ACTIVE CONTROL OF FLOW AND HEAT TRANSFER IN BOUNDARY LAYER ON THE POROUS BODY OF ARBITRARY SHAPE

by

Dragiša D. NIKODIJEVIĆ, Živojin M. STAMENKOVIĆ*, Dragan S. ŽIVKOVIĆ, Aleksandar Z. BORIČIĆ, and Miloš M. KOCIĆ

Faculty of Mechanical Engineering, University of Niš, Niš, Serbia

Original scientific paper
DOI: 10.2298/TSCI120427170N

The paper discusses the possibility of active control of flow and heat transfer using a magnetic field and suction in a generalized form. The unsteady temperature 2-D laminar magnetohydrodynamics boundary layer of incompressible fluid on a porous body of arbitrary shape is analyzed. Outer electric filed is neglected, magnetic Reynolds number is significantly lower than one i. e. the considered problem is in inductionless approximation. Characteristic properties of fluid are constant and it is assumed that a uniform suction or injection of a fluid, same as the fluid in primary flow, can take place through the body surface. The boundary-layer equations are generalized such that the equations and the boundary conditions are independent of the particular conditions of the problem, and this form is considered as universal. Obtained universal equations are numerically solved using the "progonka" method. Numerical results for the dimensionless velocity, temperature, shear stress and heat transfer as functions of introduced sets of parameters are obtained, displayed graphically and used to carry out general conclusions about the development of temperature magnetohydrodynamics boundary layer.

Key words: *magnetohydrodynamics, boundary layer, heat transfer, generalized similarity*

Introduction

The idea of boundary layer control first appeared when Prandtl formed the theory, and this idea came from Prandtl [1] himself. Boundary-layer control usually means either an attempt to change the overall flow field to reduce pressure drag and/or to increase lift, or an attempt to control the position of boundary layer separation point. Since then, many passive and active techniques have been developed for the prevention or delay of flow separation: admitting the body motion in streamwise direction, increasing the boundary layer velocity, boundary layer suction, second gas injection, profile laminarization, body cooling. The interest in the effect of outer magnetic field on heat-physical processes appeared in 1960s [2].

A large number of theoretical investigations dealing with magnetohydrodynamics (MHD) flows of viscous fluids have been performed during the last decades due to their rapidly increasing applications in many fields of technology and engineering, such as MHD power generation, MHD flow meters, MHD pumps, magneto-biological and medical processes [3]. Precise and active control of heat transfer and fluid flow under extreme conditions is important for fu-

* Corresponding author; e-mail: zikas@masfak.ni.ac.rs

ture science and technology. Many mathematical models have been proposed to explain the behaviors of the viscous MHD flow under different conditions. Generally, the fundamental equations governing the flow of a viscous electrically conducting fluid have a very complicated form. Solutions of the mentioned models were followed by a rapid increase of analytical papers and experimental procedures about heat transfer in the MHD boundary layer [4-8].

For the sake of enriching the above research, this paper considers an unsteady temperature, 2-D laminar MHD boundary layer of incompressible fluid on the porous surface body. Externally applied magnetic field is still in relation to the fluid in outer flow and perpendicular to the body. Further on, it is assumed that there is no outer electric field and magnetic Reynolds number is significantly lower than one *i. e*. the considered problem is in inductionless approximation. Body surface temperature is the function of longitudinal coordinate and the fluid is injected or sucked with a constant velocity through the porous surface. The velocity of the flow is considered much lower than the speed of light, and the usual assumption that the temperature difference is small (under $50^{o}C$) in temperature boundary layer calculation is used, with the characteristic properties of fluid being constant (viscosity, thermal conductivity, electrical conductivity, magnetic permeability, mass heat capacity). The introduced assumptions simplify the considered problem, however, the obtained physical model is interesting from the practical point of view, since its relation with a large number of MHD flows is significant for technical practice.

In the described flow problem the porous contour and externally applied magnetic field are used to control the flow in the boundary layer. Partial differential equations, which mathematically describe the considered problem, can be solved in every particular case using numerical methods. Many exact and approximate analytical methods have been developed for solving problems on MHD flow over bodies of different shape. The bibliography of earlier studies on the finite conductivity MHD flows can be found in [9]. All of the mentioned MHD boundary layer flows are treated separately for given particular problems.

This paper presents quite a different approach based on the ideas given in papers [10-12], which are extended in papers [13-15]. The essence of this approach lies in introducing adequate transformations and sets of parameters in starting equations of a laminar 2-D unsteady temperature MHD boundary layer of incompressible fluid on porous contour, which transform the equations system and corresponding boundary conditions into a form unique for all particular problems, which is considered universal. The solution of universal equations, obtained using modern numerical methods, can be employed in the derivation of general conclusions about developing the described temperature MHD boundary layer, and for calculation of special cases of the observed problem. In order to solve particular problems, it is necessary to determine the impulse equation using the obtained universal solutions.

Mathematical model

The described 2-D problem of MHD unsteady temperature boundary layer in inductionless approximation is mathematically presented with the equation system:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}\ u-U \tag{1}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2}{\rho c_p}\ u-U\ ^2 \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

and corresponding boundary and initial conditions:

$$u=0,\ \ v=v_w(x,t),\ \ T=T_w(x)\quad \text{for}\quad y=0 \tag{4}$$

$$u\to U(x,t),\ \ T\to T_\infty\ \ \text{for}\ \ y\to\infty \tag{5}$$

$$u=u_0(x,y),\ \ T=T_0(x,y)\quad \text{for}\quad t=t_0 \tag{6}$$

$$u=u_1(t,y),\ \ T=T_1(t,y)\quad \text{for}\quad x=x_0 \tag{7}$$

For further consideration, velocity difference $v(x,y,t)$ and stream function $\Psi(x,y,t)$ are introduced with the following relations:

$$v_1=v-v_w,\ \ \frac{\partial\Psi}{\partial x}=-v_1,\quad \frac{\partial\Psi}{\partial y}=u \tag{8}$$

which transform eqs. (1) and (2) into the system:

$$\frac{\partial^2\Psi}{\partial t\partial y}+\frac{\partial\Psi}{\partial y}\frac{\partial^2\Psi}{\partial x\partial y}+\left(v_w-\frac{\partial\Psi}{\partial x}\right)\frac{\partial^2\Psi}{\partial y^2}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^3\Psi}{\partial y^3}-\frac{\sigma B^2}{\rho}\left(\frac{\partial\Psi}{\partial y}-U\right) \tag{9}$$

$$\frac{\partial T}{\partial t}+\frac{\partial\Psi}{\partial y}\frac{\partial T}{\partial x}+\left(v_w-\frac{\partial\Psi}{\partial x}\right)\frac{\partial T}{\partial y}=\frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial^2\Psi}{\partial y^2}\right)^2+\frac{\sigma B^2}{\rho c_p}\left(\frac{\partial\Psi}{\partial y}-U\right)^2 \tag{10}$$

Boundary and initial conditions are transformed into conditions:

$$\Psi=0,\ \ \frac{\partial\Psi}{\partial y}=0,\ \ T=T_w\ \ x\quad \text{for}\ \ y=0 \tag{11}$$

$$\frac{\partial\Psi}{\partial y}\to U\ \ x,t\ \ ,\ \ T\to T_\infty\ \ \text{for}\ \ y\to\infty \tag{12}$$

$$\frac{\partial\Psi}{\partial y}=u_0(x,y),\ \ T=T_0\ \ x,y\quad \text{for}\quad t=t_0 \tag{13}$$

$$\frac{\partial\Psi}{\partial y}=u_1(t,y),\ \ T=T_1\ \ t,y\quad \text{for}\quad x=x_0 \tag{14}$$

The first equation of system (9) does not depend on the second equation (10), and it can be solved independently. For solving the second equation system, the solution of the first equation is used.

