NUMERICAL INVESTIGATION INTO NATURAL CONVECTION HEAT TRANSFER ENHANCEMENT OF COPPER-WATER NANOFLUID IN A WAVY WALL ENCLOSURE

by

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Numerical investigations are performed into the natural convection heat transfer characteristics within a wavy-wall enclosure filled with Cu-water nanofluid. In the paper, the bottom wall of the enclosure has a wavy geometry and is maintained at a constant high temperature, while the top wall is straight and is maintained at a constant low temperature. The left and right walls of the enclosure are both straight and insulated. In performing the simulation, the Boussinesq approximation is used to model the governing equations. The study examines the effect of the nanoparticle volume fraction, the Rayleigh number, the wave amplitude, and the wavelength on the heat transfer characteristics. It is shown that the heat transfer performance can be enhanced as the volume fraction of nanoparticles increases. It is also shown that for a given Rayleigh number, the heat transfer effect can be optimized via an appropriate changing of the geometry conditions.

Key words: *nanofluid, natural convection, enclosure, heat transfer*

Introduction

Natural convection heat transfer in regular enclosures (*e. g*., square or rectangular enclosures) is widely investigated due to its importance in many engineering applications, including electronic cooling devices, heat exchangers, MEMS devices, electric machinery, solar energy collectors, and so forth [1]. Many investigations have discussed the flow behavior and corresponding heat transfer characteristics in regular enclosures [2-4]. Natural convection heat transfer in irregular enclosures (*e. g*., wavy-wall enclosures) is often encountered in many engineering applications due to the potential for an improved heat transfer performance. However, because of geometrical complexity, the study of the natural convection heat transfer in irregular enclosures was limited.

Mahmud *et al*. [5] have studied the natural convection heat transfer characteristics within an enclosure with wavy structures. Rostami [6] investigated the problem of an unsteady natural convection heat transfer in an enclosure with wavy walls. Oztop *et al*. [7] has stu-

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died the effects of volumetric heat sources on the natural convection heat transfer in a wavy-walled enclosure. Overall, there results presented in [5-7] showed that the heat transfer effect depended on the wavy geometry parameters and flow conditions.

The investigations presented above considered that the working fluid was air or water. However, such fluids have a low thermal conductivity, and thus the heat transfer effect is inherently limited. In order to improve the heat transfer performance, Choi [8] added metallic nanoparticles with a high thermal conductivity (*e. g*., Cu or Al_2O_3 nanoparticles) to the working fluid (*e. g*., water). Note that the fluid is generally called nanofluid. Recently, the application of nanofluids for the heat transfer enhancement has been investigated in a wide range as summarized in the review papers [9, 10]. Overall, the results showed that the nanofluids can effectively improve the heat transfer performance.

In the study, a numerical simulation is performed to investigate the natural convection heat transfer performance within a wavy-wall enclosure filled with a Cu-water nanofluid. In performing the simulations, the Boussinesq approximation is used to characterize the behavior of the natural convection heat transfer. The simulations focus specifically on the effects of the nanoparticle volume fraction, Rayleigh number, and wave amplitude and wavelength of the wavy-wall enclosure on the flow streamlines, isotherm distribution, and Nusselt numbers within the enclosure.

Mathematical formulation and numerical solution procedure

The flow behavior and heat transfer characteristics in the enclosure shown in fig. 1 can be governed by the continuity, Navier-Stokes, and energy equations. To simplify the governing equations, the following assumptions are made: the nanofluid is Newtonian, incompressible and laminar, the thermophysical properties of the nanofluid are all constant other than the density which varies in accordance with the Boussinesq approximation, the base fluid and nanoparticles are in thermal equilibrium and have the same flow velocity, the flow field is 2-D and in a steady-state, and the viscous dissipation effect is ignored.

Let the following non-dimensional quantities be introduced:

$$x^* = \frac{x}{H}, \quad y^* = \frac{y}{H}, \quad u^* = \frac{u}{\frac{\alpha_{bf}}{H}}, \quad v^* = \frac{v}{\frac{\alpha_{bf}}{H}}, \quad p^* = \frac{pH^2}{\rho_{bf}\alpha_{bf}^2}, \quad \theta = \frac{T - T_L}{T_H - T_L} \tag{1}$$

