EVALUATION OF NATURAL CONVECTION FLOW OF A NANOFLUID OVER A LINEARLY STRETCHING SHEET IN THE PRESENCE OF MAGNETIC FIELD BY THE DIFFERENTIAL TRANSFORMATION METHOD

by

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In the present study, the convective flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of a magnetic field are investigated. The governing partial differential equations with the auxiliary conditions are reduced to ordinary differential equations with the appropriate corresponding conditions via scaling transformations. The semi-analytical solutions of the resulting ordinary differential equations are obtained using differential transformation method coupled with Pade approximation. Comparison with published results is presented which reveals that the applied method is sufficiently accurate for engineering applications.

Key words: *nanofluid, magnetic field, stretching sheet, natural convection, scaling transformations, differential transformation method*

Introduction

The study of magnetic field effects has important applications in physics, chemistry and engineering. In recent years, we find several applications in the polymer industry (where one deals with stretching of plastic sheets) and metallurgy where hydro-magnetic techniques are being used. Nanofluid is envisioned to describe a fluid in which nanometer sized particles are suspended in convectional heat transfer basic fluids. Convectional heat transfer fluids, including oil, water, and ethylene glycol mixture are poor heat transfer fluids, since the thermal conductivity of these fluids play important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Therefore numerous methods have been taken to improve the thermal conductivity of these fluids by suspending nano/micro sized particle materials in liquids. Several recent numerical studies on the modeling of natural convection heat transfer in nanofluids have been published [1-4]. The differential transformation method (DTM) was first

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applied in the engineering domain by Zhou [5]. DTM obtains an analytical solution in the form of a polynomial by means of an iterative procedure. DTM is an alternative procedure for obtaining analytic Taylor series solution of the differential equations. This method is well addressed in [6, 7]. In the present paper, we study the effect of a magnetic field on the free convection flow of a nanofluid over a linear stretching by using scaling group of transformations. The reduced coupled ordinary differential equations are solved using a semi-analytical method. Recently many authors used analytical methods successfully in different engineering problems [8-10]. The effects of the parameters governing the problem are studied and discussed.

Formulation of the problem

Consider influence of a constant magnetic field of strength B_0 which is applied normally to the sheet. The temperature at the stretching surface takes the constant value T_w, while the ambient value, attained as y tends to infinity, takes the constant value T_∞. It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field (as the magnetic Reynolds number is small). The fluid is a water based nanofluid containing different types of nanoparticles: Cu, Al_2O_3, Ag, and TiO_2. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo physical properties of the nanofluid are given in tab. 1 [11]. Under the above assumptions, the boundary layer equations governing the flow and concentration field can be written in dimensional form as:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\rho_{\eta f}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \mu_{\eta f}\frac{\partial^2 \bar{u}^2}{\partial \bar{y}^2} - \sigma \beta_0{}^2 \bar{u} \tag{2}$$

$$(\rho C_p)_{\eta f}\left(\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right) = k_{\eta f}\frac{\partial^2 T^2}{\partial \bar{y}^2} \tag{3}$$

where x and y are the co-ordinates along and perpendicular to the sheet, u and v – the velocity components in the x-and y-directions, respectively. T is the local temperature of the fluid, B_0 – the magnetic parameter, and σ – the electric conductivity.

Table 1. Thermo-physical properties of water and nanoparticles [11]

	ρ [kg m^{-1}]	C_p [Jkg^{-1}K^{-1}]	k [Wm^{-1}K^{-1}]	$\beta\cdot 10^5$ [K^{-1}]
Pure water	997.1	4179	0.613	21
Copper (Cu)	8933	385	401	1.67
Silver (Ag)	10500	235	429	1.89
Alumina (Al_2O_3)	3970	765	40	0.85
Titanium oxide (TiO_2)	4250	686.2	8.9538	0.9