For further consideration of the described problem, new variables are introduced:

$$x=x,\ \ t=t,\ \eta=\frac{Dy}{h\ \ x,t},\ \ \Phi\ \ x,t,\eta\ \ =\frac{D\Psi\ \ x,y,t}{U\ \ x,t\ \ h\ \ x,t},\ \ \Theta\ \ x,t,\eta\ \ =\frac{T_w-T}{T_w-T_\infty} \tag{15}$$

where D is the normalizing constant and $h(x,t)$ is the characteristic linear scale of the transversal coordinate in the boundary layer.

According to the introduced variables, system of eqs. (9) and (10) is transformed into the new form:

$$D^2\frac{\partial^3\Phi}{\partial\eta^3}+f_{1,0}\left(\Phi\frac{\partial^2\Phi}{\partial\eta^2}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2+1\right)+\ f_{0,1}+g_{1,0}\ \left(1-\frac{\partial\Phi}{\partial\eta}\right)+$$

$$+\frac{1}{2}\ F\Phi+\eta g\ \frac{\partial^2\Phi}{\partial\eta^2}+D\lambda_{0,0}\frac{\partial^2\Phi}{\partial\eta^2}=z\frac{\partial^2\Phi}{\partial t\partial\eta}+UzX(\eta,x) \tag{16}$$

$$\frac{D^2}{P_r}\frac{\partial^2\Theta}{\partial\eta^2}-D^2E_c\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2-E_cg_{1,0}\left(1-\frac{\partial\Phi}{\partial\eta}\right)^2+\ 1-\Theta\ l_1\frac{\partial\Phi}{\partial\eta}+$$

$$+\frac{1}{2}\ F+2f_{1,0}\ \Phi\frac{\partial\Theta}{\partial\eta}+D\lambda_{0,0}\frac{\partial\Theta}{\partial\eta}+\frac{1}{2}\eta g\frac{\partial\Theta}{\partial\eta}=z\frac{\partial\Theta}{\partial t}-UzY(x,\eta) \tag{17}$$

where for the sake of shorter expression, the notations are introduced:

$$z=\frac{h^2}{\nu},\quad g=\frac{\partial z}{\partial t},\quad N=\frac{\sigma B^2}{\rho},\quad g_{1,0}=Nz,\quad F=U\frac{\partial z}{\partial x},$$

$$f_{1,0}=z\frac{\partial U}{\partial x},\quad f_{0,1}=\frac{z}{U}\frac{\partial U}{\partial t},\quad \lambda_{0,0}=-v_w\sqrt{\frac{z}{\nu}},\quad l_1=\frac{Uz}{T_w-T_\infty}\frac{\mathrm{d}T_w}{\mathrm{d}x} \tag{18}$$

$$\mathrm{Pr}=\frac{\nu\rho c_p}{\lambda}\ \text{Prandtl number},\ \mathrm{E_c}=\frac{U^2}{c_p\ \ T_w-T_\infty}\ \text{Eckert number},$$

$$X\ \ x_1,x_2\ =\frac{\partial\Phi}{\partial x_1}\frac{\partial^2\Phi}{\partial\eta\partial x_2}-\frac{\partial\Phi}{\partial x_2}\frac{\partial^2\Phi}{\partial x_1\partial\eta},\ Y\ \ x_1,x_2\ =\frac{\partial\Phi}{\partial x_1}\frac{\partial\Theta}{\partial x_2}-\frac{\partial\Phi}{\partial x_2}\frac{\partial\Theta}{\partial x_1}$$

Now we introduce sets of parameters (dynamical, magnetic, temperature, blowing/suction):

$$\left.\begin{aligned}
f_{k,n}&=U^{k-1}\frac{\partial^{k+n}U}{\partial x^k\partial t^n}z^{k+n},\quad (k,n=0,1,2,...;k\vee n\neq 0)\\
f_{k,n}&=U^{k-1}\frac{\partial^{k+n}U}{\partial x^k\partial t^n}z^{k+n},\quad (k,n=0,1,2,...;k\vee n\neq 0)\\
g_{k,n}&=U^{k-1}\frac{\partial^{k-1+n}N}{\partial x^{k-1}\partial t^n}z^{k+n},\quad (k,n=0,1,2,...,;k\neq 0)\\
l_k&=\frac{U^k}{q}\frac{d^kq}{dx^k}z^k,\quad (k=1,2,...)\ \text{where}\ q=T_w-T_\infty\\
\lambda_{k,n}&=-U^k\frac{\partial^{k+n}v_w}{\partial x^k\partial t^n}\frac{z^{k+n+\frac{1}{2}}}{\nu},\quad (k,n=0,1,2,...,)
\end{aligned}\right\} \tag{19}$$

and the constant parameter:

$$g = \frac{\partial z}{\partial t} = \text{const.} \tag{20}$$

which can have different values.

It can be noticed that the first parameters are given in terms (18). The introduced sets of parameters reflect the nature of velocity change on the outer edge of the boundary layer, the nature of injection (ejection) velocity, the alteration characteristic of variable *N* and the temperature change on the body surface, and apart from that, in the integral form (by means of *z* and $\partial z/\partial t$), the pre-history of flow in the boundary layer.