The superscript (*) indicates the non-dimensional quantity, the subscript *bf* indicates the base fluid, H is the height of the enclosure; u and v are the velocity components along x- and y-axes, respectively, α is the thermal diffusivity, p – the pressure, ρ – the density, θ – the dimensionless temperature, and T_H and T_L are the high temperature and low temperature, respectively. The continuity equation, *x*- and *y*-momentum equations, and energy equation can then be written in the non-dimensionalized forms:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{2}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\rho_{bf}}{\rho_{nf}} \frac{\partial p^*}{\partial x^*} + \frac{\nu_{nf}}{\nu_{bf}} \mathrm{Pr} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) \tag{3}$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\rho_{bf}}{\rho_{nf}} \frac{\partial p^*}{\partial y^*} + \frac{\nu_{nf}}{\nu_{bf}} \mathrm{Pr} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) +$$

$$+\mathrm{Ra}\,\mathrm{Pr} \frac{(1-\varphi)(\rho\beta)_{bf} + \varphi(\rho\beta)_p}{\rho_{nf}\beta_{bf}} \theta \tag{4}$$

$$u^* \frac{\partial \theta}{\partial x^*} + v^* \frac{\partial \theta}{\partial y^*} = \frac{\alpha_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 \theta}{\partial x^{*2}} + \frac{\partial^2 \theta}{\partial y^{*2}} \right) \tag{5}$$

where subscript *nf* indicates the nanofluid, subscript *p* indicates the nanoparticle, $\mathrm{Ra} = g\beta_{bf}H^3(T_H - T_L)/\nu_{bf}\,\alpha_{bf}$ is the Rayleigh number, $\mathrm{Pr} = \nu_{bf}/\alpha_{bf}$ – the Prandtl number, g – the gravitational acceleration, β – the thermal expansion coefficient, and ν – the kinematic viscosity. In addition, the effective properties of the nanofluids can be estimated as follows [11]:

$$\rho_{nf} = (1-\varphi)\rho_{bf} + \varphi\rho_p, \quad \mu_{nf} = \frac{\mu_{bf}}{(1-\varphi)^{2.5}}, \quad (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_{bf} + \varphi(\rho C_p)_p,$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \text{and} \quad \frac{k_{nf}}{k_{bf}} = \frac{(k_p + 2k_{bf}) - 2\varphi(k_{bf} - k_p)}{(k_p + 2k_{bf}) + \varphi(k_{bf} - k_p)}$$

Note that μ is the dynamic viscosity, k – the thermal conductivity, and φ indicates the volume fraction of nanoparticles in the base fluid.

In the present study, the convection heat transfer performance is estimated via the Nusselt number (Nu), *i. e.*, $\mathrm{Nu} = -(k_{nf}/k_{bf})(\partial\theta/\partial n^*)_w$. The mean Nusselt number (Nu_m) along the bottom wavy-wall can be obtained as:

$$\mathrm{Nu}_m = \frac{1}{l_w^*} \int_{-\lambda/2}^{\lambda/2} \mathrm{Nu}\, \mathrm{d}\xi \tag{6}$$

where l^*_w is the non-dimensional length of the wavy-wall.

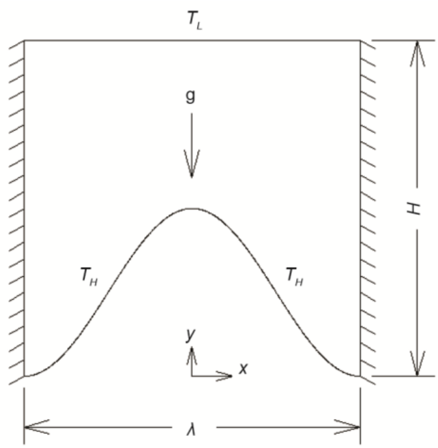

Figure 1. Schematic illustration of the wavy-wall enclosure. Note that the λ and *H* indicate the width and height of the enclosure, respectively

In simulating the flow behavior and heat transport characteristics within the wavy-wall enclosure shown in fig. 1, it is assumed that the bottom wavy-wall has a constant high temperature (θ = 1), while the top straight wall has a constant low temperature (θ = 0). In addition, the left and right straight walls are assumed to be insulated.

Finally, a no-slip impermeable velocity boundary condition is applied at all of the wall surfaces.

