SERIES SOLUTION OF ENTROPY GENERATION TOWARD AN ISOTHERMAL FLAT PLATE

by

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Original scientific paper
DOI: 10.2298/TSCI1205289M

The steady 2-D boundary layer flow over a flat plate is studied analytically by homotopy perturbation method to analyze the entropy generation inside the boundary layer with constant wall temperature. By the transformations of governing equations including continuity, momentum, and energy by similarity variables, a dimensionless equation for entropy generation inside the boundary layer is obtained. The effects of important parameters such as Reynolds and Eckert numbers are investigated and the physical interpretations of the results are explained in details.

Key words: *boundary layer, entropy generation, homotopy perturbation method*

Introduction

In the age of technology, we are observing revolutionary changes in thermodynamics. The concept of entropy is one of the most visible forms of this change. It plays an essential role in understanding of diverse phenomena in many fields [1] especially in energy conversion processes which usually lead to an irreversible increase in entropy; thus reducing the generated entropy will result in more efficient designs of energy systems. Bejan [2] presented a method named Entropy Generation Minimization (EGM) to measure and optimize the disorder or disorganization generated during a process. Later this method was employed in many engineering and physics problems [3-5]. On the other hand, the classical concept of boundary layer flow which first was studied by Blasius [6] has been perused by other researchers. Due to the importance of viscous forces inside the boundary layer which may affect the engineering process of producing, there is a certain need to investigate this phenomenon. A large amount of literatures about this problem has been cited in the books by Schlichting and Gersten [7], Leal [8]. Like

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many other phenomena in engineering, entropy generation is a non-linear equation which usually does not have an exact solution and there is a certain need to find a technique to solve it.

To overcome this problem, in this paper we have employed homotopy perturbation method (HPM) [9] which was presented by He in 1998. The application of HPM has been raised exponentially in engineering which can be gauged from Ganji's studies [10-25]. In order to explain the physical interpretations, Bejan number is computed for different cases and the effects of Ec, Re, and Pr on entropy generation have been shown graphically.

Governing equations

In this paper we consider an incompressible viscous flow over a flat plate, as shown in fig. 1. The wall temperature T_w, is uniform and constant and is greater than the free stream temperature, T_∞. It is assumed that the free stream velocity, U_∞, is also uniform and constant. In addition it is assumed that the flow in the laminar boundary layer is 2-D, and that the temperature gradients resulting from viscous dissipation are small, with the scale analysis, the continuity, momentum, and energy equations can be expressed as:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial^2 x} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial^2 x} \tag{3}$$

and the boundary conditions are:

$$\text{at } y=0: \ u=v=0 \ \text{ and } \ T=T_w, \quad \lim_{y\to\infty} u=U_\infty \ \text{ and } \ \lim_{y\to\infty} T=T_\infty \tag{4}$$

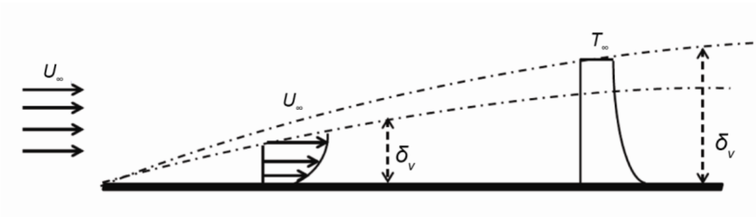

Figure 1. Velocity and thermal boundary layers over a flat plate

A stream function ψ (x, y), is introduced as:

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}$$

Obviously the continuity equation will be satisfied. We look for a similarity solution of the eqs. (1)-(3) with the boundary conditions, eq. (4), of the form:

$$f(\eta)=\frac{\psi}{\sqrt{\nu x u_\infty}},\quad \eta=\sqrt{\frac{u_\infty}{\nu x}}y,\quad \theta=\frac{T-T_w}{T_\infty-T_w} \tag{5}$$