Using the introduced sets of parameters (19) like new independent variables instead of *x* and *t*, and differentiating operators for *x* and *t*:

$$\frac{\partial}{\partial \varphi} = \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty} \frac{\partial f_{k,n}}{\partial \varphi}\frac{\partial}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty} \frac{\partial g_{k,n}}{\partial \varphi}\frac{\partial}{\partial g_{k,n}} + \sum_{k,n=0}^{\infty} \frac{\partial \lambda_{k,n}}{\partial \varphi}\frac{\partial}{\partial \lambda_{k,n}} + \begin{cases} 0, \ \text{for} \ \Phi \\ \sum_{k=1}^{\infty} \frac{\partial l_k}{\partial \varphi}\frac{\partial}{\partial l_k}, \ \text{for} \ \Theta \end{cases} \tag{21}$$

where $\varphi = x, t$ and parameter derivatives along co-ordinate *x* and time *t* are obtained by differentiation of equations (19):

$$\left.\begin{aligned}
\frac{\partial f_{k,n}}{\partial x} &= \frac{1}{Uz}\left\{(k-1) f_{1,0} f_{k,n} + (k+n) F f_{k,n} + f_{k+1,n}\right\} = \frac{1}{Uz} Q_{k,n} \\
\frac{\partial f_{k,n}}{\partial t} &= \frac{1}{z}\left\{(k-1) f_{0,1} f_{k,n} + (k+n) g f_{k,n} + f_{k,n+1}\right\} = \frac{1}{z} E_{k,n} \\
\frac{\partial g_{k,n}}{\partial x} &= \frac{1}{Uz}\left\{(k-1) f_{1,0} g_{k,n} + (k+n) F g_{k,n} + g_{k+1,n}\right\} = \frac{1}{Uz} K_{k,n}
\end{aligned}\right\} \tag{22a}$$

$$\left.\begin{aligned}
\frac{\partial g_{k,n}}{\partial t} &= \frac{1}{z}\left[(k-1) g_{k,n} f_{0,1} + (k+n) g g_{k,n} + g_{k,n+1}\right] = \frac{1}{z} L_{k,n} \\
\frac{\partial l_k}{\partial x} &= \frac{1}{Uz}\left\{\left[k f_{1,0} - l_1 + kF\right] l_k + l_{k+1}\right\} = \frac{1}{Uz} M_k \\
\frac{\partial l_k}{\partial t} &= \frac{1}{z}\left[k (f_{0,1} + g) l_k\right] = \frac{1}{z} N_k \\
\frac{\partial \lambda_{k,n}}{\partial x} &= \frac{1}{Uz}\left[k f_{1,0}\lambda_{k,n} + \left(k+n+\frac{1}{2}\right) F \lambda_{k,n} + \lambda_{k+1,n}\right] = \frac{1}{Uz} R_{k,n} \\
\frac{\partial \lambda_{k,n}}{\partial t} &= \frac{1}{z}\left[k f_{0,1}\lambda_{k,n} + \left(k+n+\frac{1}{2}\right) g \lambda_{k,n} + \lambda_{k,n+1}\right] = \frac{1}{z} S_{k,n}
\end{aligned}\right\} \tag{22b}$$

where $Q_{k,n}$, $E_{k,n}$, $K_{k,n}$, $L_{k,n}$, M_k, N_k, $R_{k,n}$, $S_{k,n}$ are terms in curly brackets in the obtained equations. It is important to notice that $Q_{k,n}$, $K_{k,n}$, M_k, $R_{k,n}$ depend on value $U\partial z/\partial x = F$, beside depending on the parameters. Using parameters (19), operators (21), terms (22a) and (22b), system of eqs. (16) and (17) is transformed into equations:

$$\Im_1 = \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\left[E_{k,n}\frac{\partial^2\Phi}{\partial\eta\partial f_{k,n}} + Q_{k,n}X(\eta, f_{k,n})\right] + \sum_{\substack{k=1\\ n=0}}^{\infty}\left[L_{k,n}\frac{\partial^2\Phi}{\partial\eta\partial g_{k,n}} + K_{k,n}X(\eta, g_{k,n})\right] +$$
$$+\sum_{k,n=0}^{\infty}\left[S_{k,n}\frac{\partial^2\Phi}{\partial\eta\partial\lambda_{k,n}} + R_{k,n}X(\eta,\lambda_{k,n})\right] \tag{23}$$

$$\Im_2 = \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\left[E_{k,n}\frac{\partial\Theta}{\partial f_{k,n}} + Q_{k,n}Y(\eta, f_{k,n})\right] + \sum_{\substack{k=1\\ n=0}}^{\infty}\left[L_{k,n}\frac{\partial\Theta}{\partial g_{k,n}} + K_{k,n}Y(\eta, g_{k,n})\right] +$$
$$+\sum_{k=1}^{\infty}\left[M_kY(\eta,l_k) + N_k\frac{\partial\Theta}{\partial l_k}\right] + \sum_{k,n=0}^{\infty}\left[S_{k,n}\frac{\partial\Theta}{\partial\lambda_{k,n}} + R_{k,n}Y(\eta,\lambda_{k,n})\right] \tag{24}$$

where the following markings are used for shorter statement: $\Im_1$ – the left side of the first equation of system (16), $\Im_2$ – the left side of the second system of equation (17).

In order to make eqs. (23) and (24) universal, it is necessary to show that value F can be expressed by means of introduced parameters. To prove the abovementioned, we start from the impulse equation of the described problem:

$$\frac{\partial}{\partial t}(U\delta^*) + \frac{\partial}{\partial x}(U^2\delta^{**}) + U\left(\frac{\partial U}{\partial x} + N\right)\delta^* - v_wU - \frac{\tau_w}{\rho} = 0 \tag{25}$$

where

$$\delta^*(x,t) = \int_0^\infty\left(1-\frac{u}{U}\right)\mathrm{d}y \text{ – displacement thickness} \tag{26}$$

$$\delta^{**}(x,t) = \int_0^\infty\frac{u}{U}\left(1-\frac{u}{U}\right)\mathrm{d}y \text{ – momentum thickness} \tag{27}$$

$$\tau_w(x,t) = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ – wall shear stress} \tag{28}$$

Introducing dimensionless characteristic functions:

$$H^*(x,t) = \frac{\delta^*}{h},\quad H^{**}(x,t) = \frac{\delta^{**}}{h},\quad \xi(x,t) = \frac{\tau_w h}{\mu U} \tag{29}$$

which, according to eqs. (15), (26), (27), and (28), can be expressed in the form:

$$H^*(x,t) = \frac{1}{D}\int_0^\infty\left(1-\frac{\partial\Phi}{\partial\eta}\right)\mathrm{d}\eta,\quad H^{**}(x,t) = \frac{1}{D}\int_0^\infty\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\mathrm{d}\eta,$$
$$\xi(x,t) = D^2\left.\frac{\partial^2\Phi}{\partial\eta^2}\right|_{\eta=0},\quad \xi_t = D\left.\frac{\partial\Theta}{\partial\eta}\right|_{\eta=0} \tag{30}$$

After the transition to new independent variables (introduced parameters) in terms (30), values H^*, H^{**}, ξ, and ξ_t become functions only of parameters $f_{k,n}$, $g_{k,n}$, l_k, $\lambda_{k,n}$, and g.

