INVESTIGATIONS ON THE INTERNAL SHAPE OF CONSTRUCTAL CAVITIES INTRUDING A HEAT GENERATING BODY

by

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This paper deals with the influence that the internal shape of open cavities exerts on the Constructal design of a heat generating body. Several shapes of cavity are studied: triangular, elliptical, trapezoidal and Y-shaped cavities intruding into a trapezoidal shaped solid with uniform heat generation. The trapezoidal solid is commonly used in round electronic devices. The geometric aspect ratios of the cavities and the solid are free to vary while the total volume occupied by the solid and the cavity are fixed. The objective is minimizing the peak (hot spot) temperature with respect to the geometrical parameters of the system. Finite element method is employed to calculate the peak temperature of the solid. With respect to the Constructal thermal design, the numerical results prove that, utilizing the triangular and Y-shaped cavities can result more reliable and effective rather than other studied cavities.

Key words: *internal shape, cavity, Constructal design, heat generating body, electronic cooling*

Introduction

The natural tendency of flow with the best possible configuration means how to optimally distribute imperfection throughout the flow, so that it moves best with the minimal loss and minimal dissipation, in other words, it moves within the least expensive route. This is the essential concept of constructal theory that states that "for a finite-size system to persist in time, it must evolve in such a way that provides easier access to the imposed currents flowing through it" [1]. Using Constructal theory, this natural behavior can be conceived as a phenomenon of configuration generation. Constructal theory has been proposed, because of its applicability to natural and engineering flow systems, and a lot of work has been done based on this theory.

In fluid mechanics, tree-shaped networks have been proposed to obtain the minimum pressure drop in a flow system [1, 2]. Tree shaped conductive networks with variable cross-section were studied by Zhou *et al.* [3]. Constructal theory was also used in thermodynamics; it posed novel approaches to improve thermodynamically the performance of a flowing system. For example, employing Constructal theory, several classes of simple flow systems consisting

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of T- and Y-shaped assemblies of ducts, channels and streams in a laminar and fully developed flow regime were studied. They demonstrated how the minimum entropy generation can be obtained [4].

Regarding to really effective and simple principles of this theory, it soon found its place. For example, during the last decades, the Constructal design of the fins was studied by several authors under different circumstances. Alebrahim and Bejan [5] explored the optimization of circular fins that enhanced both conductive and convective heat transfer. Almogbel and Bejan [6] did a research in maximizing the overall conductance of cylindrical and T-shaped assemblies of pin fins subjected to fixed total volume and the amount of fin material as well. Constructal design of fins was discussed further under different constraints, different mechanisms of heat transfer and different applications such as heat exchangers led up to valuable results [7-16]. Lorenzini *et al*. [16], for example, carried out a numerical solution to seek for the best geometry of a complex assembly of fins, *i. e*., an assembly where there is a cavity between the two branches of the T-Y-assembly of fins and two additional extended surfaces.