The dimensionless equations can be obtained as follows:

$$\frac{\mathrm{d}^3 f(\eta)}{\mathrm{d}\eta^3}+\frac{1}{2}f(\eta)\frac{\mathrm{d}^2 f(\eta)}{\mathrm{d}\eta^2}=0 \tag{7}$$

$$\frac{\mathrm{d}^2\theta(\eta)}{\mathrm{d}\eta^2}+\frac{\Pr}{2}f(\eta)\frac{\mathrm{d}\theta(\eta)}{\mathrm{d}\eta}=0$$

With boundary conditions of:

$$f(0)=f^{'}(0)=\theta(0)=0\,,\quad f^{'}(\infty),\ \theta(\infty)=1 \tag{8}$$

Entropy generation

Following Bejan [2], entropy generation equation is:

$$S_{gen}^{'''}=\frac{K}{T^2}(\nabla T)^2+\frac{\mu}{T}\phi \tag{9}$$

In the above equation, the first term is because of heat transfer and the second one is due to fluid friction. We call them S_h and S_f, respectively. Proposed Bejan number is:

$$\mathrm{Be}=\frac{S_h}{S_h+S_f} \tag{10}$$

The dimensionless entropy generation number may be defined by the following relationship:

$$S=\frac{\nu^2\,\Delta T}{u_\infty^4\,\mu}S_g \tag{11}$$

Substituting eq. (6) into eq. (9) we can obtain the non-dimensional entropy generation equation over a stationary flat plate as follows:

$$S_{gen}^{'''}=\frac{f^{''2}}{\mathrm{Re}(\theta+\theta_\infty-1)}+\frac{\theta^{'2}}{\Pr\mathrm{Ec}\,(\theta+\theta_\infty-1)^2}\left(\frac{\eta^2}{4\mathrm{Re}^2}+\frac{1}{\mathrm{Re}}\right) \tag{13}$$

Homotopy–perturbation method

Homotopy–perturbation structure is shown as:

$$H(\nu,P)=L(\nu)-L(u_0)+PL(u_0)+P\ \ N(\nu)-f(r) \tag{14}$$

where $\nu(r, P)$: Ω [0, 1] $\rightarrow \Re$ and P = [0, 1] is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. The process of changes in p from zero to unity is that of $\nu(r, P)$: changing from u_0 to $u(r)$. We consider ν, as following:

$$v=\sum_{i=0}^{\infty}P^i v_i = v_0 + Pv_1 + P^2 v_2 + P^3 v_3 + \ldots \quad (16)$$

and the best approximation for solution is $u=\lim_{p\to 1} v = v_0 + v_1 + v_2 + \ldots$.

The convergence is discussed in [16-21]. To solve eqs. (7) and (13) with HPM, we assumed that $u_0 = 0$, substituting eq. (17) in eq. (7) to the order of (P^{20}) and re-arranging the obtained relation based on like powers of *p* we have:

$$f(\eta)=\sum_{i=0}^{20} f_i(\eta)=0.1722\eta^2+0.0003\eta^5+5.456310^{-7}\eta^8+\ldots \quad (18)$$

$$\theta(\eta)=\sum_{i-0}^{20}\theta_i(\eta)=0.55419\eta+0.01080\eta^4+0.0001\eta^7+\ldots \quad (19)$$

Substituting eqs. (18) and (19) in eq. (13) the entropy generation rate will be obtained. According to the order of *P* which was 20, the resulting equations for *f*(*η*) and *θ*(*η*), and consequently *S''* was too long to be mentioned so we skipped the complete relations and graphical results are presented.

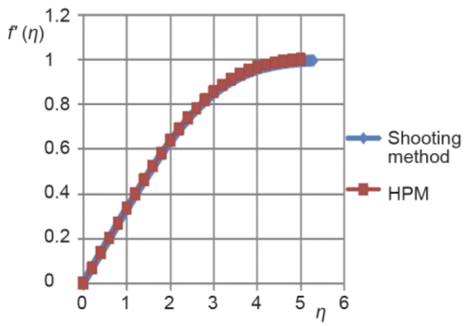

Figure 2. Dimensionless velocity distribution using schooting method and HPM (for color image see journal web site)

