ENHANCEMENT OF NATURAL CONVECTION HEAT TRANSFER IN A U-SHAPED CAVITY FILLED WITH AL_2O_3-WATER NANOFLUID

by

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This paper investigates the natural convection heat transfer enhancement of Al_2O_3-water nanofluid in a U-shaped cavity. In performing the analysis, the governing equations are modeled using the Boussinesq approximation and are solved numerically using the finite-volume numerical method. The study examines the effects of the nanoparticle volume fraction, the Rayleigh number and the geometry parameters on the mean Nusselt number. The results show that for all values of the Rayleigh number, the mean Nusselt number increases as the volume fraction of nanoparticles increases. In addition, it is shown that for a given length of the heated wall, extending the length of the cooled wall can improve the heat transfer performance.

Key words: *nanofluid, heat transfer enhancement, natural convection, cavity*

Introduction

Nanofluids are consisted of nanoparticles with high thermal conductivity (*e. g.*, Al_2O_3 or Cu) suspended in a base fluid with low thermal conductivity (*e. g.*, water or oil) [1]. In recent years, many researchers have proposed various theory models to estimate the thermophysical properties of nanofluids and then to evaluate the heat transfer performance [2-4]. Overall, the results have shown that nanofluids yield an effective improvement in the heat transfer performance compared to traditional working fluids. Natural convection heat transfer in cavities has many important applications in engineering systems, including electronic cooling devices, heat exchangers, micro-electro-mechanical systems (MEMS) devices, solar energy collectors, and so on [5]. In recent years, the problem of the natural convection heat transfer in square/rectangular cavities filled with nanofluids has been widely investigated [6-8]. Overall, the results have shown that the heat transfer can be enhanced by increasing the volume fraction of nanoparticles in the base fluid. Recently, the problem of natural convection heat transfer in non-square/non-rectangular cavities filled with nanofluid has attracted significant attention due to the potential for an enhanced heat transfer effect. For example, Mahmoodi [9] studied the natural convection of Cu-water nanofluid in an L-shaped cavity. Cho *et*

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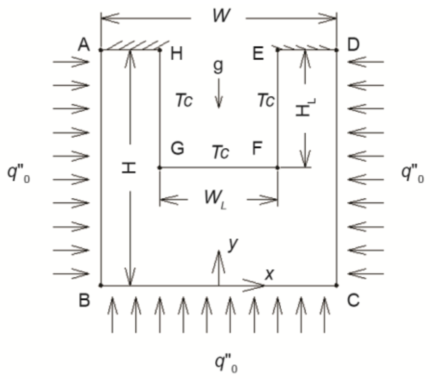

Figure 1. Schematic illustration of the U-shaped cavity

al. [10] examined the heat transfer performance in a complex-wavy-wall cavity filled with Al_2O_3-water nanofluid. The results presented in [9, 10] showed that the heat transfer performance depends on the geometry parameters of cavity and the heat transfer effect can be enhanced as the volume fraction of nanoparticles increases.

In the study, the natural convection heat transfer performance of Al_2O_3-water nanofluid in a U-shaped cavity is investigated numerically. In the simulations, the governing equations described the fluid flow and heat transfer performance in the cavity are modeled using the Boussinesq approximation. The effects on the mean Nusselt number of geometry parameters of the U-shaped cavity, Rayleigh number and nanoparticle volume fraction are discussed.

Mathematical formulation

Figure 1 illustrates the U-shaped cavity considered in the present study. As shown, the cavity has a width *W* and a height *H*. The gravitational force (g) is assumed to act in the negative y-direction. To simplify the governing equations, the following assumptions are made: the flow field is 2-D, steady-state and laminar; nanofluids are Newtonian and incompressible; the thermophysical properties of the nanofluid are all constant other than the density which varies in accordance with the Boussinesq approximation; the base fluid and nanoparticles are in thermal equilibrium, and no relative motion occurs between them; and viscous dissipation effect is ignored. Given these assumptions, the continuity, momentum, and energy equations have the forms:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\rho_{bf}}{\rho_{nf}} \frac{\partial p^*}{\partial x^*} + \frac{\nu_{nf}}{\nu_{bf}} \mathrm{Pr} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right), \tag{2}$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\rho_{bf}}{\rho_{nf}} \frac{\partial p^*}{\partial y^*} + \frac{\nu_{nf}}{\nu_{bf}} \mathrm{Pr} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) +$$

$$+ \mathrm{Ra}\,\mathrm{Pr} \frac{(1-\varphi)(\rho\beta)_{bf} + \varphi(\rho\beta)_p}{\rho_{nf}\beta_{bf}} \theta \tag{3}$$

$$u^* \frac{\partial \theta}{\partial x^*} + v^* \frac{\partial \theta}{\partial y^*} = \frac{\alpha_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 \theta}{\partial x^{*2}} + \frac{\partial^2 \theta}{\partial y^{*2}} \right) \tag{4}$$