Another issue that attracted the attention of researchers was Constructal design of the open cavities which is very similar to the design of the fins. Open cavities are defined as the regions formed between adjacent fins and stand for the essential promoters of nucleate boiling or condensation: see, for example, the Vapotron effect [17] that occurs as a consequence of the thermal interaction between a non-isothermal finned surface and a fluid locally subjected to a transient change of phase. Open cavities are also important morphological features in physiology [18]. In the category of thermal design, Rocha *et al.* [19] proposed Constructal design of a rectangular cavity intruding into a conducting trapezoidal solid with uniform internal heat generation for the first time. They optimized the trapezoidal body and the rectangular cavity geometric aspect ratios in order to achieve the maximum thermal performance namely the minimum overall thermal resistance between the volume of the entire system (cavity and solid) and the surroundings. Later, Rocha *et al*. [20] studied four shapes of cavity: rectangular, elliptical, triangular, and a T-shaped cavity penetrated into a rectangular, solid, conducting wall with uniform internal heat generation. They demonstrated that the rectangular cavity performs better than the elliptical and triangular ones. C-shaped and H-shaped cavities were proposed by Biserni *et al*. [21]. They proved that the performance of the H-shaped cavity is better than C-shaped, T-shaped, and rectangular cavities. Lorenzini *et al*. [22, 23] proved that the performance of optimized geometry of a Y-shaped cavity embedded into a solid conducting wall is superior to that of other basic geometries such as T-shaped and C-shaped cavities. Lorenzini and Rocha [24] focused on a T-Y-shaped cavity in a rectangular wall with uniform heat generation on the solid wall. When compared with the C-shaped cavity under the same thermal conditions and the same area of the cavity, the T-Y-shaped was found superior. Later, Lorenzini *et al.* [25] considered the geometrical optimization of a complex cavity, namely a T-Y-shaped cavity with two additional lateral intrusions into a solid conducting wall and demonstrated that the new complex cavity is superior to the basic T-Y- shaped cavity. To conduct an extended investigation, Rocha *et al.* [26] considered the case where heat transfer on the internal surface of a C-shaped cavity was accounted for by a constant heat transfer coefficient unlike [19-25] that assumed isothermal cavities. Xie *et al.* [27] determined the optimal aspect ratios of a T-shaped cavity intruded into a trapezoidal solid wall with uniform heat generation by applying Constructal theory. Recently, Lorenzini *et al*. [28] have applied Constructal design to cavities inserted into a cylindrical solid body.

Although several works have been devoted to design the optimal shapes and structures of cavities intruding to the solid heat generating bodies, [19-28], a few works [19, 27] have so far focused on the trapezoidal solids. Since trapezoidal heat generating solids can be assembled into "round" constructs such as hexagons, with which one can cover an entire 2-D domain [19], it is noteworthy to study the design of cavities with different shapes intruding to a trapezoidal heat generating solid to fill the present gap in the field.

In this paper, several shapes of cavity are studied: triangular, elliptical, trapezoidal, and Y-shaped cavities intruding to a trapezoidal heat generating solid. Optimization procedure consists of two steps. In the first step, optimal geometric aspect ratios of the trapezoidal solid and the cavities with a given shape (triangular, elliptical, trapezoidal or Y-shaped) are determined. The optimization objective is to minimize the overall thermal resistance between the volume of the entire system (cavity and solid) and the surroundings. In the next step, the optimized cavities for the three studied shapes are compared with each other so that the superior shape can be revealed. The described optimization procedure is carried out by using a finite elements approach of MATLAB PDE Toolbox [29] to numerically calculate the temperature fields in the fin.

Problem definition and mathematical formulation

Consider two-dimensional conducting body intruded by cavities of several types as sketched in fig. 1. The total volume occupied by the entire body (cavity and solid) is fixed:

$$V=(H_{\mathrm{e}}+H)\frac{LW}{2}=\text{const.} \tag{1}$$

where H and H_{e} are two heights of the trapezoidal conducting solid, L is the length of the body, and W – the thickness of the body, which is perpendicular to the plane of fig. 1. The external dimensions (H, H_{e}, L) of solid wall and the dimensions of cavities ($H_0, H_{\mathrm{e}0}, L_0, L_{\mathrm{e}0}$) are free to vary. The inclined edges of the trapezoidal and Y-shaped cavities are assumed parallel to those of the solid body. For the sake of simplicity, the changes of all parameters along the W dimension are assumed negligible. Thus, the area can also be considered fixed. The cavities volume is fixed as well. The described area/volume constraints are expressed by the relations:

$$\phi=\frac{V_0}{V}=\frac{H_0L_0}{(H_{\mathrm{e}}+H)L}=\text{const.}\ \text{(triangular cavity)} \tag{2}$$

$$\phi=\frac{V_0}{V}=\frac{\pi H_0L_0}{2(H_{\mathrm{e}}+H)L}=\text{const.}\ \text{(elliptical cavity)} \tag{3}$$