Now, using the parameters from impulse eq. (25), after simple transformations, the following equation is obtained:

$$F = \frac{P}{Q} \tag{31}$$

where for the sake of shorter expression, the following marks are used:

$$\begin{gathered} P = \xi - f_{1,0}\ \ 2H^{**} + H^{*}\ \ - \sum_{\substack{k,n=0 \\ k\vee n\neq 0}}^{\infty} \left\{ E_{k,n} \frac{\partial H^{*}}{\partial f_{k,n}} + \left[\ k-1\ \ f_{1,0} f_{k,n} + f_{k+1,n} \right] \frac{\partial H^{**}}{\partial f_{k,n}} \right\} - \\ - \left(f_{0,1} + g_{1,0} + \frac{1}{2} g \right) H^{*} - \lambda_{0,0} - \sum_{k=1}^{\infty} \left\{ L_{k,n} \frac{\partial H^{*}}{\partial g_{k,n}} + \left[\ k-1\ \ f_{1,0} g_{k,n} + g_{k+1,n} \right] \frac{\partial H^{**}}{\partial g_{k,n}} \right\} - \\ - \sum_{k,n=0}^{\infty} \left\{ S_{k,n} \frac{\partial H^{*}}{\partial \lambda_{k,n}} + \left[k f_{1,0} \lambda_{k,n} + \lambda_{k+1,n} \right] \frac{\partial H^{**}}{\partial g_{k,n}} \right\} \end{gathered} \tag{32}$$

$$\begin{gathered} Q = \frac{1}{2} H^{**} + \sum_{\substack{k,n=0 \\ k\vee n\neq 0}}^{\infty}\ \ k+n\ \ f_{k,n} \frac{\partial H^{**}}{\partial f_{k,n}} + \\ + \sum_{\substack{k=1 \\ n=0}}^{\infty}\ \ k+n\ \ g_{k,n} \frac{\partial H^{**}}{\partial g_{k,n}} + \sum_{k,n=0}^{\infty} \left(k+n+\frac{1}{2} \right) \lambda_{k,n} \frac{\partial H^{**}}{\partial \lambda_{k,n}} \end{gathered} \tag{33}$$

The last two equations define function F in terms of values, which depends only on the introduced parameters. Equation system (23)-(24) is now a universal system of equations of the described problem. Boundary conditions, which are also universal, are given with terms:

$$\Phi = 0, \frac{\partial \Phi}{\partial \eta} = 0, \quad \Theta = 0 \ \text{ for } \ \eta = 0; \quad \Phi \to 1, \ \Theta \to 1 \ \text{ for } \ \eta \to \infty$$

$$\Phi = \Phi_0\ \ \eta\ \ , \Theta = \Theta_0\ \ \eta\ \ \text{ for} \left\{ \begin{array}{l} f_{k,n} = 0 \quad k,n = 0,1,2,..., k \vee n \neq 0 \\ g_{k,n} = 0 \quad k,n = 0,1,2,..., k \neq 0 \\ l_k = 0 \quad k = 1,2,...\ ;\ \lambda_{k,n} = 0 \quad k,n = 0,1,2,...\ ;\ g = 0 \end{array} \right\} \tag{34}$$

where $\Phi_0(\eta)$ is the Blasius solution for the stationary boundary layer on the plate and $\Theta_0(\eta)$ – the solution of the following equation:

$$\frac{D^2}{P_r} \frac{\mathrm{d}^2 \Theta_0}{\mathrm{d}\eta^2} - D^2 E_c \left(\frac{\mathrm{d}^2 \Phi_0}{\mathrm{d}\eta^2} \right)^2 + \frac{\xi_0}{H^{**}} \Phi_0 \frac{\mathrm{d}\Theta_0}{\mathrm{d}\eta} = 0 \tag{35}$$

The universal system of eqs. (23)-(24) with boundary conditions (34) is strictly for wide class of problems in which $z = At + C(x)$, where A is the arbitrary constant and $C(x)$ is a function of longitudinal coordinate.

Equation system (23)-(24) can be integrated in m-parametric approximation for all possible situations. The obtained characteristic values can be used to yield general conclusions about the development of the described boundary layer and to solve any particular problem.

Before the integration for the scale of transversal coordinate in boundary layer $h(x,t)$, a characteristic value is chosen. In this case it is $h = \delta^{**}$, and, according to eq. (30) $H^{**} = 1$, $H^{*} = \delta^{*}/\delta^{**} = H$, equality (31) now has the form:

$$F = 2\left[\xi - f_{1,0}(2+H) - \left(f_{0,1} + g_{1,0} + \frac{1}{2}g\right)H - \sum_{\substack{k,n=0 \\ k\vee n\neq 0}}^{\infty} E_{k,n}\frac{\partial H}{\partial f_{k,n}} - \right.$$
$$\left. -\lambda_{0,0} - \sum_{\substack{k=1 \\ n=0}}^{\infty} L_{k,n}\frac{\partial H}{\partial g_{k,n}} - \sum_{k,n=0}^{\infty} S_{k,n}\frac{\partial H}{\partial \lambda_{k,n}}\right] \tag{36}$$

Taking parameters $f_{k,n} = 0$, $g_{k,n} = 0$, $\lambda_{k,n} = 0$, and $g = 0$, the first equation of system (13) is simplified into the form:

$$\frac{\mathrm{d}^3\Phi_0}{\mathrm{d}\eta^3} + \frac{\xi_0}{D^2}\Phi_0\frac{\mathrm{d}^2\Phi_0}{\mathrm{d}\eta^2} = 0 \tag{37}$$

and if $D^2 = \xi_0$, then the previous equation becomes a well-known Blasius equation. According to the previous statement, the value of 0.47 must be chosen for normalizing constant D. For selected value h, eq. (35) for determining variable Θ_0 becomes:

$$\frac{1}{P_r}\frac{\mathrm{d}^2\Theta_0}{\mathrm{d}\eta^2} + \Phi_0\frac{\mathrm{d}\Theta_0}{\mathrm{d}\eta} - E_c\left(\frac{\mathrm{d}^2\Phi_0}{\mathrm{d}\eta^2}\right)^2 = 0 \tag{38}$$

In this paper, adequate approximations of system (23)-(24) are given, in which the influence of parameters $f_{1,0}$, $f_{0,1}$, $g_{1,0}$, l_1, $\lambda_{0,0}$, and g is detained, while the influence of parameters $f_{0,1}$, l_1, $\lambda_{0,0}$ derivatives is disregarded. System (23)-(24) is simplified into the following form:

$$\Im_1 = Ff_{1,0}X(\eta; f_{1,0}) + gf_{1,0}\frac{\partial^2\Phi}{\partial\eta\partial f_{1,0}} + Fg_{1,0}X(\eta; g_{1,0}) + gg_{1,0}\frac{\partial^2\Phi}{\partial\eta\partial g_{1,0}} \tag{39}$$

$$\Im_2 = Ff_{1,0}Y(\eta; f_{1,0}) + gf_{1,0}\frac{\partial\Theta}{\partial f_{1,0}} + Fg_{1,0}Y(\eta; g_{1,0}) + gg_{1,0}\frac{\partial\Theta}{\partial g_{1,0}} \tag{40}$$

where function F in the same approximation obtained from eq. (36) has the form:

$$F = 2\left[\xi - f_{1,0}(2+H) - \left(f_{0,1} + g_{1,0} + \frac{1}{2}g\right)H - \lambda_{0,0} - gf_{1,0}\frac{\partial H}{\partial f_{1,0}} - gg_{1,0}\frac{\partial H}{\partial g_{1,0}}\right] \tag{41}$$

