction, sorption, bridging, diffusion, gravitational segregation, etc. [3]. In the current work, only the size exclusion (straining) capture, where a particle is retained by a smaller pore only, is considered.

The large scale deep bed filtration is described by governing equations of mass balance and capture kinetics for two unknowns – suspended and retained concentrations. These equations can be obtained by exact averaging of mono-size suspension transport in stochastic porous media [8].

Despite the importance of size exclusion population balance in many industrial applications, the validation of population balance models with particle straining, to the best of our knowledge, is not available in the literature. In the present work, we provide new experimental results from the laboratory study on colloidal transport in engineered porous media, aiming at the validation of the population balance model. The new modification of the size exclusion suspension transport accounting for the pore accessibility and flow fraction in the capture kinetics term, in the inlet and outlet conditions of particle mass balance, is discussed.

---

\* Corresponding author; e-mail: zhenjiang.you@adelaide.edu.au

Good agreement between the experimental results and model prediction validates the mathematical model.

### Stochastic micro model for particle size exclusion

Net repulsion condition between particles and rock matrix indicates that the size exclusion is the only particle capture mechanism. The pore space geometry is represented by the bundle of parallel capillary intercalated by the mixing chambers. Derivations of the equations for suspension transport in porous media can be found in [8, 9]. The particle population balance equation is written as:

$$\frac{\partial}{\partial t} \phi_a(H, r_s) C(r_s, x, t) + \Sigma(r_s, x, t) + U \frac{\partial}{\partial x} C(r_s, x, t) f_a(H, r_s) = 0 \quad (1)$$

where the accessible porosity  $\phi_a$  and accessible flow fraction  $f_a$  are functional of pore concentration distribution  $H$  and particle size  $r_s$ ,  $C$  and  $\Sigma$  – concentration distributions for suspended and retained particles, respectively. The total flux  $U$  is independent of the coordinate  $x$  due to the incompressibility of particulate suspension. The particle capture rate is proportional to the advective particle flux:

$$\frac{\partial \Sigma(r_s, x, t)}{\partial t} = \frac{1}{l} U C(r_s, x, t) f_a(H, r_s) f_{ns}(H, r_s) \quad (2)$$

where  $f_{ns}$  is the flux fraction via smaller pores. The plugging rate of the pores is derived under the assumption that one particle plugs one pore:

$$\frac{\partial H(r_p, x, t)}{\partial t} = - \frac{k_1(r_p)}{k} U H(r_p, x, t) \int_{r_p}^{\infty} C(r_s, x, t) f_a(H, r_s) dr_s \quad (3)$$

Introduction of inaccessible porosity and accessible fractional flow is analogous to two-phase flow in porous media [10, 11], inaccessible large pores corresponding to ganglia of non-wetting phase [12]. Equations (1)-(3) are similar to the system of two-phase multicomponent flow in porous media [1, 10].

The initial conditions  $t = 0$ :  $C(r_s, x, t) = 0$ ,  $H(r_p, x, t) = H_0(r_p)$ , are applied to the clean bed without suspension where  $H_0(r_p)$  is the initial pore size distribution of the medium. The suspension flux with the injected concentration entering larger pores is equal to that being transported through accessible pore space, which results in the following boundary condition at the inlet:

$$x = 0: C^0(r_s, t) f_a(H, r_s) + f_{nl}(H, r_s) U = C(r_s, 0, t) f_a(H, r_s) U \quad (4)$$

Let us formulate boundary conditions at the core inlet. The particles approaching smaller pores can be either swept by the tangent flux component parallel to the core edge and finally enter the larger pores, or stay captured in deep larger throats of the thin pores. The particles approaching smaller pores are more likely to be redirected into larger pores for the clean cut inlet core surface. In the case of rough inlet surface, the particles approaching smaller pores are more likely to remain in the deep entrances into small pores. If  $\alpha$  is the fraction of swept particles,  $C^0(r_s, t)[1 - f_{ns}(H, r_s) + \alpha f_{ns}(H, r_s)]U$  is the carrier water flux carrying par-

ticles into larger pores. The entering particles are transported via porous medium by the accessible flux  $f_a U$ . The continuity of the particle flux at the outlet yields the following boundary condition:  $x = 0: C^0(r_s, t)[1 - f_{ns}(H, r_s) + \alpha f_{ns}(H, r_s)]U = C(r_s, 0, t)f_a(H, r_s)U$ . Further in the text, the case  $\alpha = 0$  is assumed, i. e. the suspension flux with the injected concentration entering larger pores is equal to that being transported through accessible pore space, which results in the boundary condition at the inlet (4). On the other side, the particle suspension at the outlet is diluted in the overall water flux after passing the core outlet, corresponding to the pre-outlet condition:

$$x = L: C(r_s, L, t)f_a(r_s)U = C^L(r_s, t)U \quad (5)$$

Particularly, for the case of low retention filtration, time variation of the pore size distribution during the straining can be ignored. The steady state suspension concentration profile is obtained:

$$C(r_s, x) = \frac{C^0(r_s) f_a(r_s) + f_{nl}(r_s)}{f_a(r_s)} \exp\left[-f_{ns}(r_s) \frac{x}{l}\right] \quad (6)$$