where superscript (*) indicates the non-dimensional quantity, subscripts *nf*, *bf*, and *p* indicate the nanofluid, base fluid, and nanoparticle, respectively, *u* and *v* are the velocity components in the x– and y-directions, respectively, *p* is the pressure, ρ – the density; ν – the kinematic viscosity, α – the thermal diffusivity, θ – the non-dimensional temperature, β – the thermal expansion coefficient, φ – the nanoparticle volume fraction, and Ra and Pr are the Rayleigh number and Prandtl number, respectively. Note that the non-dimensional quantities presented in eqs. (1)-(4) are defined as:

$$u^* = \frac{u}{\frac{\alpha_{bf}}{W}},\ v^* = \frac{v}{\frac{\alpha_{bf}}{W}},\ p^* = \frac{p}{\frac{\rho_{bf}\alpha_{bf}^2}{W^2}},\ \theta = \frac{T - T_C}{\frac{q''_0 W}{k_{bf}}},\ \mathrm{Ra} = \frac{\frac{g\beta_{bf}W^3 q''_0 W}{k_{bf}}}{\nu_{bf}\alpha_{bf}},\ \mathrm{Pr} = \frac{\nu_{bf}}{\alpha_{bf}} \tag{5}$$

where T_C indicates the low temperature, k is the thermal conductivity, and q''_0 – the heat flux. Note also that the effective properties of the nanofluids can be estimated as [7, 10]: $\rho_{nf} = (1 - \varphi)\,\rho_{bf} + \varphi\rho_p$, $\mu_{nf} = \mu_{bf}/(1 - \varphi)^{2.5}$, $(\rho C_p)_{nf} = (1 - \varphi)\,(\rho C_p)_{bf} + \varphi(\rho C_p)_p$, $\alpha_{nf} = k_{bf}/(\rho C_p)_{nf}$, and $k_{nf}/k_{bf} = [(k_p + 2k_{bf}) - 2\varphi\,(k_{bf} - k_p)]/[(k_p + 2k_{bf}) - \varphi(k_{bf} - k_p)]$. Note finally that C_p is the specific heat and μ is the dynamic viscosity.

As shown in fig. 1, the U-shaped cavity is heated by constant heat fluxes (q''_0) and is cooled by constant wall temperatures (T_C). The dimensionless boundary conditions are summarized as:

– boundary AB, BC and CD: $u^* = v^* = 0,\ \partial\theta/\partial\vec{n}^* = k_{bf}/k_{nf}$,
– boundary EF, FG and GH: $u^* = v^* = 0$, $\theta = 0$,
– boundary DE and HA: $u^* = v^* = 0,\ \partial\theta/\partial\vec{n}^* = 0$,
– note that $\vec{n}^*$ indicates the normal vector.

In the present study, the convection heat transfer performance is estimated via the Nusselt number (Nu), *i. e.*, Nu = hW/k_{bf}. Note that h is the convection heat transfer coefficient and is defined as $h = q''_0/(T_s - T)$, where subscript s indicates the wall surface. The Nusselt number can be rewritten in a non-dimensional form as:

$$\mathrm{Nu} = \frac{1}{\theta_s} \tag{6}$$

Furthermore, the mean Nusselt number (Nu_m) along the heated wall surfaces can be obtained as:

$$\mathrm{Nu}_m = \frac{\int_A^B \mathrm{Nu}\,\mathrm{d}x^* + \int_B^C \mathrm{Nu}\,\mathrm{d}y^* + \int_C^D \mathrm{Nu}\,\mathrm{d}x^*}{l^*}, \tag{7}$$

where l^* is the non-dimensional total length of the line AB, BC, and CD.

The governing equations and boundary conditions were discretized using the finite-volume numerical method [11]. The velocity and pressure fields were coupled using the SIMPLE C algorithm [11]. Finally, the discretized algebraic equations were solved iteratively using the TDMA scheme. To validate the numerical model and the solution procedure described above, the numerical results obtained for the variation of the mean Nusselt number with the Rayleigh number in a square cavity filled with air were compared with the presented in [12]. Table 1 summarizes the results. It is seen that for all values of the Rayleigh number, the present results are in good agreement with those presented in the literature.