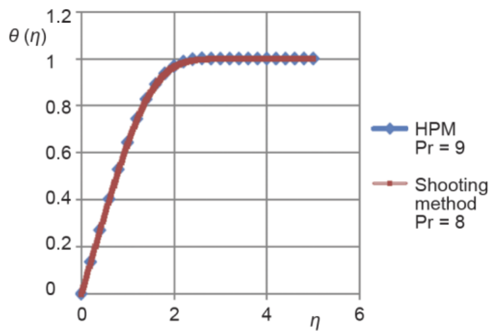

Figure 3. Comparison between dimensionless temperature distribution for different values of Pr, by two methods (for color image see journal web site)

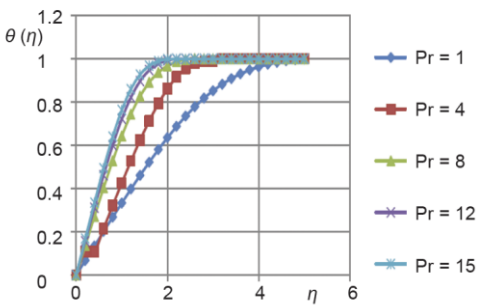

Figure 4. Dimensionless temperature distribution for different values of Pr number (for color image see journal web site)

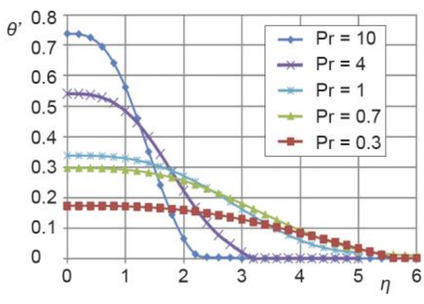

Figure 5. Dimensionless temperature gradient for different values of Pr number (for color image see journal web site)

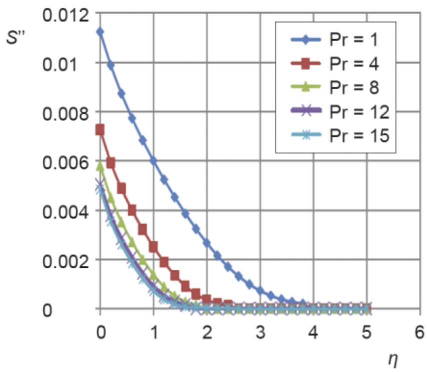

Figure 6. Dimensionless entropy generation for different values of Pr number (for color image see journal web site)

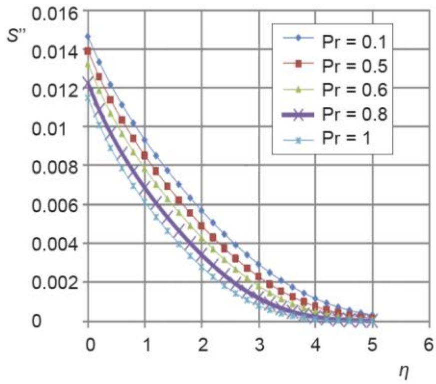

Figure 7. Dimensionless entropy generation for different values of Pr number (for color image see journal web site)

Results and discussion

The non-linear ordinary differential eq. (7) subject to the boundary conditions (8) has been solved analytically using HPM. The obtained results are in excellent agreement with shooting method which is shown in figs. 2 and 3. It is obvious that the velocity and temperature gradients outside the boundary layer are zero as well as the entropy generation. Figure 4 shows the variation of temperature profile with Pr.

As it is clearly seen, while Pr increases, the boundary layer gets thinner so thermal gradients climb up which is supported in fig. 5. In order to study entropy generation in details, the effects of Prandtl number are illustrated in figs. 6 and 7. For the case Pr > 1 thermal gradients are greater than that of velocity so heat transfer dominates in entropy generation, on the other hand, increasing in Pr is equal to a reduction in thermal diffusion term which has the boldest effect in entropy generation; hence, although the temperature gradients increases, the entropy generation decreases. Scale analysis for Pr is another approach to explain this phenomenon. Assume that θ_∞, Re, and Ec are constants so:

$$S \propto \frac{1}{\delta_T^2 \,\mathrm{Pr}}, \quad \mathrm{Pr} \gg 1, \quad \delta_T \sim \frac{1}{\sqrt{\mathrm{Pr}}} \to S \propto \frac{1}{1}, \quad \mathrm{Pr} \ll 1, \quad \delta_T \sim \frac{1}{\sqrt[3]{\mathrm{Pr}}} \to S \propto \frac{1}{\sqrt[3]{\mathrm{Pr}}}$$