$$\phi=\frac{V_0}{V}=\frac{(H_{\mathrm{e}0}+H_0)L_0}{(H_{\mathrm{e}}+H)L}=\text{const.}\ \text{(trapezoidal cavity)} \tag{4}$$

$$\phi=\frac{V_0}{V}=\frac{2(H_0L_0+2H_{\mathrm{e}0}L_{\mathrm{e}0})}{(H_{\mathrm{e}}+H)L}=\text{const.}\ \text{(Y-shaped cavity)} \tag{5}$$

where ϕ represents the volume fraction occupied by the cavity. The solid is assumed isotropic with constant thermal conductivity k, and generates heat uniformly at the volumetric rate q''' [Wm^{-3}]. The outer surfaces of the heat generating body are perfectly insulated. The generated heat flow per unit length ($q'''A$.) is removed by cooling the wall of the cavity. For the sake of simplicity and clarity, the heat transfer coefficient on the cavity wall is assumed sufficiently large such that the convective resistance can be ignored in comparison to the solid conduction resistance and hence, the cavity wall temperature can be assumed fixed at the lower level of the

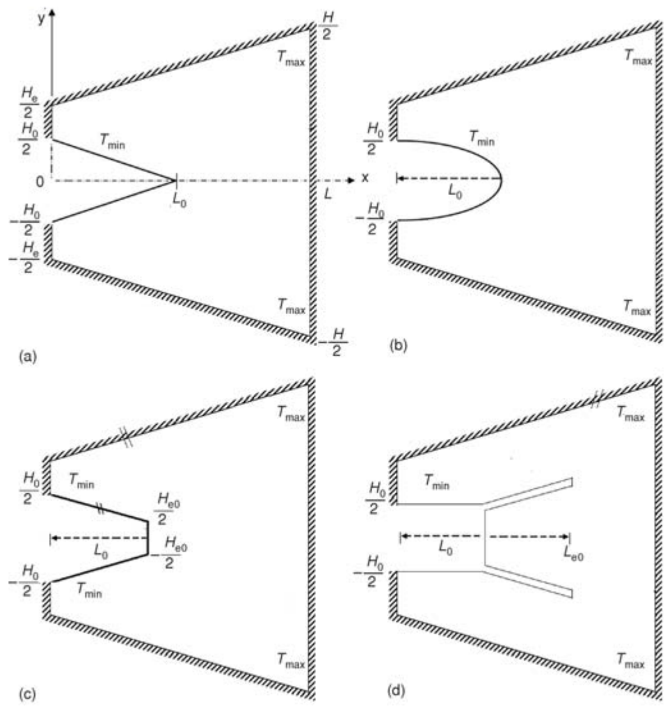

Figure 1. Isothermal (a) triangular, (b) elliptical, (c) trapezoidal, and (d) Y-shaped cavity intruding into a 2-D heat generating solid

surroundings, T_{min}. This hypothesis can be associated with a large class of examples where high density of heat transfer is required, such as in the cooling packages of small-scale electronics when phase change material is used as cooling fluid [19]. Due to the solid conduction resistance, temperature level in the solid rise to levels higher than T_{min} such that the highest temperatures (the hot spots) are registered at points on the insulated perimeter, for example, in the two corners labeled by T_{max} in fig. 1. The hot spot's temperature of the solid may exceed the allowable temperature level. Knowing that, the performance of equipment has a direct relationship with its temperature, it is important to keep it at an acceptable temperature level. This is synonymous to make T_{max} a constraint. Therefore, the design objective is represented by the maximization of the global thermal conductance, $q'''A\,(T_{max} - T_{min})$, or by the minimization of the global thermal resistance $(T_{max} - T_{min})/(q'''A)$. The numerical optimization of geometric parameters of the cavity and the solid consist of determining the temperature field in a large number of configurations (with different geometric parameters), calculating the global thermal resistance for each configuration, and selecting the configuration with the smallest global resistance. Symmetry allows us to perform calculations in only half of the domain, $y \geq 0$. In order to perform the described procedures, it is necessary to first formulate the physical problem.