Table 1. Comparison of present results for mean Nusselt number with published results

	Ra = 10^3	Ra = 10^4	Ra = 10^5	Ra = 10^6
Present results	1.118	2.247	4.537	8.931
De Vahl Davis [12]	1.118	2.243	4.519	8.799

Results and discussion

The simulations considered the Al_2O_3-water nanofluid. The thermophysical properties of the water were specified as [7, 10]: specific heat, C_p = 4179 J/kgK, density, ρ = 997.1 kg/m^3, thermal conductivity, k = 0.613 W/mK, and thermal expansion coefficient, $\beta = 21 \cdot 10^{-5}$ K^{-1}. Meanwhile, the thermophysical properties of the Al_2O_3 nanoparticles were given as [7, 10]: specific heat, C_p = 765 J/kgK, density, ρ = 3970 kg/m^3, thermal conductivity, k = 40 W/mK, and thermal expansion coefficient, $\beta = 0.85 \cdot 10^{-5}$ K^{-1}. In addition, the Prandtl number is given as Pr = 6.2. Finally, it is assumed that the width of the cavity is equal to the height of the cavity, *i. e.*, $W = H$.

Figures 2(a) and 2(b) show the flow streamlines in the U-shaped cavity for Rayleigh number of Ra = 10^3 and Ra = 10^6, respectively, and a constant nanoparticle volume fraction of φ = 4%. Since the U-shaped cavity is heated by the outside walls and is cooled by the inside walls, the fluid rises upwind from the outside walls and the consequent falling when the fluid constants with the inside walls. Therefore, for the two values of the Rayleigh number, symmetric counter-rotating recirculations are formed in the cavity. In addition, as the Rayleigh number increases, the buoyancy strengthens. As a result, the intensity of the recirculations increases and thus the core of the recirculations moves upwind, see fig. 2(b).

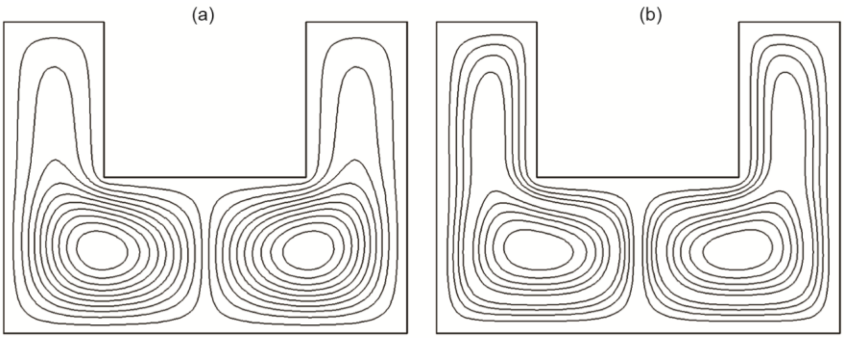

Figure 2. Flow streamlines in U-shaped cavity given nanoparticle volume fraction of φ = 4% and Rayleigh number of: (a) Ra = 10^3 and (b) Ra = 10^6; note that H_L = 0.5 W, W_L = 0.5 W and φ = 4%

Figures 3(a) and 3(b) show the isotherm distributions in the U-shaped cavity for Rayleigh number of Ra = 10^3 and Ra = 10^6, respectively, and a constant nanoparticle volume fraction of φ = 4%. At a low value of Rayleigh number, since a weaker buoyancy effect is induced, the conduction dominates the heat transfer effect. As a result, the isotherms have no significant perturbation and follow the geometry of the cavity, see fig. 3(a). However, under a high Rayleigh number condition, since a strong buoyancy is generated, the convection dominates the heat transfer behavior. Consequently, a significant twisting of the isotherms takes place, see fig. 3(b).

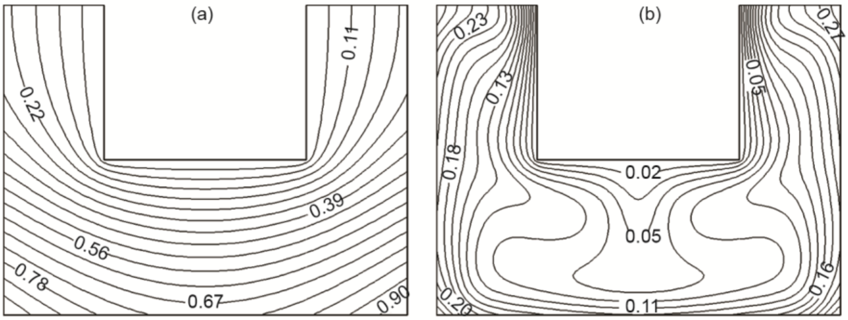

Figure 3. Isotherms in U-shaped cavity given nanoparticle volume fraction of φ = 4% and Rayleigh number of: (a) Ra = 10^3 and (b) Ra = 10^6; note that H_L = 0.5 W, W_L = 0.5 W and φ = 4%

Figure 4 shows the variation of the mean Nusselt number with the Rayleigh number as a function of the volume fraction of nanoparticle. As described above, under low Rayleigh number conditions, a weak buoyancy effect is induced and the heat transfer is dominated by the effect of the conduction. As a result, the mean Nusselt number is low. However, as the Rayleigh number increases, the effect of the convection dominates the heat transfer. Thus, the fluid in the cavity is perturbed strongly. Consequently, a high value of mean Nusselt number is obtained. In addition, the addition of nanoparticles to the pure water increases the thermal conductivity. As a result, the heat transfer effect of nanofluid is higher than that of pure water. Furthermore, it is shown that the effect of nanoparticle addition in the base fluid on the heat transfer performance is better at low Rayleigh numbers than at high Rayleigh numbers. For example, given a nanoparticle volume fraction of φ = 8%, the mean Nusselt number is around 25% higher than that for pure water for a Rayleigh number of Ra = 10^3, and 3% higher than that for pure water for a Rayleigh number of Ra = 10^6.