### Laboratory study on suspension transport through engineered porous media

The glass beads are sieved using stainless steel sieves, before being packed in column at wet conditions with a theoretical porosity of 39.6%. Colloidal suspension is then delivered through packed column at constant rate by dual-pump/syringe system. Concentrations of injected and collected suspensions  $C^0(r_{si})$  and  $C^L(r_{si})$  are measured by the particle counter (fig. 1).

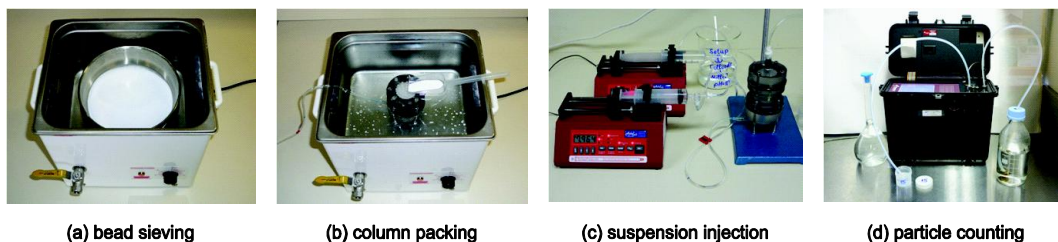


Figure 1. Schematic for laboratory set-up on size exclusion suspension transport in porous media

During the experiment,  $C^0(r_{si})$  and  $C^L(r_{si})$ , are measured for each test (here,  $i = 1, 2, \dots, n$ ). The “ $n$ ” tests result in the system of “ $n$ ” transcendental equations for three unknowns – mean pore radius  $\langle r_p \rangle$ , standard deviation  $\sigma_0$  and dimensionless inter-chamber distance  $l$ . The least squares method is applied to minimize the total quadratic deviation between the experimental data and those predicted by the analytical model (6):

$$R = \min_{\langle r_p \rangle, \sigma_0, l/L} \sum_{i=1}^n \left\{ [f_a(r_{si}) + f_{nl}(r_{si})] \exp\left[-f_{ns}(r_{si}) \frac{L}{l}\right] - \frac{C^L(r_{si})}{C^0(r_{si})} \right\}^2 \quad (7)$$

For a porous medium PM1 with glass beads size range of 20~31.5  $\mu\text{m}$ , the optimized mean pore size  $\langle r_p \rangle = 3.58 \mu\text{m}$ , the standard deviation  $\sigma_0 = 1.82 \mu\text{m}$ , and the inter-

chamber distance  $l = 0.55$  mm. For a porous medium PM2 with glass beads size ranging from 31.5 to 45  $\mu\text{m}$ ,  $\langle r_p \rangle = 5.11$   $\mu\text{m}$ ,  $\sigma_0 = 2.42$   $\mu\text{m}$ , and  $l = 0.15$  mm. The results of data treatment are shown in fig. 2. The six star points from laboratory test data for PM1, fig. 2(a) and five test data points for PM2, fig. 2(b) match well with the curve predicted by the model (6). The dashed curves in figs. 2(a) and 2(b) are from the classical model [6], which does not account for the concentration increase at the inlet (4), dilution effect at the outlet (5) and accessible flux in the capture rate expression (2). The deviation between the two models can be expected for intermediate sized particles.