The 2-D conduction equation for the domain occupied by the heat generating solid under steady-state condition is:

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} + 1 = 0 \tag{6}$$

where the dimensionless variables and parameters are defined as:

$$\tilde{T} = \frac{T - T_{min}}{\dfrac{q'''A}{k}} \tag{7}$$

and

$$(\tilde{x}, \tilde{y}, \tilde{H}, \tilde{L}, \tilde{H}_e, \tilde{H}_0, \tilde{L}_0, \tilde{H}_{e0}, \tilde{L}_{e0}) = \frac{(x, y, H, L, H_e, H_0, L_0, H_{e0}, L_{e0})}{\sqrt{A}} \tag{8}$$

The optimization objectives can be expressed in the dimensionless form as the minimization of:

$$\tilde{T}_{\max} = \frac{T_{\max} - T_{\min}}{\dfrac{q'''A}{k}} \tag{9}$$

Solution

The governing partial differential eq. (5), is solved numerically using the finite elements analysis of partial-differential-equations (PDE) toolbox of embedded in MATLAB [29], which converts eq. (5) into the system of discrete algebraic equations based on triangular elements. The element layout is non-uniform in both $\tilde{x}$ and $\tilde{y}$ co-ordinates, and is varied from one geometry to the next. As is customarily done, the appropriate mesh size is determined by means of successive refinements, increasing the number of elements between consecutive mesh sizes, until the strongest convergence criterion, $|(\tilde{T}_{\max}^{j} - \tilde{T}_{\max}^{j+1})/\tilde{T}_{\max}^{j}| < 10^{-4}$ is satisfied. Here, $\tilde{T}_{\max}^{j}$ stands for the hot spot temperature calculated using the current mesh size, and $\tilde{T}_{\max}^{j+1}$ corresponds to the hot spot temperature calculated using the next mesh size where the number of elements is increased by a fourfold factor. The optimization results are calculated by using a range between 10,000 and 45,000 triangular elements. For the verification of the present numerical work, the numerical results obtained using our code in MATLAB [29] PDE are compared with those obtained by FIDAP package [30], in tab. 1 for the case of $H/L = 0.6$, $H_0/L_0 = 1$, and several values of H_e/H.

Table 1. Comparison the numerical results obtained using our code in MATLAB PDE with those obtained by FIDAP package for an elliptical cavity ($H/L = 0.6$, $H_0/L_0 = 1$)

H_e/H	$\theta_{\max}$(FIDAP)	$\theta_{\max}$(MATLAB-PDE) Four refinements	$\|(\theta_{\max}^{FIDAP} - \theta_{\max}^{MATLAB})/\theta_{\max}^{FIDAP}\|$
0.5	0.6080	0.608026	4.2763e-005
1	0.4905	0.490525	5.0966e-005
4	0.3495	0.349521	6.0086e-005

Results and discussion

The effect of the geometric aspect ratios of a trapezoidal heat generating solid, H/L and H_e/H on the hot spot temperature, is depicted in figs. 2, 3, 4, and 5 with reference to the case of: triangular, elliptical, trapezoidal, and Y-shaped cavities, respectively, when the aspect ratio of the four cavities are fixed, $H_0/L_0 = 1$. In the four cases, it can be observed that there is an optimal H/L, which minimizes the hot spot temperature, $\tilde{T}_{\max}$ at several values of H_e/H. Figures 2, 3, 4, and 5 also show that there is an alternative opportunity of optimization with respect to H_e/H. Therefore, the results shown in figs. 2, 3, 4, and 5 are summarized in figs. 6, 7, 8, and 9, respectively, to address the optimal values of H/L namely $(H/L)_{opt}$, and the minimized maximum temperature, $\tilde{T}_{\max,\min}$ for a wide range of H_e/H.