Boundary conditions, which coincide with equation system (39)-(40), are:

$$\Phi = 0,\ \frac{\partial\Phi}{\partial\eta} = 0,\ \Theta = 0 \quad \text{for} \quad \eta = 0, \quad \Phi \to 1,\ \Theta \to 1 \ \text{for} \ \eta \to \infty$$

$$\Phi = \Phi_0(\eta),\ \Theta = \Theta_0(\eta) \quad \text{for} \ \left\{\begin{matrix} f_{1,0} = 0,\ f_{0,1} = 0,\ g_{1,0} = 0 \\ l_1 = 0,\ \lambda_{0,0} = 0,\ g = 0 \end{matrix}\right\} \tag{41}$$

which is obtained from condition (34), using the same simplifications as for the system of equations. Equations (39) and (40) are five-parametric once localized approximations of eqs. (23) and (24).

Results and discussion

This section provides a part of the results obtained from the numerical integration of universal equations (39) and (40). All of the results are given for Prandtl number Pr = 6.99, initial Eckert number Ec = 0.0005, and constant parameter g = 0.05.

The integration domain of the treated system is divided in two parts – the first one from $f_{1,0} = 0$ towards the stagnation point ($f_{1,0} > 0$), and the second one from $f_{1,0} = 0$ towards the separation point ($f_{1,0} < 0$). Discretization of the equations was done by applying the implicit scheme, central for dimensionless transversal coordinate η and backward for $f_{1,0}$. As a result of such procedure, a tridiagonal system of algebraic equations was obtained.

In both integration domains, unsteadiness parameter $f_{0,1}$ takes positive or negative values corresponding to the accelerated or decelerated free stream, respectively. Magnetic parameter $g_{1,0}$ starts from zero, corresponding to the absence of an applied magnetic field, temperature parameter l_1 represents the change of body surface temperature along the longitudinal coordinate, while parameter $\lambda_{0,0}$ reflects the suction (positive values) or blowing (negative values).

Figures 1 and 2 present the variations of variables F, H and dimensionless shear stress ξ as a function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$. It may be noted that the accelerated free stream decreases the boundary layer thickness and increases the velocity gradient near the wall. Positive or unfavorable pressure gradients, that decelerate the free stream ($f_{0,1} < 0$), increase boundary layer thickness and decrease the velocity gradient at the wall. Unfavorable pressure gradients can cause boundary layer separation, which often results in drastically altered flow patterns and losses. The shear stress at the wall is less downstream (to the separation point) than upstream, indicating that the wall shear stress decreases along the body. The obtained results, indeed, show that the accelerated free stream ($f_{0,1}$ = 0.06) moves the boundary layer separation point ($\xi = 0$) downstream as expected, while deceleration has a negative influence.

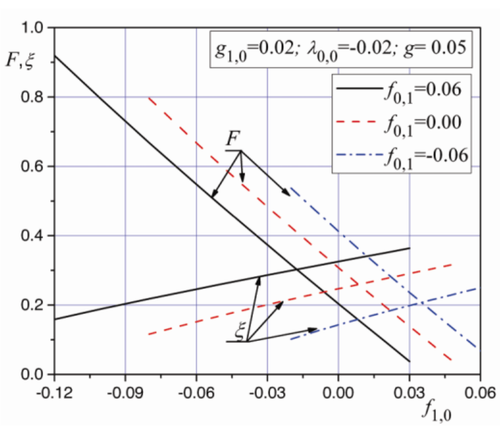

Figure 1. Function F and shear stress ξ for different values of unsteadiness parameter $f_{0,1}$

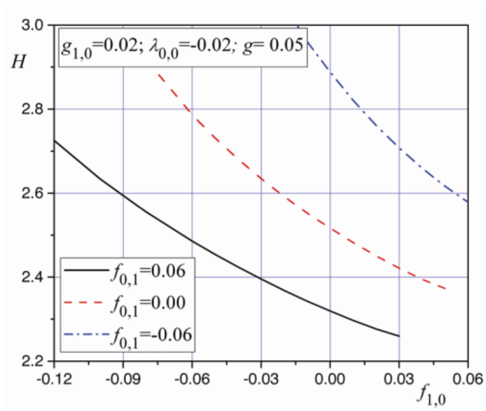

Figure 2. Function H for different values of unsteadiness parameter $f_{0,1}$

Figure 3 presents the variations of dimensionless temperature gradient ξ_t as a function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$. The decelerated free stream causes the increase in heat transfer near the separation point, while towards the stagnation point the accelerated stream has the same effect.

Figures 4 to 6 present the influence of applied magnetic field on boundary layer characteristic functions. The effect of magnetic parameter $g_{1,0}$ on functions F, H, and dimensionless shear stress ξ is shown in fig. 4 and fig. 5.

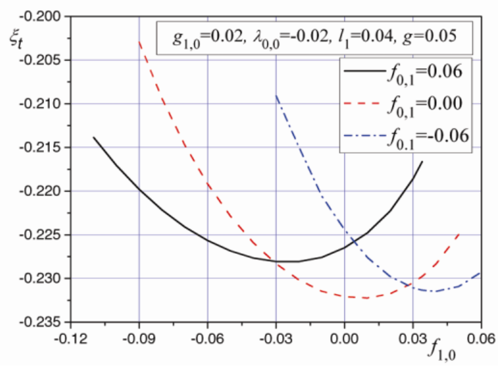

Figure 3. Dimensionless temperature gradient ξ_t as a function of dynamic parameter for different values of unsteadiness parameter $f_{0,1}$

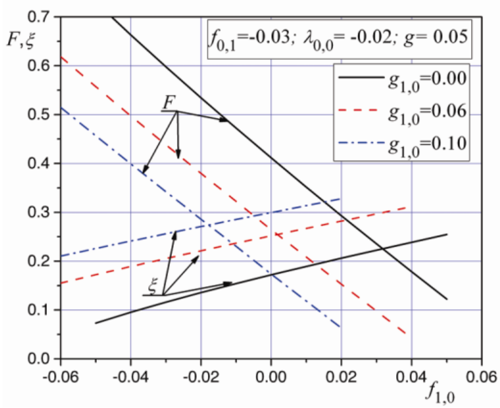

Figure 4. Function F and dimensionless shear stress ξ for different values of magnetic parameter $g_{1,0}$