Here β, in tab. 1, is the volumetric coefficient of expansion. The effective density ρ_{nf}, the effective dynamic viscosity μ_{nf}, the heat capacitance $(\rho C_p)_{nf}$, and the thermal conductivity k_{nf} of the nanofluid are given as [12]:

$$\rho_{\eta f} = (1-\varphi)\rho_f + \varphi\rho_s, \quad \mu_{\eta f} = \frac{\mu_f}{(1-\varphi)^{2.5}},$$

$$(\rho C_p)_{\eta f} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s, \quad k_{\eta f} = k_f \left[\frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)}\right] \tag{4}$$

Here, φ is the solid volume fraction. The boundary conditions of eqs. (1)-(3) are:

$$\bar{u} = \bar{u}_w \;\; \bar{x} = a\bar{x}, \;\; T = T_w, \;\; \bar{v} = 0 \qquad \text{at} \qquad y = 0$$

$$\bar{u} \to 0, \;\; T \to T_\infty \qquad \text{as} \qquad y \to \infty \tag{5}$$

where μ_f is the dynamic viscosity of the basic fluid, ρ_f and ρ_s are the densities of the pure fluid and nanoparticle, respectively, $(\rho C_p)_f$ and $(\rho C_p)_s$ – the specific heat parameters of the base fluid and nanoparticle, respectively, k_f and k_s – the thermal conductivities of the base fluid and nanoparticle, respectively, and a is constant. Introducing the following non-dimensional variables:

$$x = \frac{\bar{x}}{\sqrt{\frac{\nu_f}{a}}}, \quad y = \frac{\bar{y}}{\sqrt{\frac{\nu_f}{a}}}, \quad u = \frac{\bar{u}}{\sqrt{au_f}}, \quad \upsilon = \frac{\bar{v}}{\sqrt{a\nu_f}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Eqsuations (1) to (3) and the related conditions (5) will take non-dimensional form.

By introducing the stream function ψ, which is defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, and using the simplified form of Lie-group transformations namely, the scaling group G of transformations [13-15], we get the similarity transformations as:

$$\eta = y, \quad \psi = xF(\eta), \quad \theta = \theta(\eta) \tag{7}$$

For the flow

In this section, the analytical solutions of the velocity components are obtained. The similarity transformations (7) maps, eq. (2), to:

$$F''' + (1-\phi)^{2.5}\left\{\left[1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right](FF'' - F'^2) - MF'\right\} = 0 \tag{8}$$

where primes denote the differentiation with respect to η. The corresponding boundary conditions become:

$$F = 0, F' = 0 \text{ at } \eta = 0,$$

$$F' \to 0 \text{ as } \eta \to 0 \tag{9}$$

The shear stress at the stretching sheet characterized by the skin friction coefficient C_f, is given by:

$$C_f = \frac{-2\mu_f}{\rho_f \;\; \bar{u}_w(\bar{x})^{\;2}}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0} \tag{10}$$

The skin friction can be written as [15]:

$$\sqrt{\mathrm{Re}_x}C_f = -2F''(0) \tag{11}$$

where $\mathrm{Re}_x = \bar{x}\bar{u}_w(\bar{x})/\bar{\nu}_f$ is the local Reynolds number based on the stretching velocity $\bar{u}_w(\bar{x})$; $\mathrm{Re}_x^{1/2}C_f$ is referred as the reduced skin friction coefficient.

Applying the differential transformation on the eq. (8) we get:

$$(k+1)(k+2)(k+3)F(k+3)+(1-\varphi)^{2.5}\cdot$$
$$\cdot\left\{\left[1-\varphi+\varphi\left(\frac{\rho_s}{\rho_f}\right)\right]\left(\begin{array}{l}\sum_{\lambda=0}^{k}F(k-\lambda)(\lambda+1)(\lambda+2)F(\lambda+2)\\ -\sum_{\lambda=0}^{k}(k-\lambda+1)F(k-\lambda+1)(\lambda+1)F(\lambda+1)\end{array}\right)\right\}-$$
$$-M(k+1)F(k+1)=0 \tag{12}$$