In performing the simulations, the non-dimensional profile of the wavy-wall was modeled as follows:

$$y^*(x^*) = \alpha\left[1-\cos\left(\frac{2\pi x^*}{\lambda^*}\right)\right], \tag{7}$$

where α is the wave amplitude and $\lambda^* = \lambda/H$ is the wavelength. Note that as shown in fig. 1, the wavelength is equal to the width of the enclosure. Finally, the body-fitted grid system used to model the wavy-wall configuration was generated by solving a set of Poisson equations [12]. Since the geometry configuration generated by eq. (7) is a non-orthogonal system, the governing equations given in eqs. (2)-(5) must be transformed from a Cartesian co-ordinate system to a generalized curvilinear co-ordinate system. The transformed governing equations have the following form [13, 14]:

$$\frac{\partial}{\partial \xi}(\rho_t U f) + \frac{\partial}{\partial \eta}(\rho_t V f) = \frac{\partial}{\partial \xi}\left[\frac{\Gamma_t}{J}\left(\alpha_t \frac{\partial f}{\partial \xi} - \beta_t \frac{\partial f}{\partial \eta}\right)\right] + \frac{\partial}{\partial \eta}\left[\frac{\Gamma_t}{J}\left(-\beta_t \frac{\partial f}{\partial \xi} + \gamma_t \frac{\partial f}{\partial \eta}\right)\right] + JS_t \tag{8}$$

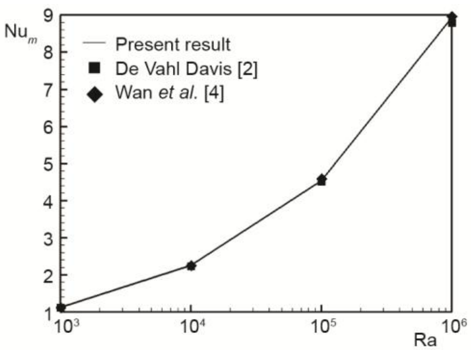

Figure 2. Present results and published results for variation of mean Nusselt number with Rayleigh number. Note that in the enclosure considered in fig. 2, the left and right wall is straight and has a constant high and low temperature, respectively, and the top and bottom wall is straight and insulated. Note also that $\lambda = H$ and Pr = 0.707

where, f is a generalized variable; ξ and η are the two axes of the transformed co-ordinate system; U and V – the velocity components in the transformed co-ordinate system, S_t is a source term; α_t, β_t and γ_t are parameters of the transformed co-ordinates, respectively, and J is the Jacobian factor.

The governing equations and boundary conditions were discretized using the finite-volume numerical method [15]. The velocity and pressure fields were coupled were coupled using the SIMPLE C algorithm [15]. Finally, the discretized algebraic equations were solved iteratively using the TDMA scheme.

Finally, to validate the numerical code, the numerical results for the variation of the mean Nusselt number with the Rayleigh number in a square cavity were compared with those presented in two other studies [2, 4]. As shown in fig. 2, a good agreement was obtained between all three sets of results. In other words, the numerical code is confirmed.

Results and discussion

In the study, the Cu-water nanofluid is considered to estimate the heat transfer performance. The thermophysical properties of the water (*i. e.*, base fluid) were specified as: specific heat, C_p = 4179 J/kgK, density, ρ = 997.1 kg/m^3, thermal conductivity, k = 0.613 W/mK, and thermal expansion coefficient, $\beta = 21\cdot10^{-5}$ K^{-1}. Meanwhile, the thermophysical properties of the Cu nanoparticles were given as follows: specific heat, C_p = 385 J/kgK, density, ρ = 8933 kg/m^3, thermal conductivity, k = 400 W/mK, and thermal expansion coefficient, $\beta = 1.67\cdot10^{-5}$ K^{-1}.

Figures 3 and 4 show the flow streamlines and isotherms within the wavy-wall enclosure giving Rayleigh numbers of Ra = 10^3 and Ra = $5 \cdot 10^5$, respectively. Note that in both figures, the nanoparticle volume fraction is φ = 5%. Because the temperature of the bottom wavy wall is higher than that of the top straight wall, the fluid rises upward from the bottom hot wavy surface and the consequent falling when they contact with the top cold wall. Therefore, for both values of the Rayleigh number, two symmetric counter-rotating vortices (*i. e.*, anti-clockwise rotation at the left half court and clockwise rotation at the right half court) are formed within the enclosure, figs. 3(a) and 4(a).