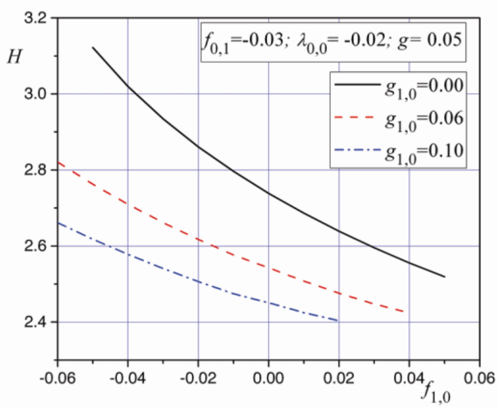

Figure 5. Function H for different values of magnetic parameter $g_{1,0}$

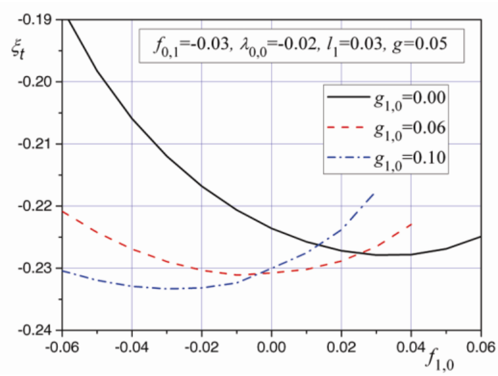

Figure 6. Dimensionless temperature gradient ξ_t as a function of dynamic parameter for different values of magnetic parameter $g_{1,0}$

These figures present the case of the decelerated outer flow ($f_{0,1}$). It is interesting to note the decrease in functions F and H with the increase in the magnetic parameter, and also with the increase in dynamic parameter $f_{1,0}$. These results confirm the delay of the boundary-layer separation, while a greater postponement is achieved with the increasing of magnetic parameter. The figures also show that increasing the magnetic filed decreases the velocity boundary layer thickness due to its damping effect. From fig. 4 one may also note that with the increase in magnetic parameter ξ magnetic field also increases. This remark leads to the conclusion that the magnetic field postpones the boundary-layer separation, and that a greater postponement is achieved with the increasing of magnetic parameter $g_{1,0}$. Figure 4 is given for the case of the decelerated outer flow ($f_{0,1}$ = –0.03), however, the same conclusion is obtained for the case of the accelerated outer flow ($f_{0,1} > 0$).

The effect of magnetic parameter $g_{1,0}$ on dimensionless temperature gradient ξ_t as a function of dynamic parameter $f_{1,0}$ is shown in fig. 6. It is obvious that in the absence of the magnetic filed ($g_{1,0} = 0$) heat transfer decreases towards the separation point. The increase of the magnetic field gives nearly uniform heat transfer from the stagnation point to the separation point.

The effects of the suction or the blowing parameter $\lambda_{0,0}$ on the behaviors of the MHD boundary layer fluid flow are presented in figs. 7-9. It can be seen that the blowing increases the boundary layer thickness and decreases the velocity gradient of flow. Nevertheless, the suction has the opposite effect on the boundary layer flow. All these results act in accordance with physical situations. Figure 9 shows that suction causes the increase in heat transfer, while blowing has the opposite effect. The results are given for the case of temperature decrease along the body ($l_1 < 0$), but the same conclusion is valid for the case of temperature increase ($l_1 > 0$).

Dimensionless stream function Φ (dimensionless velocity) is given in fig. 10 for different values of unsteadiness parameter. It can be noted that the velocity in the boundary layer tends more quickly to the free stream velocity for the case of the accelerated free stream, more slowly for the case of the decelerated flow compared with the steady outer flow ($f_{0,1} = 0$). The same conclusion is valid for other cross-sections of boundary-layer and for all values of dynamic and magnetic parameter.

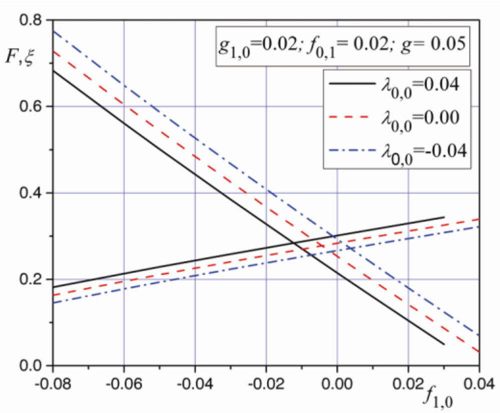

Figure 7. Function *F* and dimensionless shear stress ξ for different values of suction (blowing) parameter $\lambda_{0,0}$

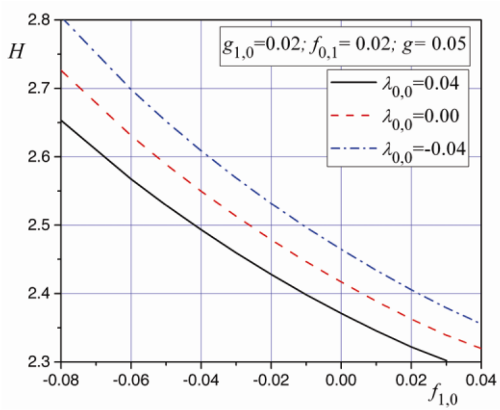

Figure 8. Function *H* for different values of suction (blowing) parameter $\lambda_{0,0}$

Figure 11 shows the dimensionless stream function Φ as a function of dimensionless transversal coordinate η for different values of magnetic parameter. From fig. 11, we observe that with the increase in the magnetic parameter this ratio also increases, and the minimal value is obtained for the case of non-conducting fluid or for the case of magnetic field absence. This analysis indicates the significant influence of magnetic field on increasing velocity in the boundary layer. The results clearly show that the magnetic field tends to delay or prevent separation.

It is interesting to note that the decelerated free stream increases the dimensionless boundary layer temperature, while the positive values of the same parameter have the opposite effect as shown in fig. 12.

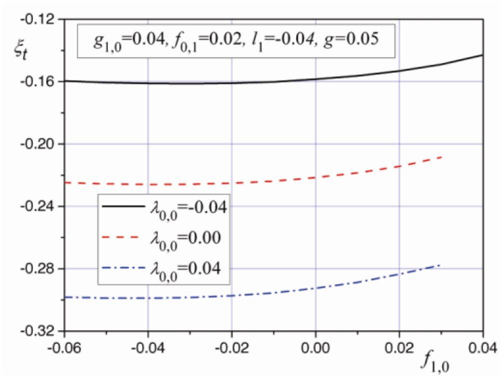

Figure 9. Dimensionless temperature gradient ξ_t as a function of dynamic parameter for different values of suction (blowing) parameter $\lambda_{0,0}$

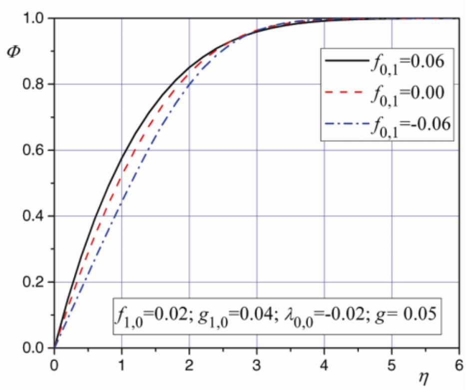

Figure 10. Effect of unsteadiness parameter $f_{0,1}$ on dimensionless velocity

Figure 13 shows the effects of the magnetic parameter on the temperature profiles. It can be seen that the dimensionless temperature decreases (thermodynamic temperature increase) with the increase in the magnetic parameter. It is important to mention that the obtained temperature function is not completely universal, since it depends on Prandtl number Pr.