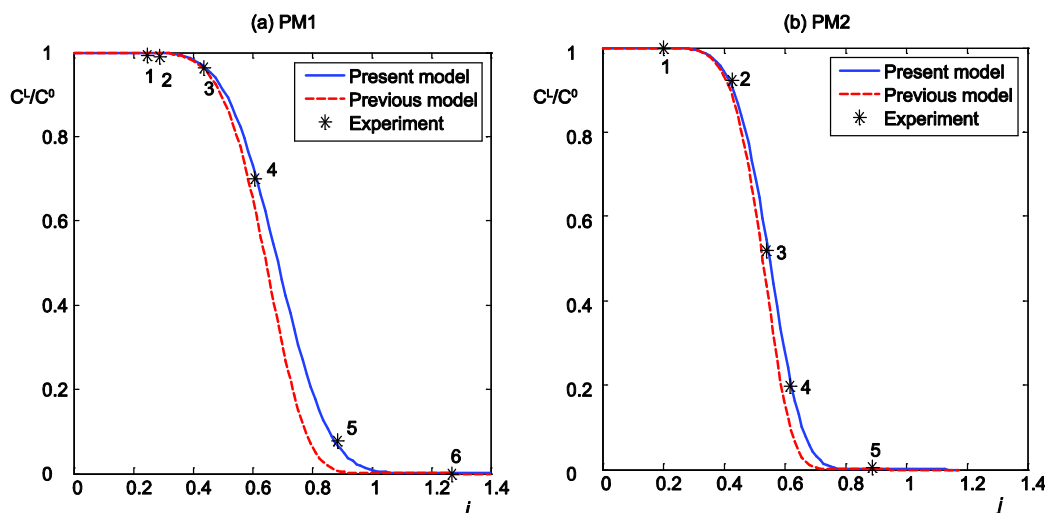


Figure 2. Normalized breakthrough concentrations vs. jamming ratio

## Conclusions

A stochastic micro model is presented to describe suspension transport under size exclusion particle capture mechanism. The model accounts for the increase of particle concentration at the inlet due to the injected flux entrance only into accessible fraction of large pores, for the decrease of particle concentration at the outlet due to dissolution of the particle suspension carried by the accessible water flux in the overall water flux outside the porous media, and for the retention rate proportional to the accessible suspension flux. The fraction of swept particles introduced in the inlet boundary condition accounts for the redirection of particles into larger pores. The data from two laboratory tests have been successfully matched by the proposed model.

## Acknowledgment

Financial supports from the Australian Research Council (ARC) Discovery Project 1094299, ARC Linkage Project 100100613 and Santos Pty Ltd are gratefully acknowledged.

## References

- [1] Bedrikovetsky, P. G., *Mathematical Theory of Oil & Gas Recovery* (With applications to ex-USSR oil & gas condensate fields), Kluwer, London, 1993

- [2] Bradford, S. A., Torkzaban, S., Wiegmann, A., Pore-Scale Simulations to Determine the Applied Hydrodynamic Torque and Colloid Immobilization, *Vadose Zone Journal*, 10 (2011), 1, pp. 252-261
- [3] Torkzaban, S., et al., Impacts of Bridging Complexation on the Transport of Surface-modified Nanoparticles in Saturated Sand, *Journal of Contaminant Hydrology*, 136-137 (2012), Aug., pp. 86-95
- [4] Yu, M., Lin, J. Z., Chan, T. L., Effect of Precursor Loading on Non-Spherical TiO<sub>2</sub> Nanoparticle Synthesis in a Diffusion Flame Reactor, *Chem. Eng. Sci.*, 63 (2008), 9, pp. 2317-2329
- [5] Yu, M., Lin, J. Z., Taylor-Expansion Moment Method for Agglomerate Coagulation due to Brownian Motion in the Entire Size Regime, *Journal of Aerosol Science*, 40 (2009), 6, pp. 549-562
- [6] Yu, M., Lin, J. Z., Chan, T. L., Numerical Simulation of Nanoparticle Synthesis in Diffusion Flame Reactor, *Powder Technology*, 181 (2008), 1, pp. 9-20
- [7] Yu, M., Lin, J. Z., Chan, T. L., A New Moment Method for Solving the Coagulation Equation for Particles in Brownian Motion, *Aerosol Science and Technology*, 42 (2008), 9, pp. 705-713
- [8] Bedrikovetsky, P., Upscaling of Stochastic Micro Model for Suspension Transport in Porous Media, *Transport in Porous Media*, 75 (2008), 3, pp. 335-369
- [9] Chalk, P., et al., Pore Size Distribution from Challenge Coreflood Testing by Colloidal Flow, *Chemical Engineering Research and Design*, 90 (2012), 1, pp. 63-77
- [10] Barenblatt, G. I., Entov, V. M., Rizhik, V. M., Theory of Fluid Flows through Natural Rocks, Kluwer, London, 1987
- [11] Ilina, T., et al., A Pseudo Two-Phase Model for Colloid Facilitated Transport in Porous Media, *Transport in Porous Media*, 71 (2008), 3, pp. 311-329
- [12] Bedrikovetsky, P. G., WAG Displacements of Oil-Condensates Accounting for Hydrocarbon Ganglia, *Transport in Porous Media*, 52 (2003), 2, pp. 229-266