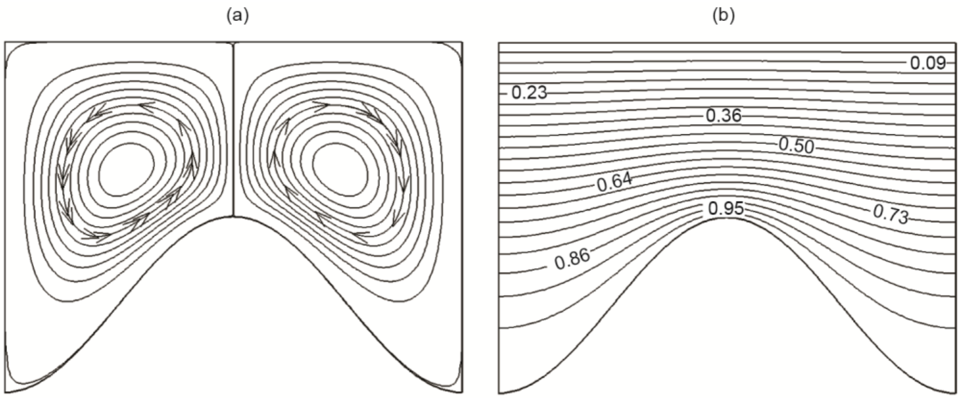

Figure 3. (a) Flow streamlines and (b) isotherms in the wavy-wall enclosure given nanoparticle volume fraction of φ = 5% and Rayleigh number Ra = 10^3; note that $\lambda = H$

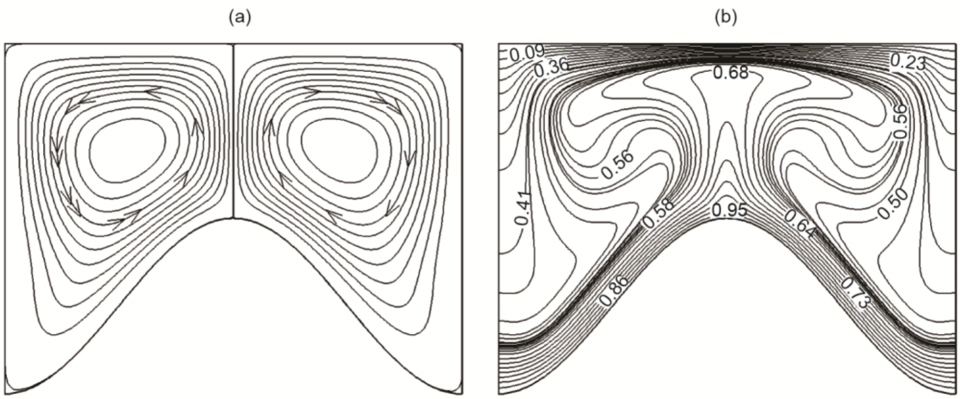

Figure 4. (a) Flow streamlines and (b) isotherms in the wavy-wall enclosure given nanoparticle volume fraction of φ = 5% and Rayleigh number of Ra = $5 \cdot 10^5$; note that $\lambda = H$

It is observed that the vortices expand and become intense as the Rayleigh number increases since the buoyancy effect increases with increasing Rayleigh number. It can be also seen that given a lower Rayleigh number (Ra = 10^3), the isotherms almost follow the geometry form, see fig. 3(b). The result indicates that the conduction dominants the heat transfer performance within the enclosure since a weak buoyancy effect occurs. However, under a higher Rayleigh number (Ra = $5 \cdot 10^5$), a significant swirling of the isotherms occurs within the enclosure since buoyancy effect becomes stronger as increasing the Rayleigh number, see fig.

4(b). In other words, under high Rayleigh number conditions, the heat transfer within the enclosure is dominated by convection rather conduction effects.

Figure 5 shows the variation of the mean Nusselt number with the Rayleigh number as a function of the nanoparticle volume fraction. It is observed that under low Rayleigh number conditions, the mean Nusselt number is almost independent of the Rayleigh number. As described above, since a weak buoyancy effect is induced, the heat transfer within the enclosure is mainly dominated by the effect of conduction. Therefore, under the same geometry conditions, the heat transfer performance in the low Rayleigh number regions is similar. As Rayleigh number increases, a great buoyancy effect is induced. Therefore, the fluid within the enclosure is strongly perturbed by the effect of convection. As a result, the mean Nusselt number fast increases and thus causes a high heat transfer effect within the enclosure. The results presented in fig. 5 also show that for all values of the Rayleigh number considered in the study, the mean Nusselt number increases with an increasing volume fraction of nanoparticles. The result implies that as the volume fraction of nanoparticles increases, the thermal conductivity of the fluid also increases, resulting in the enhancement of the mean Nusselt number. From inspection, when the nanoparticle volume fraction of φ = 10% is given, the mean Nusselt number can approximately increase 33.1% than that for pure water for a Rayleigh number of Ra = 10^3, and 12.6% than that for pure water for a Rayleigh number of Ra = $5 \cdot 10^5$. Overall, nanoparticles addition into the base fluid result in a significant improvement in the heat transfer performance.