The transform of the boundary conditions are:

$$F(0)=0\,,\ F(1)=1\,,\ F(2)=\varDelta \tag{13}$$

where $\varDelta$ is an unknown constant. For computing its value, the problem is solved with initial conditions. Pade approximation [16-20] is used and then the third boundary condition is applied. When $\phi = 0.1$ and $M = 2$ with [5,5] Pade approximation the obtained is:

$$\varDelta = -0.8537840322$$
$$F'(\eta)=\frac{1-0.795301t+0.271831t^2-0.048127t^3+0.004091t^4+0.000027t^5}{1+0.912267t+0.371141t^2+0.085261t^3+0.011244t^4+0.00069t^5} \tag{14}$$

For the heat transfer

Substituting from (7) into (3), we get:

$$\frac{1}{\mathrm{Pr}}\frac{1}{1-\phi+\phi\left[(\rho C_p)_s/(\rho C_p)_f\right]}\left(\frac{k_{\eta f}}{k_f}\right)\theta''+\frac{1}{m}(1-\mathrm{e}^{-m\eta})\theta'=0 \tag{15}$$

where primes denote the differentiation with respect to η. The boundary conditions become:

$$\theta(0)=1,\ \theta(\infty)=0 \tag{16}$$

The quantity of practical interest, in this section is the Nusselt number Nu_x which is defined as:

$$\mathrm{Nu}_x=\frac{\overline{x}\overline{q}_w}{k(T_w-T_\infty)} \tag{17}$$

where q_w is the local surface heat flux given by:

$$\bar{q}_w=-k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} \tag{18}$$

We have the following reduced Nusselt number [18]:

$$\frac{1}{\sqrt{\mathrm{Re}_x}}\mathrm{Nu}_x = -\theta'\ 0 \tag{19}$$

Kuznetsov and Nield [31] referred to $\mathrm{Re}_x^{-1/2}\,\mathrm{Nu}_x$ as the reduced Nusselt number. Applying the differential transformation on the equation (15) we get:

$$\frac{1}{\mathrm{Pr}}\frac{1}{1-\phi+\phi\left[(\rho C_p)_s/(\rho C_p)_f\right]}\left(\frac{k_{\eta f}}{k_f}\right)(k+1)(k+2)\theta(k+2)+$$

$$+\sum_{\lambda=0}^{k}F(k-\lambda)(\lambda+1)\theta(\lambda+1)=0 \tag{20}$$

The transform of the boundary conditions are:

$$\theta(0)=0\,,\ \ \theta(1)=\zeta \tag{21}$$

where ζ is an unknown constant. For computing its value, the problem is solved with initial conditions. Pade approximation [16-20] is used and then the third boundary condition is applied. When ϕ = 0.1 and M = 2 with [10,10] Pade approximation the obtained is:

$$\zeta = -1.239951$$

$$\theta(\eta)=\frac{1-0.730175t+0.083051t^2-0.309841t^3+0.000304t^4+0.034997t^5\cdots}{1+2.05275t+2.79799t^2+2.49382t^3+1.75527t^4+0.926606t^5\cdots} \tag{22}$$

Results and discussion

The distributions of the velocity $F'(\eta)$, the temperature $\theta(\eta)$, Nusselt number, and skin friction in the case of Cu-water are shown in figs. 1-6. Computations are carried out for various values of the magnetic parameter and the nanoparticles volume fraction for different types of nanoparticles, when Pr = 6.2 (water). Magnetic parameter M is varied from 0 to 2, nanoparticles volume fraction φ is varied from 0 to 0.2. The nanoparticles used in the study are from Cu, Ag, Al_2O_3, and TiO_2.

Conclusions

It is found that with the increase of the magnetic parameter, the momentum boundary layer thickness decreases, while the thermal boundary layer thickness increases. The heat transfer rates decrease as the nanoparticle volume fraction φ increases. For a selected value of φ, the heat transfer rates decrease as M increases. The reduced skin friction increases as M increases for selected values of φ; for large values of M the reduced skin friction decreases as φ increases. All obtained results are in good agreement with the previous similar works which shows the high accuracy of the proposed analytical method.