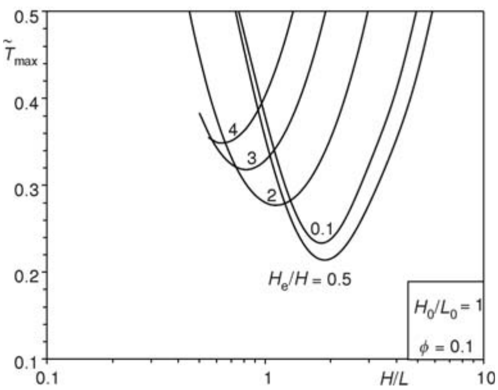

Figure 2. The minimization of the hot spot temperature with respect to *H/L* for several values of H_e/H when the internal shape of the cavity, H_0/L_0, is fixed and the cavity is triangular

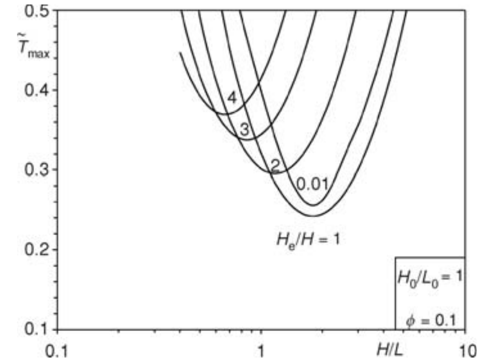

Figure 3. The minimization of the hot spot temperature with respect to *H/L* for several values of H_e/H when the internal shape of the cavity, H_0/L_0, is fixed and the cavity is elliptical

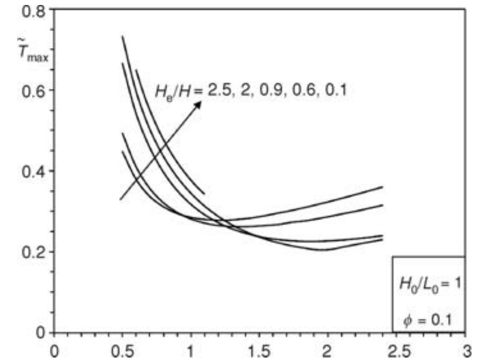

Figure 4. The minimization of the hot spot temperature with respect to *H/L* for several values of H_e/H when the internal shape of the cavity, H_0/L_0, is fixed and the cavity is trapezoidal

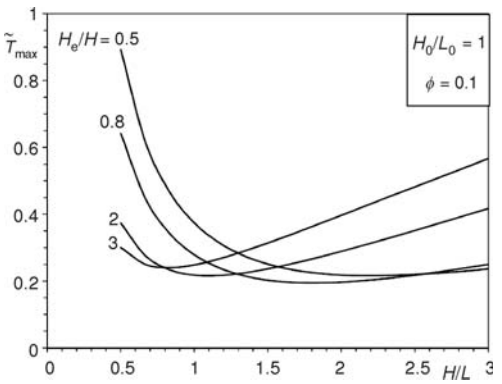

Figure 5. The minimization of the hot spot temperature with respect to *H/L* for several values of H_e/H when the internal shape of the cavity, H_0/L_0, is fixed for Y-shaped cavity with $L_{e0}/L_0 = 0.5$ and $H_{e0}/H_e = 0.2$

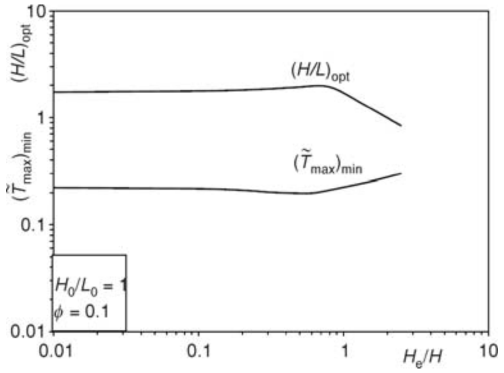

Figure 6. The optimized values of *H/L* and the minimized hot spot temperature for a wide range of H_e/H when the internal shape of the cavity, H_0/L_0, is fixed and the cavity is triangular