Figure 6 shows the variation of the mean Nusselt number with the Rayleigh number as a function of the wavelength. It is shown that at the region of the low Rayleigh number, the mean Nusselt number increase with the wavelength.

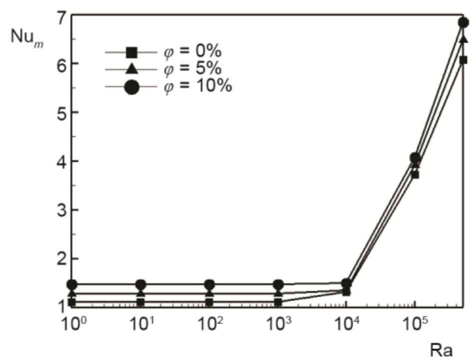

Figure 5. Variation of mean Nusselt number with Rayleigh number as function of nanoparticle volume fraction; note that α = 0.25 and λ = *H*

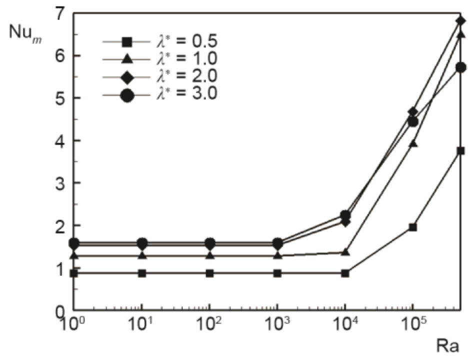

Figure 6. Variation of mean Nusselt number with Rayleigh number as function of wavy surface wavelength; note that α = 0.25 and φ = 5%

The results can be expected that because the conduction area is increased as increasing the wavelength, the mean Nusselt number increases with the wavelength. However, at the region of high Rayleigh number, the mean Nusselt number increases as the wavelength increases form λ = 0.5 to λ = 2, but then decreases as the wavelength further increases to λ = 3. The results indicate that if the wavelength is too length, the variation in the geometry form is more gentle. As a result, the increase of the Nusselt number induced by the convection effect is limited. Therefore, the mean Nusselt number is lower than the case of a smaller wavelength.

Figure 7 shows the variation of the Nusselt number with the Rayleigh number as a function of the wave amplitude. It is shown that at the region of low Rayleigh numbers, the mean Nusselt number increases with an increasing the wave amplitude because conduction dominates the heat transfer effect. However, at the region of the high Rayleigh numbers, the mean Nusselt number is higher in the case of a lower wave amplitude. It is necessary to be noted that when Ra = $5\cdot10^5$, the mean Nusselt number of α = 0.15 is smaller than that of α = 0.25 and α = 0.35. This is because in a smaller wave amplitude, secondary vortexes are generated in the corner of the bottom wavy wall and straight vertical wall. The secondary vortex affects the strength of the main vortexes, and thus reduces the heat transfer performance. Note that at large wave amplitudes, a stronger convection effect is induced due to reducing of the enclosure space, and therefore suppresses the generation of the secondary vertexes.

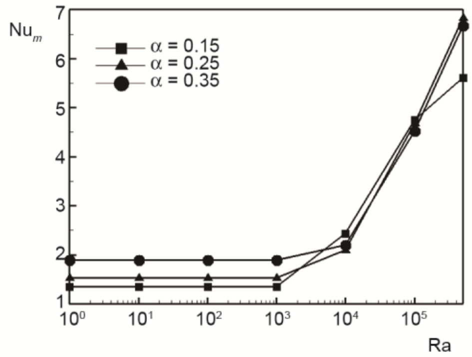

Figure 7. Variation of Nusselt number with Rayleigh number as function of wavy surface amplitude; note that φ = 5% and λ = 2H

Conclusions

In this paper the natural convection heat transfer characteristics of Cu-water nanofluid in a wavy-wall enclosure has been investigated. In the considered enclosure, the wavy wall is located on the bottom of the enclosure and is maintained at a constant high temperature, while the top of the enclosure is straight and assumed to have a constant low temperature. In addition, the left and right walls are straight and insulated. In performing the study, the governing equations of the natural convection were modeled using the Boussinesq approximation. The effects of wavy surface amplitude and wavelength, nanoparticle volume fraction, and Rayleigh number on the mean Nusselt number have examined using numerical simulations. The results have shown that the mean Nusselt number increases as increasing the nanoparticle volume fraction. In addition, the results have shown that the mean Nusselt number can be optimized by tuning appropriately the geometry conditions in according to the Rayleigh number.

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