Figure 5 shows the variation of the mean Nusselt number with the Rayleigh number as a function of the geometry parameter W_L. As the W_L increases, the length of the cooled walls is also increases. Since the length of the heated walls is constant in the current study, the heat can be removed fast as the W_L increases.

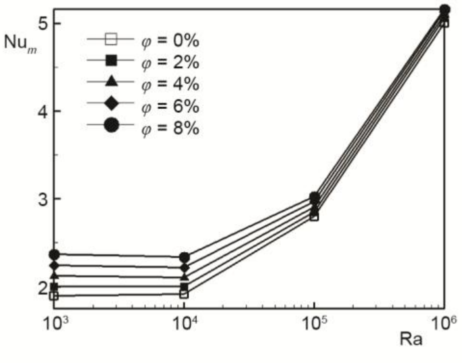

**Figure 4. Variation of mean Nusselt number with Rayleigh number as function of nanoparticle volume fraction;
note that H_L = 0.5 W and W_L = 0.5 W**

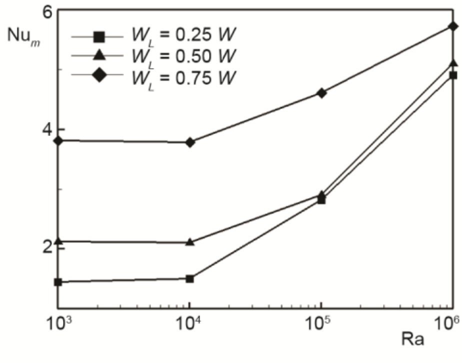

**Figure 5. Variation of mean Nusselt number with Rayleigh number as function of geometry parameter W_L;
note that H_L = 0.5 W and φ = 4%**

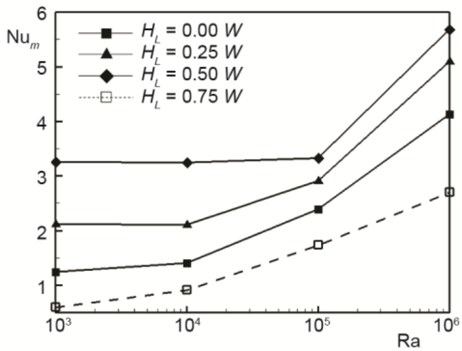

Figure 6. Variation of mean Nusselt number with Rayleigh number as function of geometry parameter H_L; note that W_L = 0.5 W and φ = 4%

Consequently, it is shown that a larger W_L has a higher mean Nusselt number.

Figure 6 shows the variation of the mean Nusselt number with the Rayleigh number as a function of the geometry parameter H_L. It is shown that as the increase of the H_L, the transition region from conduction-dominated heat transfer to convection-dominated heat transfer is postponed. For a given value of H_L, the mean Nusselt number almost maintains a constant value when the Rayleigh number is smaller than a threshold value. For example, for H_L = 0.75 W, the threshold value is Ra = 10^5, while for H_L = 0.50 W, the threshold value is Ra = 10^4. This means that when the Rayleigh number is smaller than the threshold value, the conduction dominates the heat transfer performance. However, when Rayleigh number exceeds the threshold value, the mean Nusselt number increases fast. This means that the heat transfer performance is mainly dominated by the effect of the convection. Comparing the U-shaped cavity with a square cavity (*i. e.*, the case of H_L = 0.00 W), it is shown clearly that increasing the length of H_L, the mean Nusselt number can effectively enhance, especially for high Rayleigh numbers. In other words, extending the length of the cooled wall can improve the heat transfer performance.

Conclusions

This study has performed a numerical investigation into the natural convection heat transfer improvement in a U-shaped cavity filled with Al_2O_3-water nanofluid. In performing the simulations, the governing equations have been modeled using the Boussinesq approximation and then solved using the finite volume method. The effects of the nanoparticle volume fraction, the Rayleigh number and the geometry parameters on the heat transfer performance have been examined. The results have shown that for the Rayleigh numbers considered in this study, the mean Nusselt number increases with an increasing volume fraction of nanoparticles. The results have also shown that the heat transfer performance can be enhanced by extending the length of the cooled wall.

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