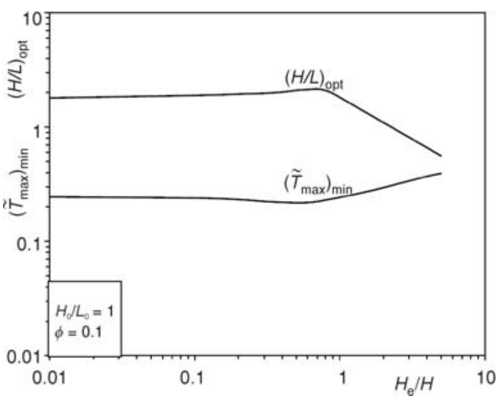

Figure 7. The optimized values of *H/L* and the minimized hot spot temperature for a wide range of H_e/H when the internal shape of the cavity, H_0/L_0, is fixed and the cavity is elliptical

The possibility of a double-minimization is observed in figs. 6, 7, 8, and 9. To clarify the preceding observation and to account for the effect of the internal aspect ratio of the cavities, H_0/L_0, a second level of numerical optimization is conducted. The scheme consists of the repetition of the preceding optimization procedure for a wide range of the internal aspect ratio of a trian-

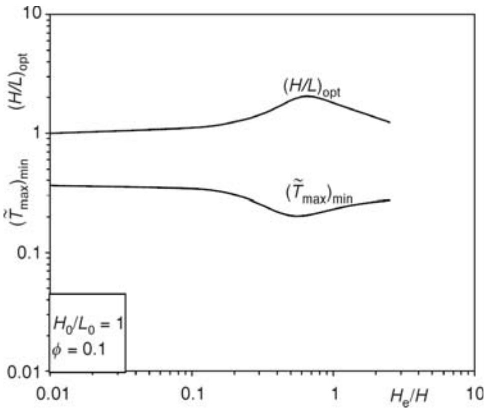

Figure 8. The optimized values of *H/L* and the minimized hot spot temperature for a wide range of H_e/H when the internal shape of the cavity, H_0/L_0, is fixed and the cavity is trapezoidal

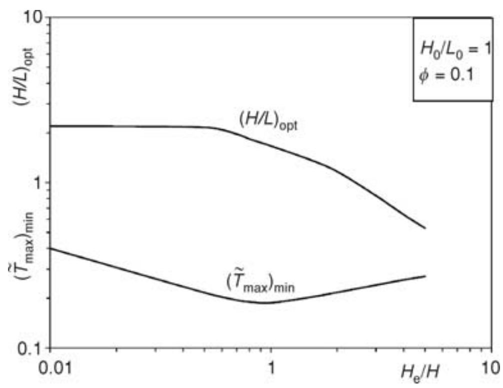

Figure 9. The optimized values of *H/L* and the minimized hot spot temperature for a wide range of H_e/H when the internal shape of the cavity, H_0/L_0, is fixed for Y-shaped cavity with $L_{e0}/L_0 = 0.5$ and $H_{e0}/H_e = 0.2$

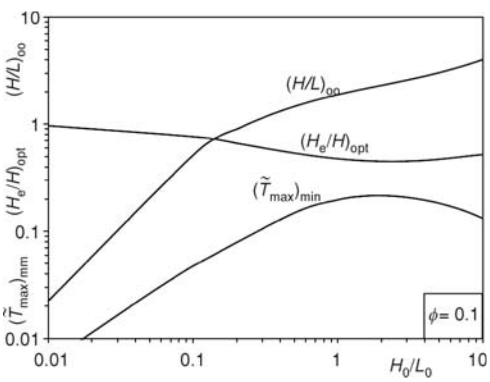

Figure 10. The double-optimized geometric aspect ratios of the trapezoidal solid and the double-minimized hot spot temperature of the aspect ratio of the cavity, H_0/L_0, vary and the cavity is triangular