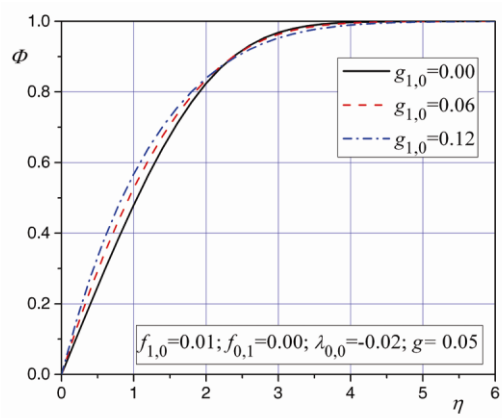

Figure 11. Effect of magnetic parameter $g_{1,0}$ on dimensionless velocity

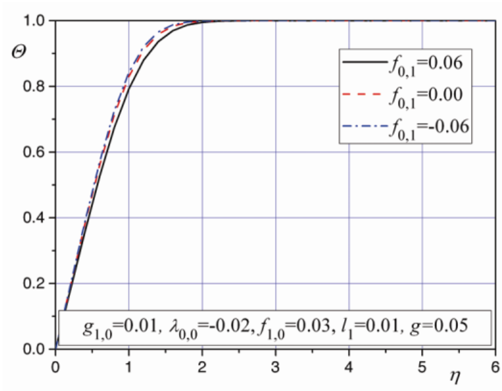

Figure 12. Effect of unsteadiness parameter $f_{0,1}$ on dimensionless temperature

The effects of the suction or the blowing parameter $\lambda_{0,0}$ on the dimensionless velocity and temperature in the MHD boundary layer are presented in figs. 14 and 15. It can be seen that blowing decreases the velocity in the boundary layer, while suction increases the velocity and decreases the boundary layer thickness. As expected, the suction tends to delay the boundary layer separation and has positive effects on the flow. All these results act in accordance with physical situations. The temperature distribution as a function of dimensionless transversal coordinate η for different values of suction (blowing) parameter $\lambda_{0,0}$ is shown in fig. 15. The middle line presents the case of the absence of suction or blowing. From this figure, it can be seen that the dimensionless temperature decreases (thermodynamic temperature increase) with the increase in the blowing parameter.

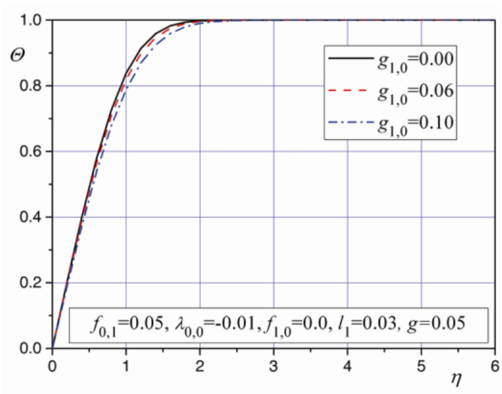

Figure 13. Effect of magnetic parameter $g_{1,0}$ on dimensionless temperature

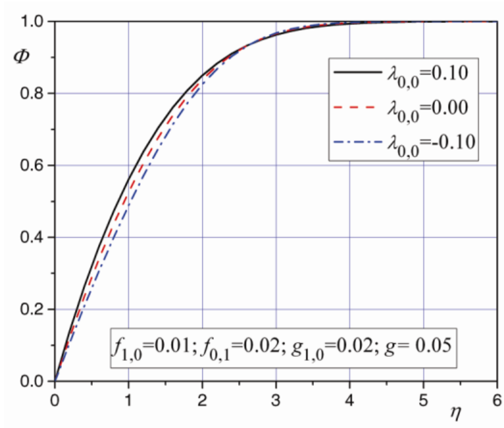

Figure 14. Effect of suction (blowing) parameter $\lambda_{0,0}$ on dimensionless velocity

Figure 16 shows the effects of the temperature parameter on the dimensionless temperature profiles. From this figure, it can be seen that the dimensionless temperature decreases (thermodynamic temperature increase) for the case of decreasing temperature along the body and vice versa. It is important to mention that the obtained results are given for the case of the absence of magnetic field, blowing, accelerated outer flow, and in the vicinity of the stagnation point.

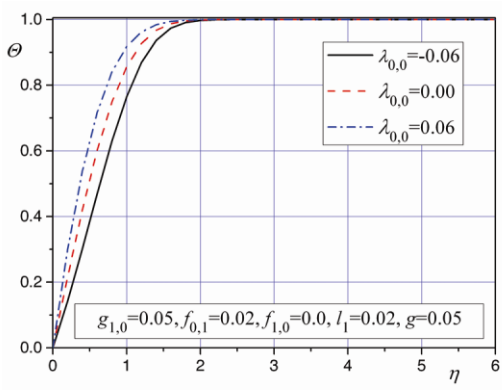

Figure 15. Effect of suction (blowing) parameter $\lambda_{0,0}$ on dimensionless temperature

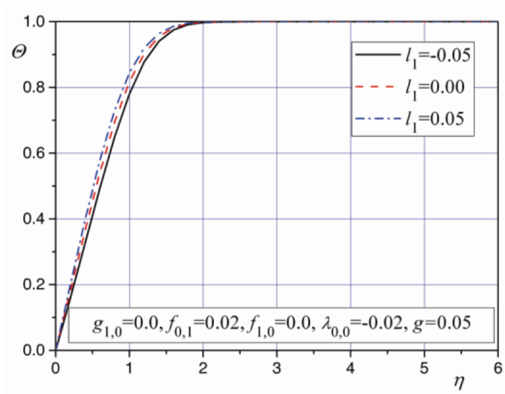

Figure 16. Effect of temperature parameter l_1 on dimensionless temperature

These results show the benefits of the presented general similarity method over other methods because it is possible to analyze the general influence of individual flow parameters without going into particular problems.