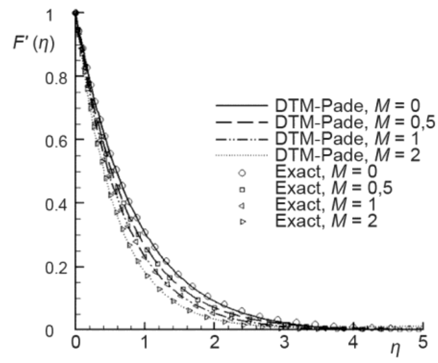

Figure 1. Effect of *M* on velocity distribution *F* '(*η*) for Pr = 6.2 and *φ* = 0.1

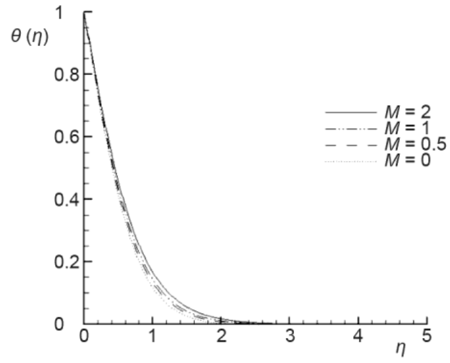

Figure 2. Effect of *M* on temperature distribution *η* for Pr = 6.2 and *φ* = 0.1

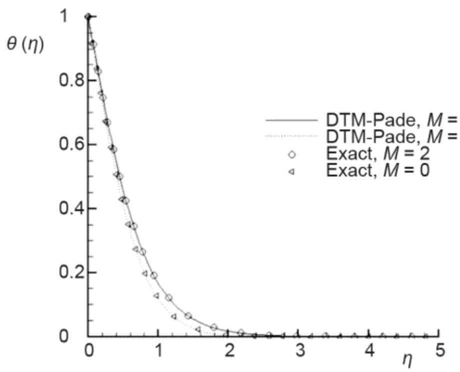

Figure 3. Validation of fig. 2 for *M* = 0,2, Pr = 6.2 and *φ* = 0.1

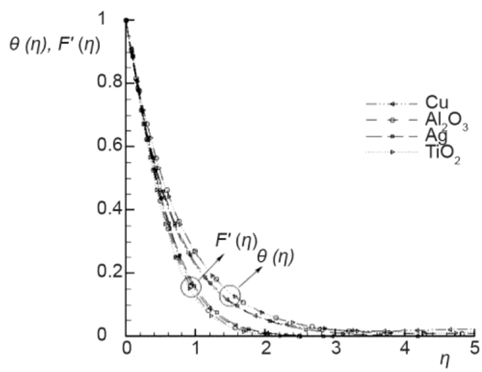

Figure 4. Velocity and temperature profiles for different types of nanofluids when *M* = 1, Pr = 6.2 and *φ* = 0.1

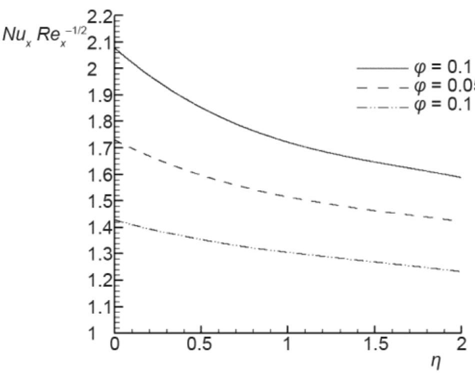

Figure 5. Effects of the nanoparticle volume fraction *φ* on dimensionless heat transfer rates

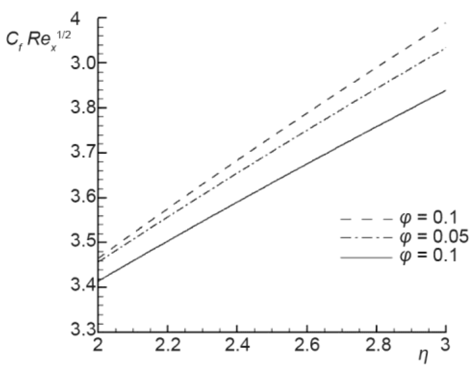

Figure 6. Effects of the nanoparticle volume fraction *φ* on reduced skin friction coefficient

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