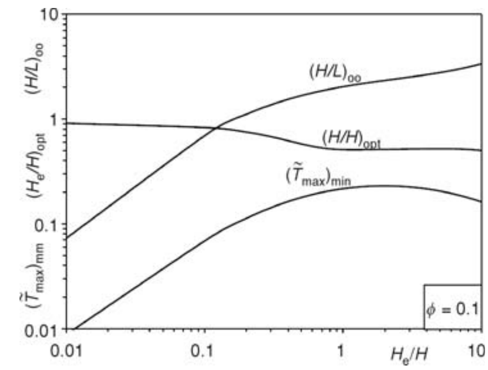

Figure 11. The double-optimized geometric aspect ratios of the trapezoidal solid and the double-minimized hot spot temperature of the aspect ratio of the cavity, H_0/L_0, vary and the cavity is elliptical

gular, elliptical, trapezoidal, and Y-shaped cavities, H_0/L_0, in figs. 10, 11, 12, and 13, respectively. Hence, the double optimized values of *H/L* and H_e/H, namely $(H/L)_{oo}$ and $(H_e/H)_{opt}$, plus the double minimized hot spot temperatures, are highlighted in figs. 10, 11, 12, and 13. It is concluded from figs. 10, 11, 12, and 13 that the thermal performance improves as the cavity shape becomes slender ($H_0 \ll L_0$). Comparison among the triangular, elliptical, trapezoidal, and Y-shaped cavities intruding into the trapezoidal solid is presented in fig. 14, where the double-minimized temperature is plotted as a function of the cavity's aspect ratio, H_0/L_0. It is observed that the superior shape (with lower peak temperature) depends on H_0/L_0. When $H_0/L_0 < 5$, trapezoidal cavity is superior to elliptical one, however when $H_0/L_0 > 5$, thermal performances of both cavities are the same. This figure also shows that trapezoidal and triangular cavities be-

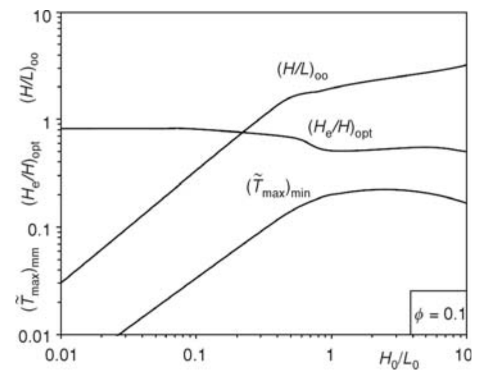

Figure 12. The double-optimized geometric aspect ratios of the trapezoidal solid and the double-minimized hot spot temperature of the aspect ratio of the cavity, H_0/L_0, vary and the cavity is trapezoidal

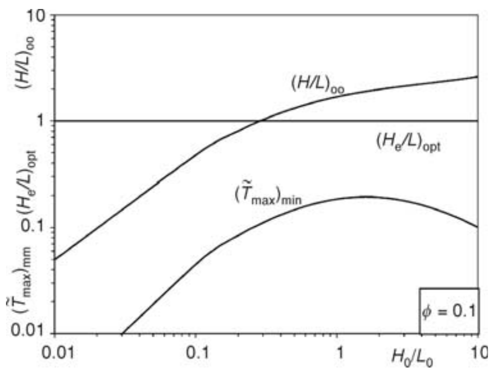

Figure 13. The double-optimized geometric aspect ratios of the trapezoidal solid and the double-minimized hot spot temperature of the aspect ratio of the cavity, H_0/L_0, vary for Y-shaped cavity with $L_{e0}/L_0 = 0.5$, $H_{e0}/H_e = 0.2$