The effect of temperature parameter l_1 on dimensionless temperature gradient ξ_t in function of dynamic parameter $f_{1,0}$ is shown in fig. 17. It is obvious that for the case of temperature decreasing along the body dimensionless heat transfer also decreases, while positive values of parameter l_1 have the opposite effect. It is interesting to note that the dimensionless temperature gradient increases in the vicinity of the stagnation and separation point.

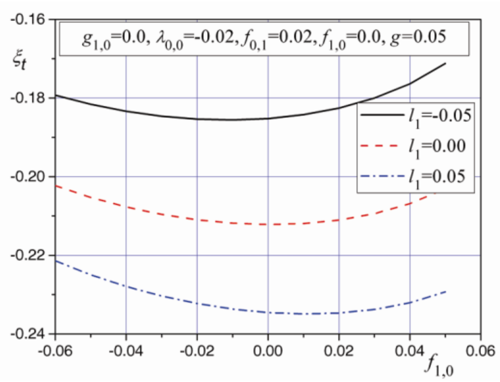

Figure 17. Effect of temperature parameter l_1 on dimensionless temperature gradient ξ_t

Conclusions

The generalized similarity solution to the problem of the temperature 2-D MHD boundary layer flow on the porous body was presented in this paper to exhibit the combined effects of the dynamic, magnetic, temperature, and suction (blowing) parameters. This problem can be analyzed for every particular case *i. e*. for a given function of free stream velocity and body temperature. Here, quite a different approach is employed in order to use the benefits of a multi-parametric method, and universal equations of observed problem are derived. These equations are solved numerically, and some of the approximation and integration results are given in the form of diagrams and conclusions. The obtained results can be used in drawing general conclusions about the boundary-layer development and in calculation of particular problems as shown in the paper.

Acknowledgments

This paper is supported by the Serbian Ministry of Education, Science and Technological Development (Project No. TR 35016; Research of MHD flow in the channels, around the bodies and application in the development of the MHD pump).

Nomenclature

B – magnetic induction, [T]
c_p – specific heat capacity, [$Jkg^{-1}K^{-1}$]
D – standardization constant [–]
Ec – Eckert number [–]
F – characteristic function, [–]
$f_{k,n}$ – dynamical parameters, [–]
g – characteristic function z derivative, [–]
$g_{k,n}$ – magnetic parameters, [–]
H – characteristic function, [–]
h – characteristic linear scale of transversal co-ordinate, [m]
H^* – characteristic function, [–]
H^{**} – characteristic function, [–]
$l_{k,n}$ – temperature parameters, [–]
N – characteristic function, [–]
Pr – Prandtl number, [–]
q – temperature difference between body surface and outer flow, [K]
T – thermodynamic temperature, [K]
t – time, [s]
U – free stream velocity, [ms^{-1}]
u – longitudinal velocity [ms^{-1}]
v – transversal velocity [ms^{-1}]
x – longitudinal coordinate, [m]
y – transversal coordinate, [m]
z – characteristic function, [s]

Greek symbols

δ^* – extrusion thickness, [m]
δ^{**} – thickness of impulse loss, [m]
Φ – dimensionless stream function, [–]
η – dimensionless transversal coordinate,[–]
λ – thermal conductivity, [$Wm^{-1}K^{-1}$]
μ – viscosity, [Nsm^{-2}]
ν – kinematic viscosity, [m^2s^{-1}]
Θ – dimensionless temperature, [–]
ρ – fluid density, [kgm^{-3}]
σ – conductivity, [$A^2s^3kgm^{-2}$]
τ – shear stress [Nm^{-2}]
Ψ – stream function, [m^2s^{-1}]
ξ – characteristic function, [–]

Subscripts

0 – initial time moment
1 – boundary layer cross section
w – body surface

References

[1] Schlichting, H., *Boundary Layer Theory*, McGraw-Hill, New York, USA, 1979
[2] Mikhailov, A. Yu., Heat and Mass Transfer in a Magnetic Field, *Magnetohydrodynamics*, *5* (1966), 1, pp. 3-10
[3] Molokov, S. *et al*., *Magnetohydrodynamics-Historical Evolution and Trends*, Springer, Berlin, Germany, 2007
[4] Lu, H. Y., Lee, C. H., Simulation of Three-Dimensional Nonideal MHD Flow at Low Magnetic Reynolds Number, *Science China Series E-Technological Sciences*, *52* (2009), 12, pp. 3690-3697
[5] Poggie, J., Gaitonde, D., Magnetic Control of Flow past a Blunt Body: Numerical Validation and Exploration, *Physics of Fluids*, *14* (2002), 5, pp. 1720-1731
[6] Shit, G. C., Haldar, R., Thermal Radiation and Hall Effect on MHD Flow, Heat and Mass Transfer over an Inclined Permeable Stretching Sheet, *Thermal Science*, *15* (2011), Suppl. 2, pp. S195-S204
[7] Ishak, A. *et al*., MHD Boundary-Layer Flow of a Micropolar Fluid Past a Wedge with Constant Wall Heat Flux, *Communications in Nonlinear Science and Numerical Simulation*, *14* (2009), 1, pp. 109–118
[8] Abbas, I. A, Palani, G., Effects of Magnetohydrodynamic Flow past a Vertical Plate with Variable Surface Temperature, *Applied Mathematics and Mechanics (English edition), 31* (2010), 3, pp. 329-338
[9] Kulikovskii, A. G., Lyubimov, G. A., *Magnitnaya gidrodinamika*, Fismatgiz, Moscow, 1962
[10] Loicijanski, L. G., Universal Equations and Parametric Approximations in Theory of Laminar Boundary-Layer, *AN SSSR Applied mathematics and mechanics*, *29* (1965), 1, pp. 70-78
[11] Saljnikov, V. N., A Contribution to Universal Solutions of the Boundary-Layer Theory, *Theoretical and Applied Mechanics*, *4* (1978), pp. 139-163
[12] Busmarin, O. N., Saraev, V. Yu., Parametric Method in Theory of Unsteady Boundary-Layer, *Journal of Engineering Physics and Thermophysics*, *27* (1974), 1, pp. 871-876
[13] Boricic, Z. *et al*., Universal Solutions of Unsteady Two-Dimensional MHD Boundary Layer on the Body with Temperature Gradient along Surface, *WSEAS Transactions on Fluid Mechanics, 4* (2009), 3, pp. 97-106
[14] Obrovic, B. *et al*., Boundary-Layer of Dissociated Gas on Bodies of Revolution of a Porous Contour, *Strojniski vestnik – Journal of Mechanical Engineering*, *55* (2009), 4, pp. 244–253
[15] Nikodijevic, D. *et al*., Parametric Method for Unsteady Two-Dimensional MHD Boundary-Layer on a Body for which Temperature Varies with Time, *Archives of Mechanics*, *63* (2011), 1, pp. 57–76

Paper submitted: April 27, 2012
Paper revised: May 5, 2012
Paper accepted: June 6, 2012