According to eq. (20) we can see that for higher Pr, entropy generation is independent of Pr, however, for lower Pr, as Pr decreases entropy generation decreases too. For further study on entropy generation we have demonstrated variation of Be for different values of Pr in figs. 8 and 9. For the case Pr > 1, as η grows, Be takes a decreasing trend till it vanishes at the boundary layer edge which reveals that the entropy generation is entirely caused by fluid friction. Considering fig. 9, we can see that due to the marked reduction of S_h at the lower values of η, Be experiences a decrease in this region and then takes an upward trend as η tends to δ.

Figures 10 and 11 check the effects of θ_∞ on the entropy generation for a constant surface temperature and Pr. Clearly an increase in θ_∞ is equal to a decrease in T_∞, as a result, entropy generation decreases. The direct relationship between Be and θ_∞ can be seen in fig. 11

where for higher values of θ_∞ and S_f gets more important and Bejan number decreases; hence, it is logical to claim that just for small θ_∞ and S_f is negligible.

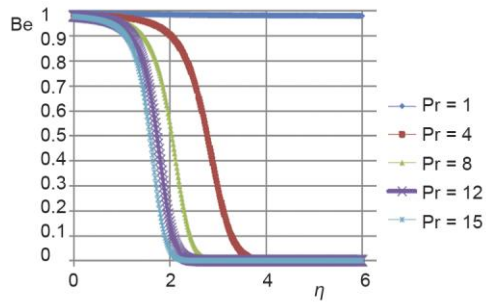

Figure 8. Bejan number for different values of Pr number
(for color image see journal web site)

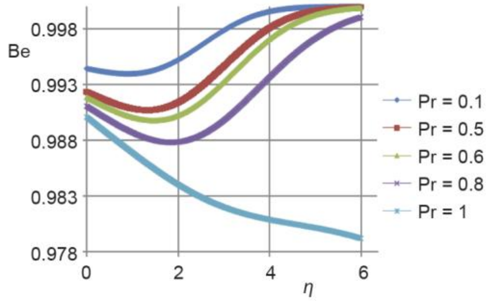

Figure 9. Bejan number for different values of Pr number
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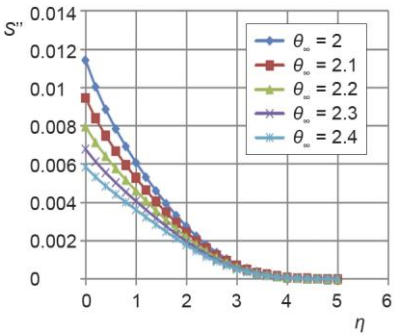

Figure 10. Dimensionless entropy generation for different values of θ_∞
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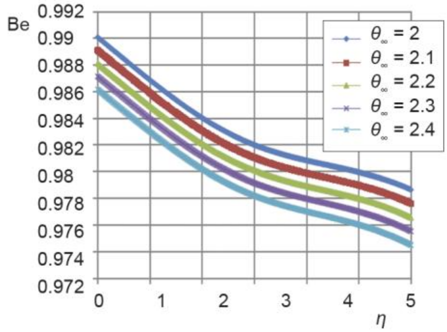

Figure 11. Bejan number for different values of θ_∞
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Conclusions

In this paper the entropy generation for a flow over a flat plate for various conditions of Ec, Re, and Pr is studied. The velocity and temperature distributions were calculated with homotopy perturbation method. To show the accuracy of the results, we have compared the answers with shooting method. The maximum error was about 4% which is acceptable. The bold outcomes are:

- When Pr increases the entropy generation decreases, but for Pr $\gg 1$, Pr does not affect the entropy generation. On the other hand, for Pr $\ll 1$ we can see strong relationship between S'' and Pr.
- When θ_∞ increases Be decreases. Also, when Re and Ec increase, S'' decreases.

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Paper submitted: August 12, 2012
Paper revised: September 7, 2012
Paper accepted: September 12, 2012