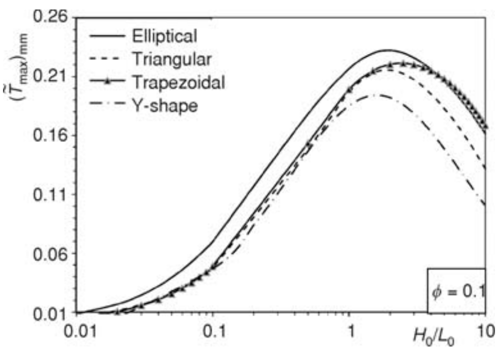

Figure 14. Comparison among the, elliptical, triangular, trapezoidal, and Y-shaped cavities intruding into the trapezoidal solid

have in the same way for $H_0/L_0 < 2$, however, the triangular cavity becomes superior to the trapezoidal one, when $H_0/L_0 > 2$. Trapezoidal, triangular and Y-shaped cavities have the same thermal performance, when $H_0/L_0 < 0.7$, while for $H_0/L_0 > 0.7$, the superiority of Y-shaped cavity over the other ones is evident.

Summary and conclusions

In the present study, Constructal theory is applied to optimize the configuration of a trapezoidal solid conducting wall intruded by an isothermal cavity. Four shapes of the cavity: triangular, elliptical, trapezoidal, and Y-shaped intruding to a trapezoidal heat generating solid are studied. Trapezoidal solid wall may be assembled into "round" constructs such as hexagons, with which one can cover an entire 2-D. In the most fundamental sense, the maximum dimensionless excess temperature is minimized with respect to three degrees of freedom, two aspect ratios of the trapezoidal solid and aspect ratio of the cavity, under the constraints that the volumes of the total body and the cavity are fixed. In the optimization process, the finite element method is employed and the degrees of freedom are relaxed one by one. The numerical results prove that there are optimal geometric aspect ratios for the external shape of the trapezoidal solid and the internal shape of the cavity that maximize the thermal performance or minimize the hot spot temperature. For the four cases, where the cavities are considered, triangular, elliptical, trapezoidal or Y-shaped, it was realized that the thermal performance improves as the cavity shape becomes slender. This conclusion can result strategic in geometric design of the cavities. Comparison among the triangular, elliptical, trapezoidal, and Y-shaped cavities intruding to the trapezoidal solid is presented. It is observed that the superior shape (with lower peak temperature) depends on H_0/L_0. For example, when H_0/L_0 is less than 0.7, trapezoidal, triangular, and Y-shaped cavities have the same thermal performance, while for $H_0/L_0 > 0.7$, the Y-shaped cavity becomes superior to the other ones.

In sum, a significant conclusion that may be drawn from this research is that: the triangular and Y-shaped cavities are reliable options among the different shapes studied in this work. There are still some considerations herein which need to be investigated more deeply in the future. For example, heat transfer on the isothermal surface of the cavity can accounted for by a constant heat transfer coefficient.

Acknowledgments

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Nomenclature

A – total area occupied by the body (solid and the cavity), [m^2]
A_0 – total area occupied by the cavity, [m^2]
H – height of the trapezoidal solid (right), [m]
H_e – height of the trapezoidal solid (left), [m]
H_0 – height of the cavity, [m]
k – conductivity of the fin, [Wm^{-1}K^{-1}]
L – length of the solid, [m]
L_0 – length of the cavity, [m]
q''' – heat generation rate per unit volume, [Wm^{-3}]
T – local temperature, [K]
T_{max} – hot spot temperature, [K]
T_{min} – temperature of the surroundings, [K]
V – total volume occupied by the body (solid and the cavity), [m^2]
V_0 – total volume occupied by the cavity, [m^2]
W – the third dimension width perpendicular to the paper, [m]
x, y – Cartesian co-ordinates defined in fig. 1, [m]

Greeks symbols

ϕ – the volume fraction occupied by cavity

Supercripts

~ – dimensionless variables

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